Scattering, Extinction, and Albedo of Impedance-Boundary Objects

Ari Sihvola, Beibei Kong, Pasi Ylä-Oijala, Dimitrios C. Tzarouchis, and Henrik Wallén

Abstract – This article presents results on the scattering and absorption behavior of lossy impedance-boundary spheres and cubes. The albedo (the ratio between the scattering and extinction cross sections) of these scatterers turns out to be a very insightful quantity in this respect. The strongly nonlinear dependence of albedo on the size of the objects is illustrated and compared with the albedo of dissipative penetrable spheres. The shape of the object seems to have surprisingly little effect on the albedo. Furthermore, the effect of losses on the resonance behavior of small impedance-boundary spheres is analyzed, leading to the observation that the dipolar mode vanishes from the albedo spectrum, while higher order multipoles remain visible. This resembles a similar phenomenon earlier shown to appear in connection with small plasmonic scatterers.

1. Introduction

Material particles and objects form inhomogeneities that disturb the propagation of electromagnetic waves. Wave energy is scattered and possibly absorbed. In this article, the focus is on scattering by objects that are characterized by the boundary condition on the surface. In the electromagnetic literature, a wide variety of boundary conditions has been studied, both theoretically [1] and as useful approximations [2, 3]. In the following, we will focus on scattering objects with the so-called *impedance boundary condition* (IBC).

The IBC is between the tangential components of electric (**E**) and magnetic (**H**) field vectors at a boundary. A relation between two two-dimensional vectors is, in general, a dyadic; hence, the IBC has to be described by four scalars. Formally, the impedance condition is written

$$\mathbf{E}_t = \overline{\overline{Z}}_s \cdot (\mathbf{n} \times \mathbf{H}_t) \tag{1}$$

where the tangential electric and magnetic fields at the boundary with the normal unit vector **n** are

$$\mathbf{E}_t = -\mathbf{n} \times (\mathbf{n} \times \mathbf{E}), \quad \mathbf{H}_t = -\mathbf{n} \times (\mathbf{n} \times \mathbf{H}) \tag{2}$$

Here, the surface impedance dyadic $\overline{\overline{Z}}_s$ is two dimensional in the plane perpendicular to **n** (in other words $\mathbf{n} \cdot \overline{\overline{Z}}_s = 0$ and $\overline{\overline{Z}}_s \cdot \mathbf{n} = 0$), which here operates on the electric and magnetic fields.

In the simplest case, the dyadic $\overline{\overline{Z}}_s$ is a multiple of the unit dyadic, whence the boundary condition simplifies to

$$\mathbf{E}_t = Z_s \, \mathbf{n} \times \mathbf{H}_t \tag{3}$$

This isotropic IBC is also known in the literature as the Leontovich boundary condition [4, 5].

This surface impedance model can be particularly interesting in many ways. Good conductors are often treated as isotropic IBC surfaces. Also, general IBC models have connections with metasurfaces, which are presently an active research interest [6]. The properties of metasurfaces are usually characterized via a sheet impedance, and this type of approach has been used to introduce concepts, such as mantle cloaking [7, 8].

Furthermore, IBC models can be used as equivalent descriptions for real phenomena, such as the plasmonic and dielectric resonances in a sphere [9]. In addition, more exotic cases of IBCs can be potentially used toward unexplored regions of electromagnetic wave manipulation. A fascinating example is the perfect electromagnetic conductor boundary and medium [10], with connections to axion electrodynamics [11] and topological insulators [12].

In our previous studies [13], we have analyzed interesting resonance spectra of spheres with a lossless impedance boundary. In this article, which is an expanded study of [14], we analyze the scattering by IBC objects whose (isotropic) surface impedance is dissipative.

2. Albedo

A dissipative scatterer perturbs and therefore attenuates a propagating wave in two ways, by scattering and by absorbing electromagnetic energy. For finite scatterers, measures for these two contributions are the scattering efficiency Q_{sca} and the absorption efficiency Q_{abs}, with the sum of the two being the extinction efficiency $Q_{\text{ext}} = Q_{\text{sca}} + Q_{\text{abs}}$ [15]. In the following analysis, a useful quantity is the albedo A of the scatterer. Albedo (whiteness) is the ratio between scattering and extinction cross sections (or efficiencies):

$$A = \frac{Q_{\text{sca}}}{Q_{\text{ext}}} \tag{4}$$

Albedo has a value between zero (for a totally absorbing body) and unity (for a completely scattering

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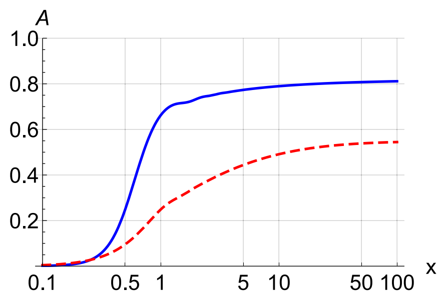

Figure 1. The albedo of a lossy IBC boundary sphere with $Z_s = 10\eta_0$ (solid blue) and $Z_s = \eta_0$ (dashed red) as a function of the electrical size parameter x.

object). In passing, note that with active scatterers, these limits for the albedo can be overcome.

