



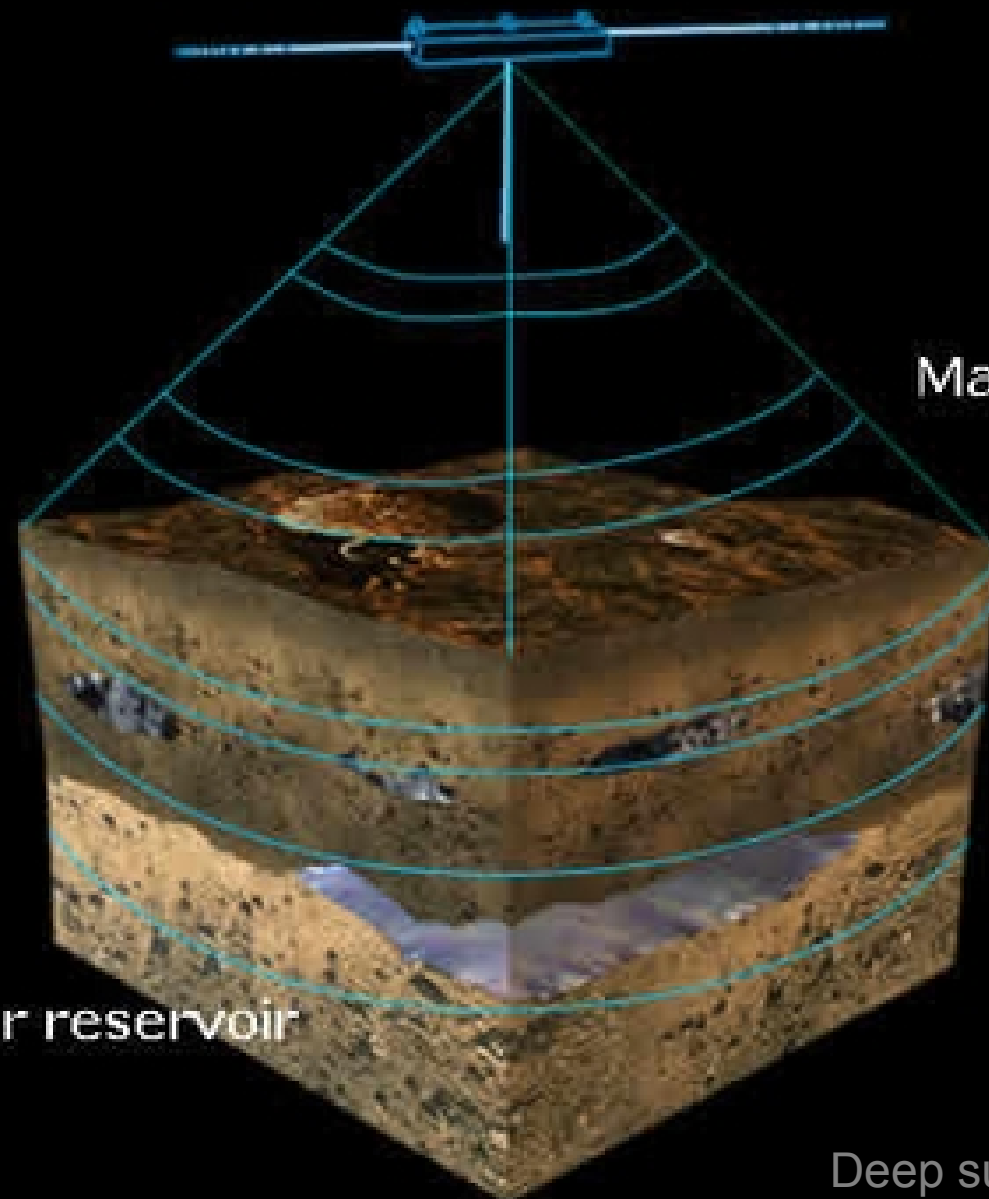
Ionospheric scintillations: impact on the HF subsurface radar sounding