3. Lossy Sphere

Let us start by the scattering and absorption characteristics of a lossy IBC sphere, defined by two parameters: its electrical size $x = 2\pi a/\lambda$ and the (possibly complex) surface impedance $Z_s = R_s + \mathrm{j}X_s$. The time harmonic notation of $(\mathrm{j}\omega t)$ is followed, the radius of the sphere is a, and λ is the wavelength.

Figure 1 displays how the albedo of a lossy IBC sphere varies when the size parameter changes. From a very low albedo, it increases with size, with a very nonlinear speed of increase. Furthermore, as expected, a sphere with surface resistance matching the free-space impedance ($Z_s = \eta_0$) has a lower albedo than one with impedance contrast. Figure 2 takes a closer look into the small size regime, also called the Rayleigh regime (in electromagnetic scattering, the Rayleigh regime is marked by the strong fourth-order dependence of the scattering on the relative size [16]). A somewhat counterintuitive observation is that for very small spheres, the albedo of a sphere with surface impedance $Z_s = \eta_0$ is higher than that of the impedance contrast sphere ($Z_s = 10\eta_0$), as shown in Figure 3. However, at the crossover point ($x \approx 0.25$), the albedos are already quite low ($A \approx 0.028$), and the inversion in the albedo behavior is hence more of an academic interest. For the increasing size of the sphere, the increase of the albedo starts to saturate, meaning that the mutual share of scattering and absorption remains unchanged.

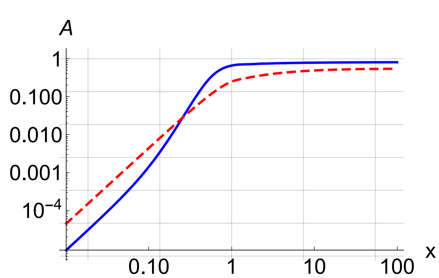

Figure 2. A logarithmic view of the albedo of IBC spheres $Z_s = 10\eta_0$ (solid blue) and $Z_s = \eta_0$ (dashed red), as a function of the electrical size parameter x.

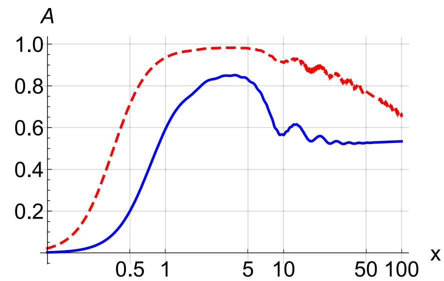

Figure 3. The albedo of a penetrable lossy sphere with relative permittivity $\varepsilon_r = 2 - \mathrm{j}0.1$ (solid blue) and $\varepsilon_r = 2 - \mathrm{j}0.01$ (dashed red) as a function of the electrical size parameter x.

It is of interest to compare this albedo behavior to that of a penetrable sphere. Figure 3 shows the albedo of a lossy dielectric sphere as its electrical size increases. Here, we can observe that although the Rayleigh regime looks similar to that of the IBC sphere, the increase of the albedo is not continuous: it is rather nonlinear. Note the maximum albedo value in the plot for the size parameter value around $x = 4$.

In the computations in Figures 1-3, the change in the size parameter x can represent the variation in absolute size, frequency, or a combination of them. However, the impedance Z_s and the permittivity ε are assumed constant throughout the range of x.

4. Effect of Scatterer Shape

How do these observations on the albedo behavior depend on the shape of the scatterer? For insight, we compute the albedo of a cube with a lossy isotropic surface impedance. The computations have been done by using the surface integral equation method on the basis of the electric field integral equation [17]. Figure 4 shows how the albedo increases as a function of the size parameter, for the two surface impedance values $Z_s = \eta_0$ and $10\eta_0$. Here, the size parameter x for the IBC cube is defined as that of the size parameter of an equivolume sphere.

It is rather surprising how similar the behaviors of the two albedo curves of the sphere and the cube are in Figure 4. One can, however, observe a slight oscillation in the albedo curve of the cube, while the corresponding behavior of the sphere shows a more smooth increase

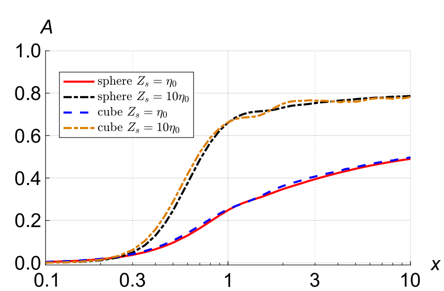

Figure 4. The albedo of a lossy IBC boundary cube with $Z_s = 10\eta_0$ (dashed orange) and $Z_s = \eta_0$ (dashed blue) as a function of the electrical size parameter x. The albedo of a lossy IBC sphere are marked with solid red line ($Z_s = \eta_0$) and dashed black line ($Z_s = 10\eta_0$).

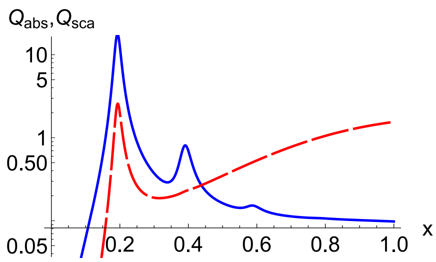

Figure 5. The absorption (solid blue) and scattering (dashed red) efficiencies of a small lossy IBC sphere as a function of the electrical size parameter x. The surface impedance is $Z_s = (0.01 - \mathrm{j}0.2)\eta_0$.

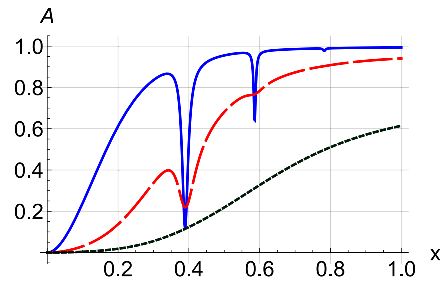

Figure 6. The albedo of a small lossy IBC sphere as a function of the electrical size parameter x. The surface impedance is $Z_s/\eta_0 = 0.001 - \mathrm{j}0.2$ (solid blue), $0.01 - \mathrm{j}0.2$ (dashed red), and $0.1 - \mathrm{j}0.2$ (dotted black).

with x. Another result from our numerical study of IBC cubes is that for a given size and surface impedance, the albedo of the cube has a very weak dependence on the incidence angle or the polarization of the incident wave vector.

5. Multipolar Peculiarities

The scatterers in the previous section were very dissipative. Consider next low-loss IBC scatterers. The fascinating message of our previous studies [13] was that small lossless IBC scatterers can support strong multipolar resonances analogously with plasmonic nanoparticles. Does this phenomenon leave its marks on the albedo? To answer this, we have computed the response of low-loss IBC spheres with varying parameters. Figure 5 shows the scattering and absorption efficiencies of spheres with a size parameter less than one and with capacitive but lossy surface impedance $Z_s = (0.01 - \mathrm{j}0.2)\eta_0$. A fairly strong resonance can be seen for size parameter values $x \approx 0.2$ and weaker ones for $x \approx 0.4$ and 0.6.

However, the resonances take a different profile in the scattering and absorption curves in Figure 5. This fact becomes very clearly illustrated when the albedo is plotted as a function of the size parameter, as in Figure 6. For the low-loss sphere (solid blue curve), the albedo curve is practically smooth through the dipole ($x \approx 0.2$), but the quadrupole ($x \approx 0.4$) has a strong dip, as does the octopole ($x \approx 0.6$), and even at a higher multipole ($x \approx 0.8$), a dip can be distinguished. Hence, at these higher order multipoles, the absorptive character of the IBC particle dominates over the scattering power. The other curves in Figure 6 show that even a fairly small value of the real part in the surface impedance will wash out the resonance structure in the albedo curves. However, unlike the case for the smooth albedo over the dipole region, this ultimate flattening is due to both the scattering and absorption cross sections losing the multipolar peaks and not due to the qualitative difference in the scattering and absorption curves.

The behavior of the resonances in Figures 5 and 6 resembles the one that has been observed earlier in connection with nanoplasmonics [18, 19]. For small plasmonic (negative permittivity) scatterers, the quadrupole has a distinct profile in the albedo maps, while the dipole is practically invisible. Admittedly, differences exist: the resonances in electrically small IBC spheres (which appear for capacitive surface reactances [13]) are of a magnetic resonance type, while for small plasmonic spheres, the localized surface plasmons are all due to electric multipoles [18].

6. Conclusion

The scattering, absorption, and extinction responses of impedance-boundary objects depend on their size and shape, in addition to their complex surface impedance. Analysis of the response functions, illustrated in this article, leads to several interesting conclusions, especially with the albedo behavior. One of these observations is the surprising similarity of the albedos of the sphere and cube, suggestive of the insensitivity of the albedo on the shape of an IBC scatterer. Also, note the strong difference in the manner in which the multipolar resonances appear in the albedo spectrum: the dipole mode is invisible, while the higher order modes can be distinguished clearly as dips in the albedo curve.

7. References

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