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MARSIS antenna beam

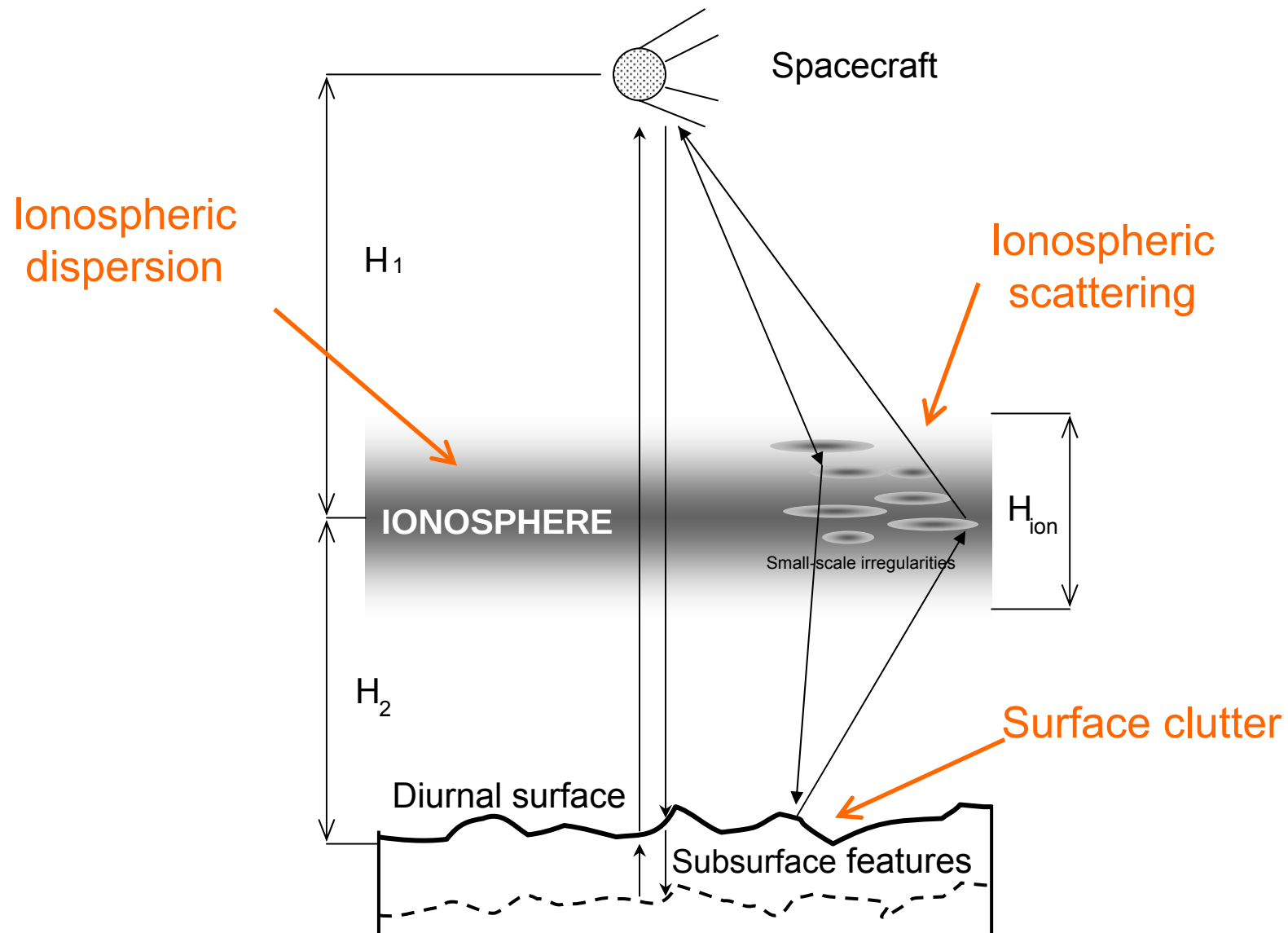


Mars crust

Water reservoir

Deep subsurface sounding

Subsurface radar sounding from the orbit: schematic depiction



UWB LFM signal processing

Compressed signal after matched filtration

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F^*(\omega) F(\omega) H(\omega) \exp(-i\omega t + \varphi(\omega) - \tilde{\varphi}(\omega)) d\omega$$

$$\varphi(\omega) = 2k \int_0^z n(z) dz \quad - \text{systematic ionospheric phase shift}$$

$$\tilde{\varphi}(\omega) \quad - \text{phase correcting function}$$

$$\omega = 2\pi f \quad n(z) = \sqrt{1 - \frac{\omega_p^2(z)}{\omega^2}}, \quad \omega_p^2 = 3392N[m^{-3}], \quad k = \frac{\omega}{c}$$

$$H(\omega) \quad - \text{spectral window function (Hanning)}$$

Two-frequency
correlation function

Amplitude mean square (mean power) of the compressed UWB LFM signal

$$|s(t)|^2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |F(\omega_1)|^2 |F(\omega_2)|^2 H(\omega_1) H(\omega_2) \Gamma(\omega_1, \omega_2)$$

$$\exp(-i(\omega_1 - \omega_2)t + (\varphi(\omega_1) - \tilde{\varphi}(\omega_1)) - (\varphi(\omega_2) - \tilde{\varphi}(\omega_2))) d\omega_1 d\omega_2$$

Criteria of the LFM signal compression quality: the contrast functions

$$C_A^2 = \frac{\int_{t_0}^{t_1} |s(t)|^2 dt - \left(\int_{t_0}^{t_1} |s(t)| dt \right)^2}{\left(\int_{t_0}^{t_1} |s(t)| dt \right)^2}$$

Amplitude contrast

$$C_I^2 = \frac{\int_{t_0}^{t_1} |s(t)|^4 dt - \left(\int_{t_0}^{t_1} |s(t)|^2 dt \right)^2}{\left(\int_{t_0}^{t_1} |s(t)|^2 dt \right)^2}$$

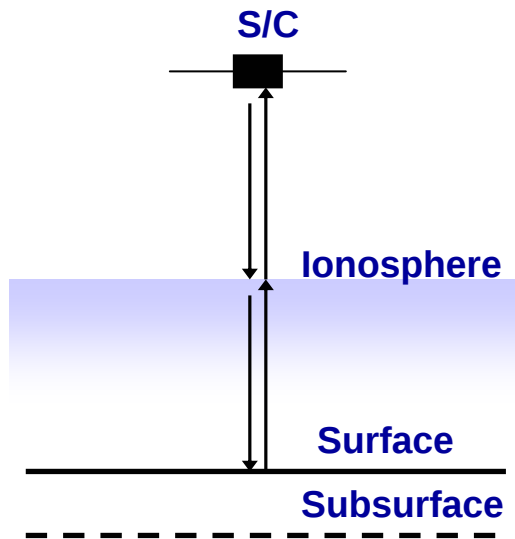
Intensity contrast

Picardi, G., et al. Martian Advanced Radar for Subsurface and Ionospheric Sounding (MARSIS): Models and System Analysis. Tech.Rep. MRS-001/005/99 V.2.0 21/05/1999.

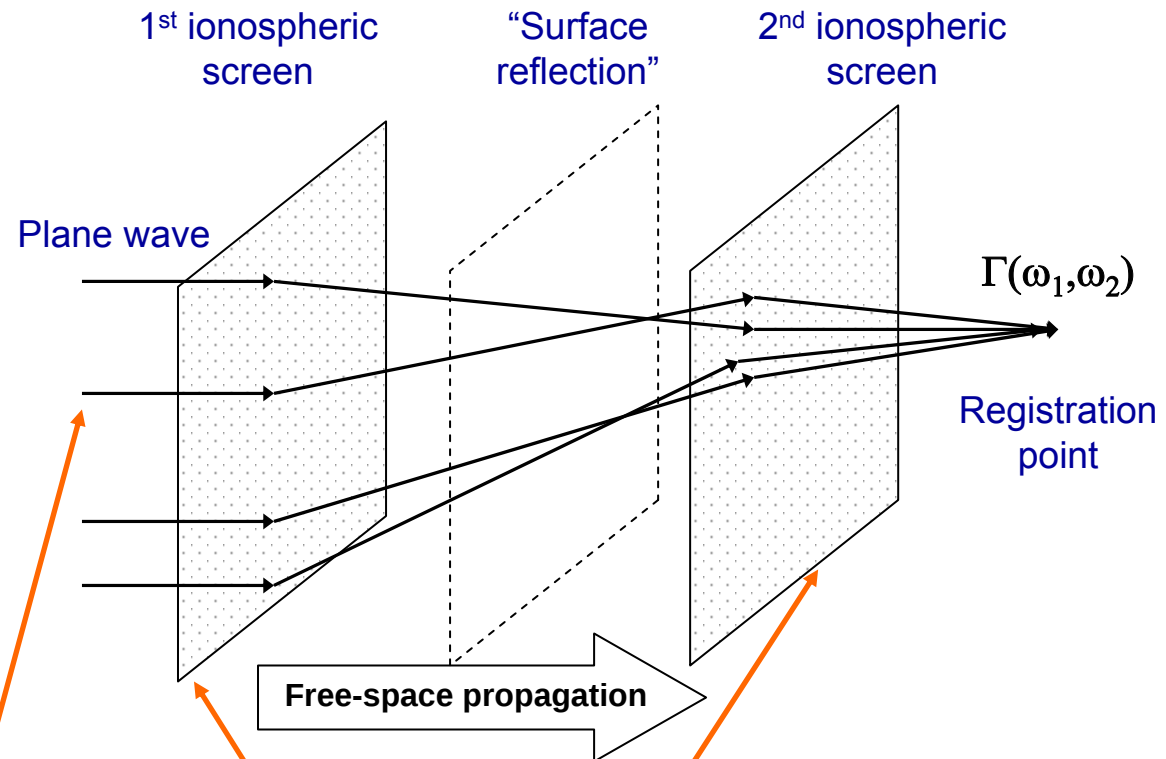
The contrast function are typically applied for estimation of the LFM signals compression quality. In fact, they are the normalized statistical moments of the signal intensity.

Markov approximation equations for the two frequency correlation function

The sketch
of the experimental geometry



The equivalent scheme for simulation



Benefits:

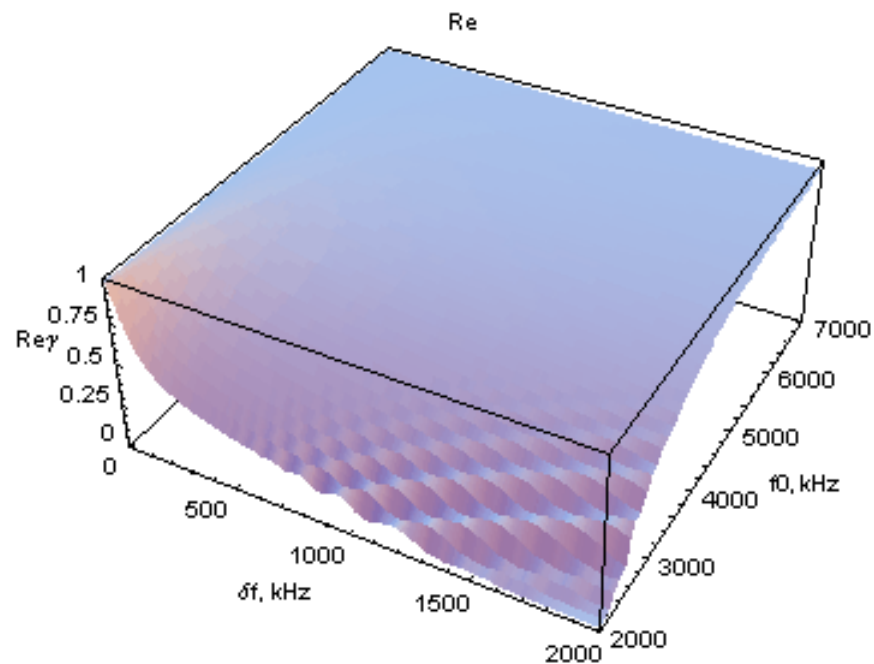
- The fast effective matrix algorithm for the solution of the Markov approximation equation for the two frequency correlation function can be applied

Disadvantages:

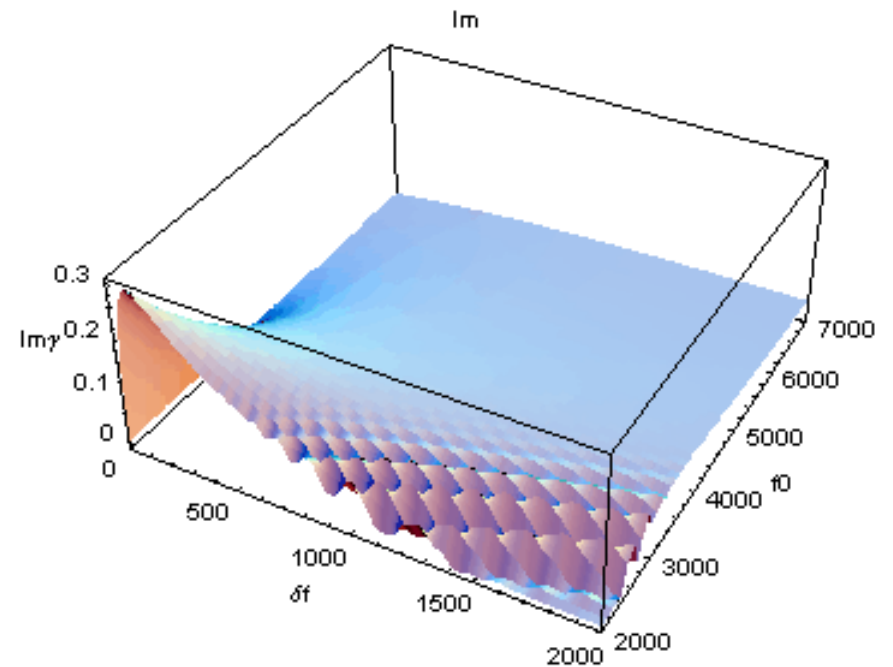
- Plane incident wave, which corresponds to remote transmitter
- Aperture synthesis cannot be simulated
- Phase fluctuations in the two screens assumed to be independent, which is in fact not true

Two frequency correlation function: weak ionospheric fluctuations

$$\Delta N/N = 0.5\%$$



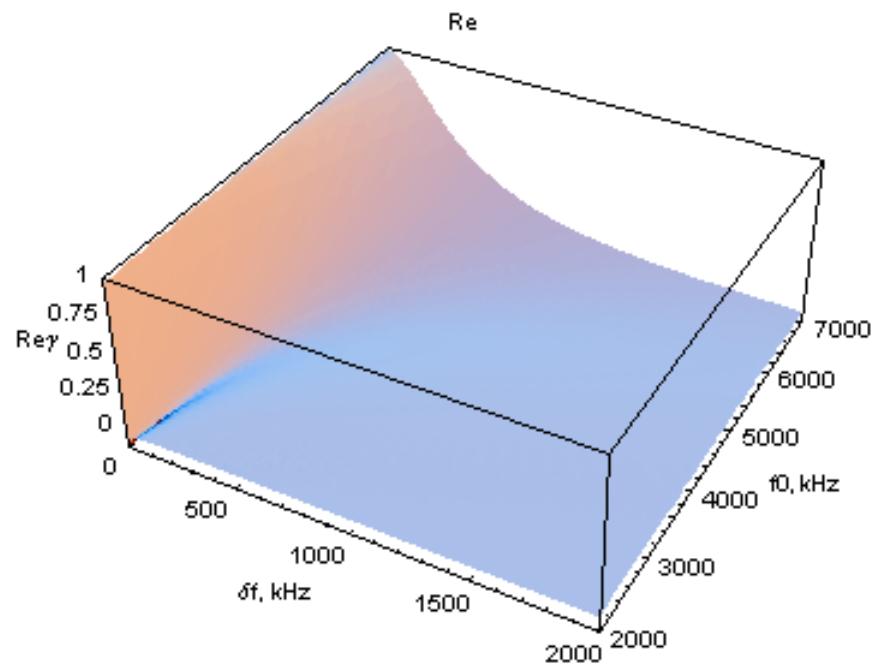
Real part



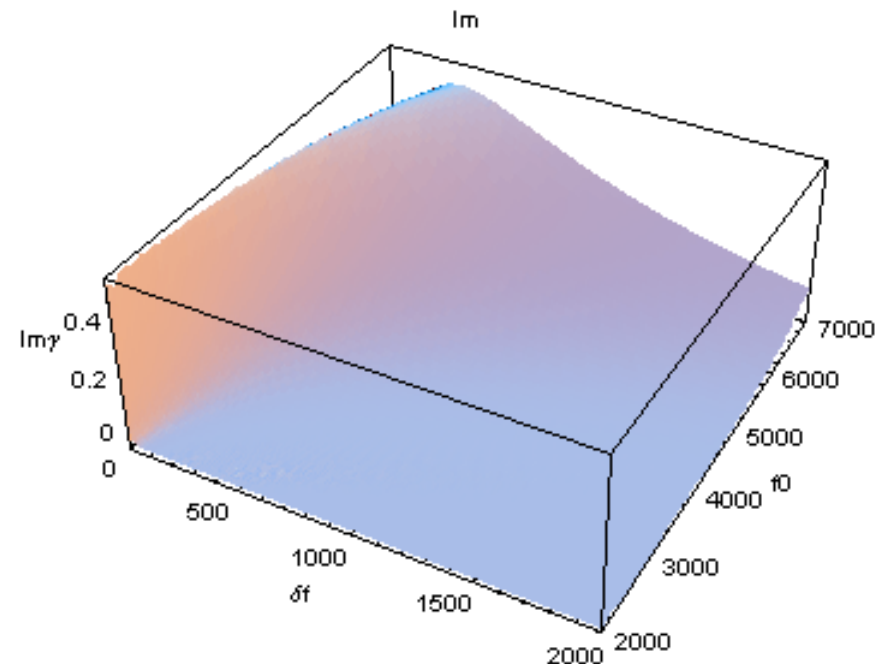
Imaginary part

Two frequency correlation function: strong ionospheric fluctuations

$$\Delta N/N = 10 \%$$

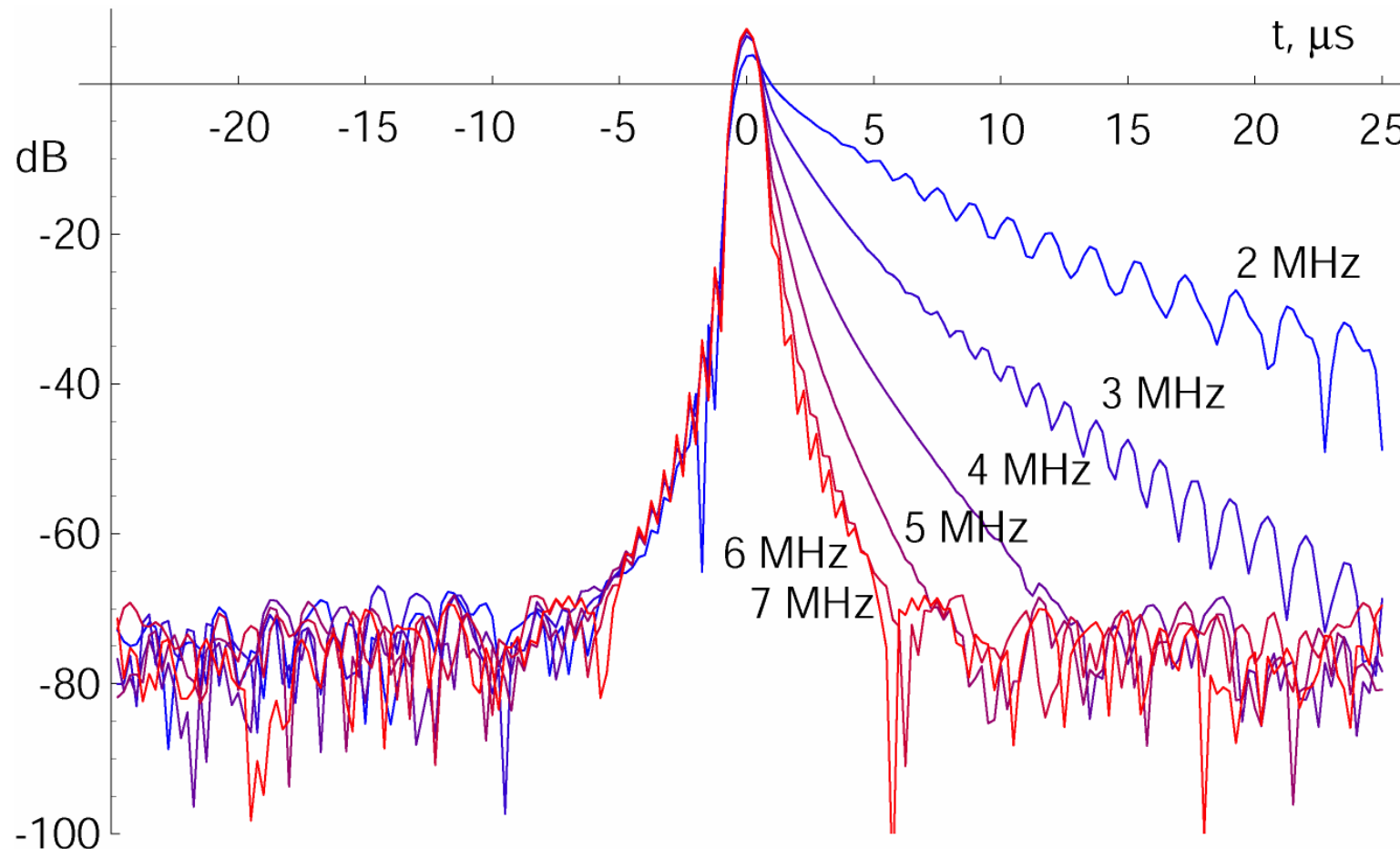


Real part



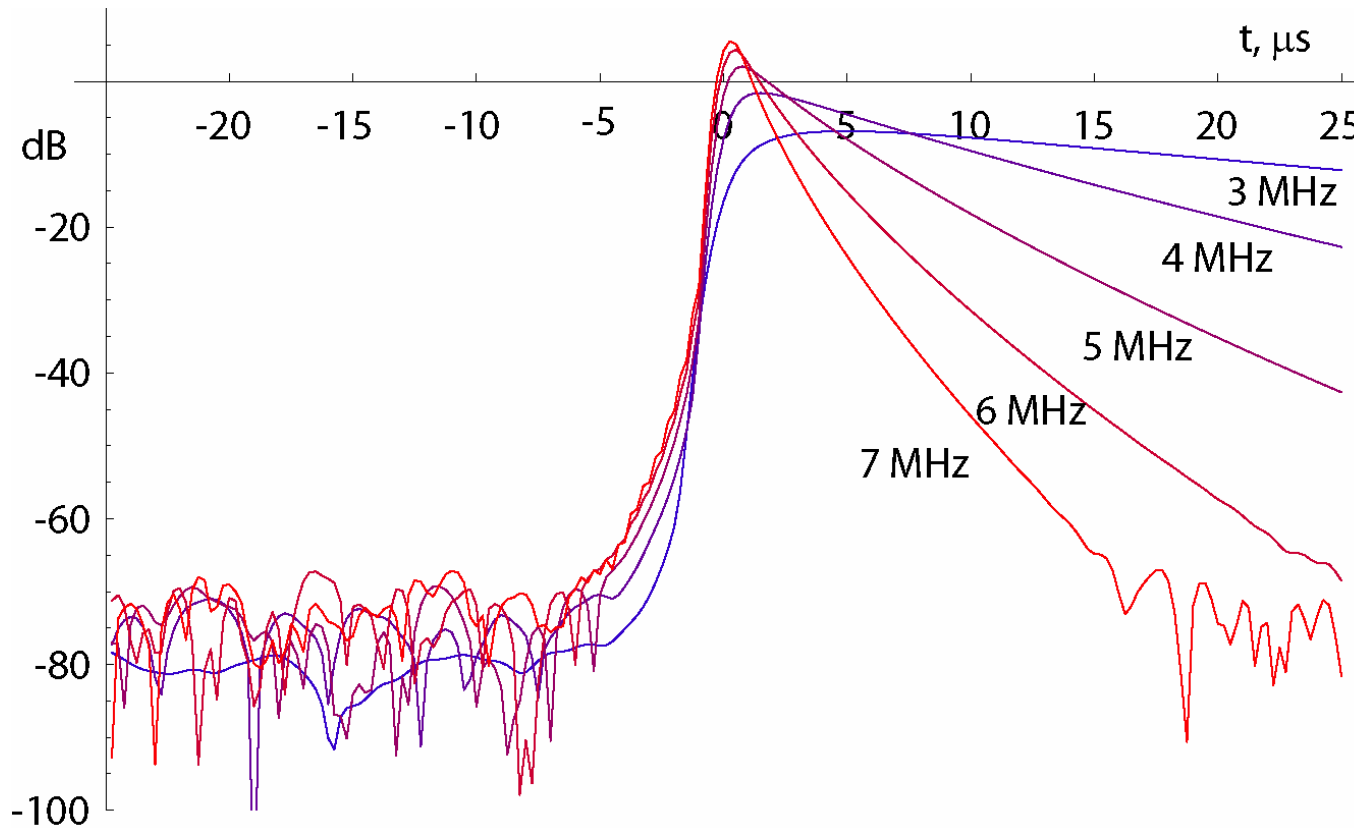
Imaginary part

Distortions of the UWB LFM signals by the small-scale stochastic ionospheric irregularities. Weak fluctuations.



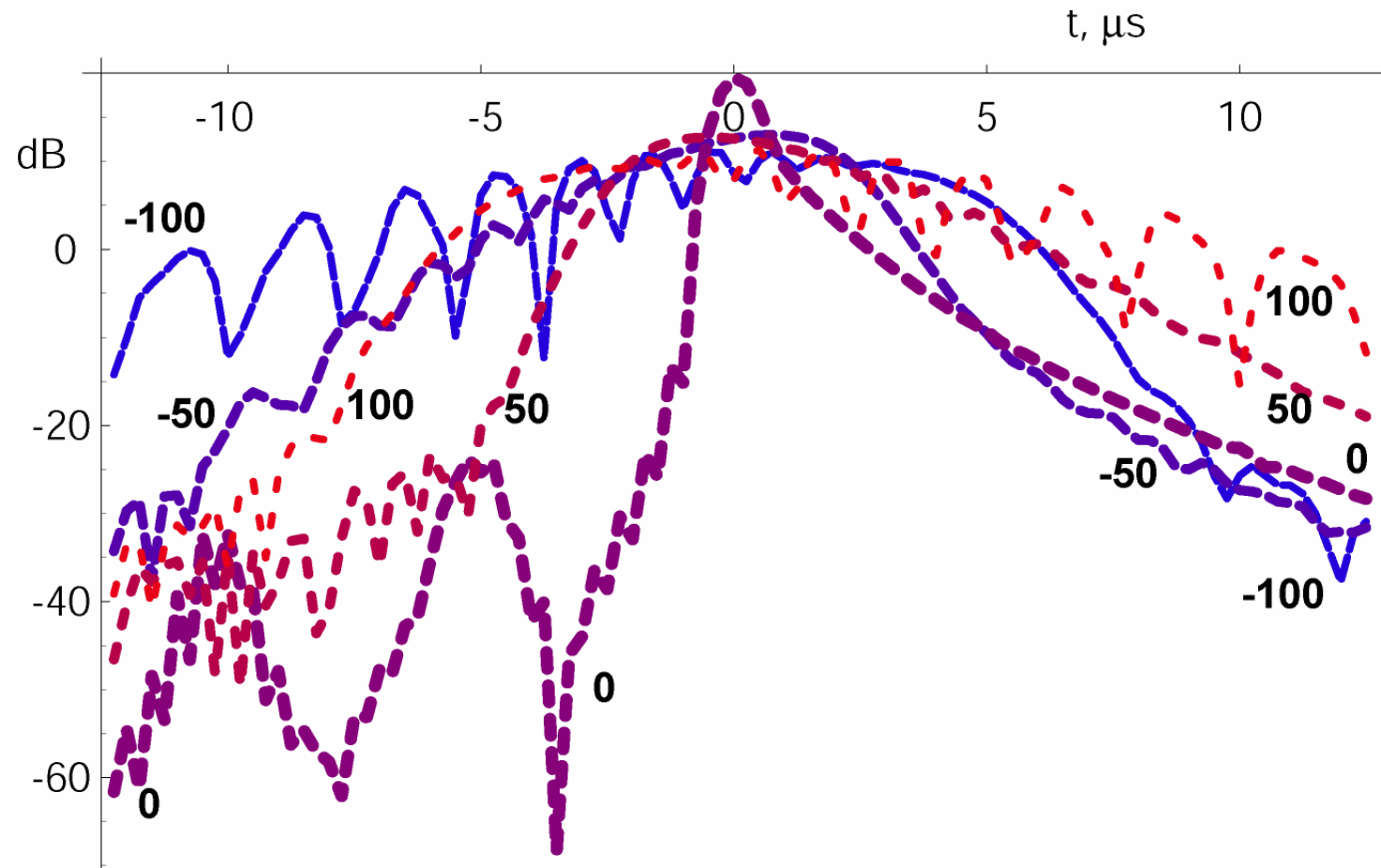
Compressed LFM UWB signals. The critical frequency of the ionosphere – 1 MHz, correlation radius of the ionospheric inhomogeneities – 1 km. LFM bandwidth 2 MHz, central frequencies of the signals are labeled near each curve. Regular phase distortions by the labeled ionosphere are completely eliminated.

Distortions of the UWB LFM signals by the small-scale stochastic ionospheric irregularities. Strong fluctuations.



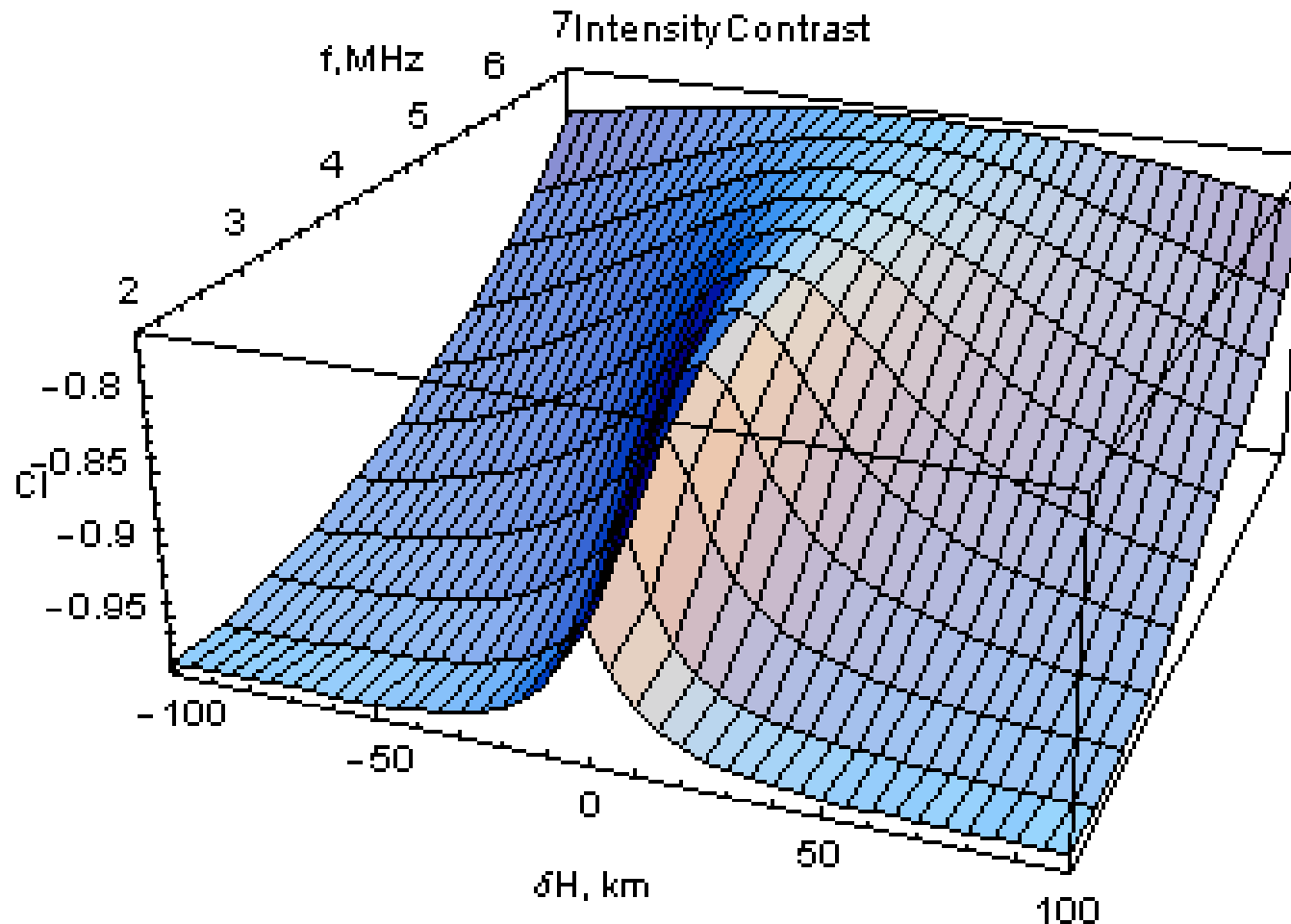
Compressed LFM UWB signals. The critical frequency of the ionosphere – 1 MHz, correlation radius of the ionospheric inhomogeneities – 1 km. LFM bandwidth 2 MHz, central frequencies of the signals are labeled near each curve. Regular phase distortions by the labeled ionosphere are completely eliminated.

Regular and stochastic ionospheric phase distortions of the UWB LFM signals working together



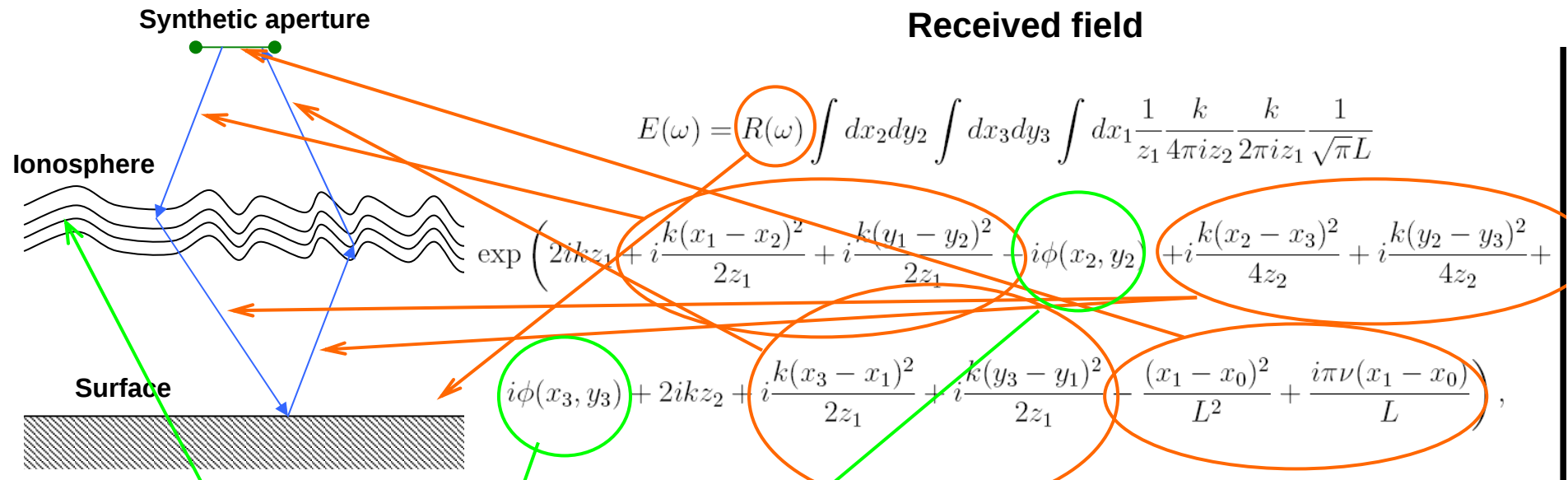
Compressed LFM UWB signals. The critical frequency both of the ionosphere and the correction layer – 1 MHz, correlation radius of the ionospheric inhomogeneities – 1 km. LFM bandwidth 2 MHz, central frequency of the LFM signal – 3 MHz. Thickness differences between the ionosphere and correcting plasma layers are shown by the numbers near each curve.

Signal compression quality test.



Variations of the intensity contrast vs uncompensated plasma layer thickness and central frequency of the LFM signals are shown in the figure. Critical frequency both of the ionosphere and correcting layer 1 MHz, LFM frequency bandwidth 2 MHz, weak plasma fluctuations ($\Delta N/N = 5\%$)

Quasi-deterministic phase screen model of the stochastic ionospheric fluctuations



Field propagation back from the spacecraft to the surface and back to the satellite is described within the paraxial (Kirchoff) approximation

Aperture synthesis is approximately simulated by the integration with Gaussian weight function

Ionospheric phase shift

Numerical simulations

We restrict our attention to the simple quasi-deterministic model of the ionospheric stochastic phase fluctuations, which is essentially 1D superposition of several sinusoidal components with phases and amplitudes

$$\phi(x) = \sum_i A_i \cos(k_i x)$$

It can be shown that the following expansion of the phase shift is valid:

$$\begin{aligned} & \exp(A_1 \cos(k_1 x) + A_2 \cos(k_2 x) + A_3 \cos(k_3 x) + \dots) = \\ & = \sum_{n_1, n_2, n_3, \dots} i^{n_1 + n_2 + n_3 + \dots} J_{n_1}(A_1) \times J_{n_2}(A_2) \times J_{n_3}(A_3) \times \dots \times \exp(ik_1 n_1 x + ik_2 n_2 x + ik_3 n_3 x + \dots) \end{aligned}$$

where $J_n(\cdot)$ are the cylindrical Bessel functions of the first kind. Substituting this expansion into the integral expression for the registered field, one gets the representation for this field in the form of the discrete sum, which can be easily evaluated with the computer:

$$\begin{aligned} E(\omega) = R(\omega) \int dx_2 dy_2 \int dx_3 dy_3 \frac{1}{z_1} \frac{k}{4\pi i z_2} \frac{k}{2\pi i z_1} \sum_{n_1, n_2, n_3, m_1, m_2, m_3} i^{n_1 + n_2 + n_3 + m_1 + m_2 + m_3} J_{n_1}(A_1) J_{n_2}(A_2) J_{n_3}(A_3) J_{m_1}(A_1) J_{m_2}(A_2) J_{m_3}(A_3) \\ \exp(ikz_1 + i \frac{k(x - x_2)^2}{2z_1} + i \frac{ky_2^2}{2z_1} + ikz_2 x_2 + 2ikz_2 + i \frac{k(x_2 - x_3)^2}{4z_2} + i \frac{k(y_2 - y_3)^2}{4z_2} + ik_3 x_3 + ikz_1 + i \frac{k(x_3 - x)^2}{2z_1} + i \frac{ky_3^2}{2z_1}) \end{aligned}$$

Obtaining of the registered field thus reduces to the evaluation of terms such that

$$\int \exp(-A_{ij} x_i x_j + B_i x_i) d^n x = \sqrt{\frac{\pi^n}{\det A_{ij}}} \exp\left(\frac{B^T A_{ij}^{-1} B}{4}\right)$$

Variables of integration are separated into two groups (x- and y-), for which the matrix A_i and the vector B_i respectively are

$$A_{ij}^{(x)} = \begin{vmatrix} \frac{1}{L^2} - \frac{ik}{z_1} & \frac{ik}{2z_1} & \frac{ik}{2z_1} \\ \frac{ik}{2z_1} & -\frac{ik(z_1 + 2z_2)}{4z_1 z_2} & \frac{ik}{4z_1} \\ \frac{ik}{2z_1} & \frac{ik}{4z_1} & -\frac{ik(z_1 + 2z_2)}{4z_1 z_2} \end{vmatrix} \quad B_i^{(x)} = \begin{vmatrix} -\frac{iL\pi\nu + 2x_0}{L^2} \\ ik_2 \\ ik_3 \end{vmatrix} \quad A_{ij}^{(y)} = \begin{vmatrix} \frac{ik(z_1 + 2z_2)}{4z_1 z_2} & \frac{ik}{4z_2} \\ \frac{ik}{4z_2} & -\frac{ik(z_1 + 2z_2)}{4z_1 z_2} \end{vmatrix}$$

$$B_i^{(y)} = 0$$

We omit the intermediate calculations and reproduce the final result:

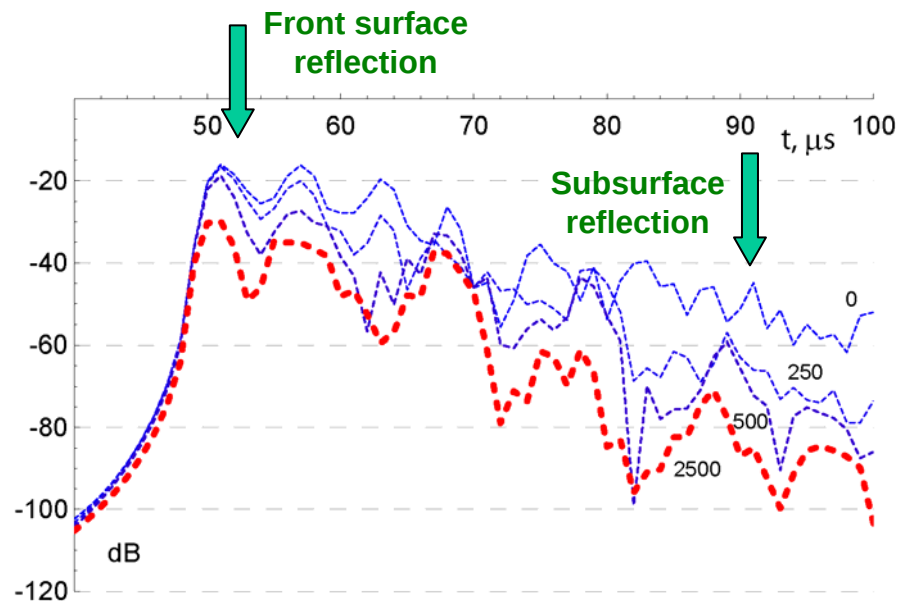
$$E(\omega) = \sum i^{(n_1 + n_2 + n_3 + m_1 + m_2 + m_3)} J_{n_1}(A_1) J_{n_2}(A_2) J_{n_3}(A_3) J_{m_1}(A_1) J_{m_2}(A_2) J_{m_3}(A_3)$$

$$\exp\left(-\frac{iz_1((z_1 + 2z_2)(k_2^2 + k_3^2) + 2k_3 k_2 z_1)L^2 + k((k_2 + k_3)L^2 + \pi\nu L - 2ix_0)^2(z_1 + z_2)}{4kL^2(z_1 + z_2)}\right)$$

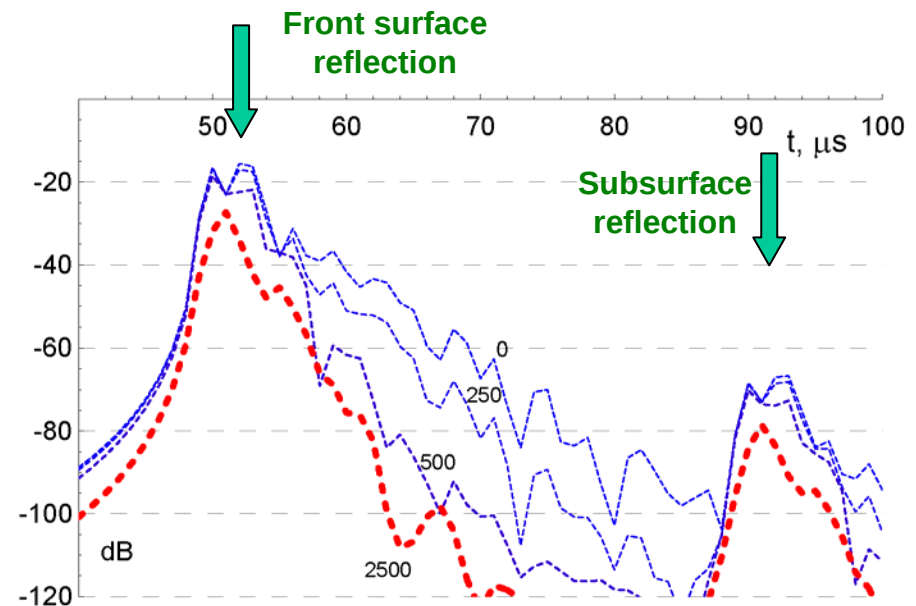
where summation is performed over all six indices $n_1, n_2, n_3, m_1, m_2, m_3$

Dependence on the synthetic aperture length.

Strong phase fluctuations $\langle \phi^2 \rangle = 25$



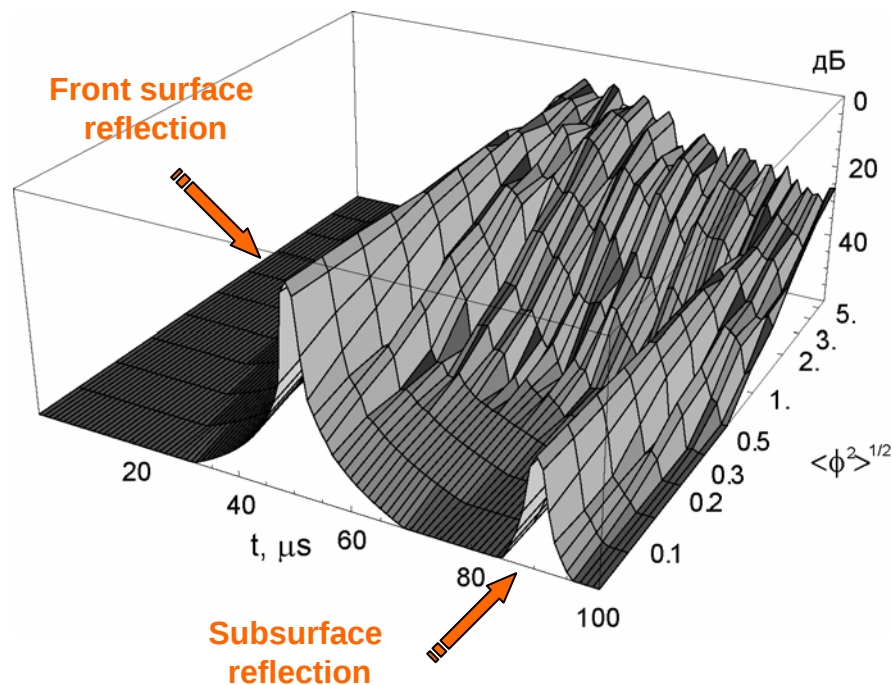
Moderate phase fluctuations $\langle \phi^2 \rangle = 4$



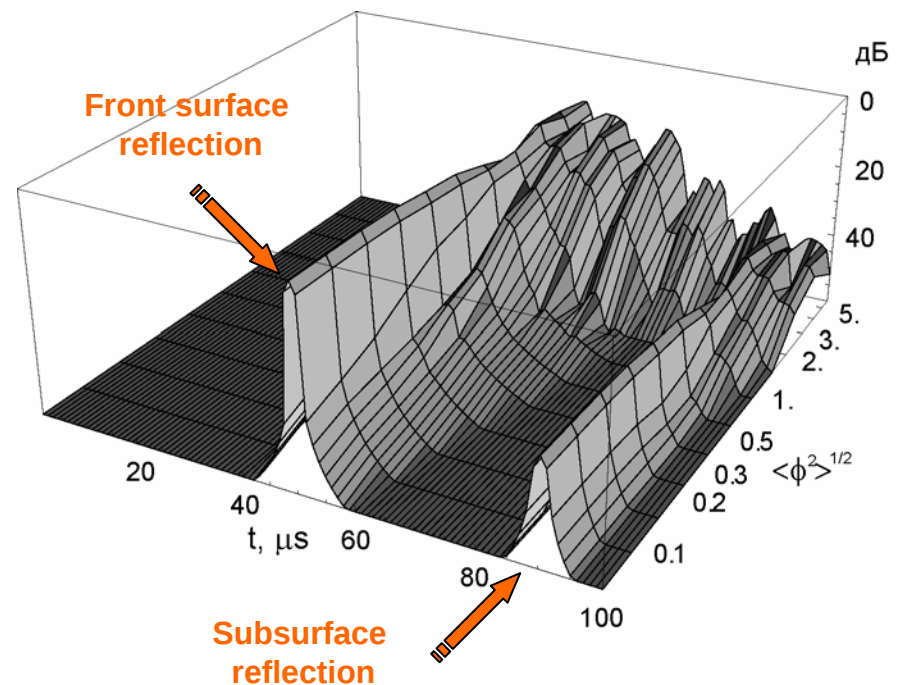
The compressed UWB LFM signals with various synthetic aperture lengths, reflected from the multi-layered subsurface structure, are shown in the figures. The longer the synthetic aperture, the better is the suppression of diffracted peaks in the signals. Extension of the synthetic aperture over the optimal length (half the Fresnel zone size at the central frequency of the LFM band) does not lead to further growth of the suppression.

Aperture synthesis vs. no aperture synthesis

Single pulse (no aperture synthesis)

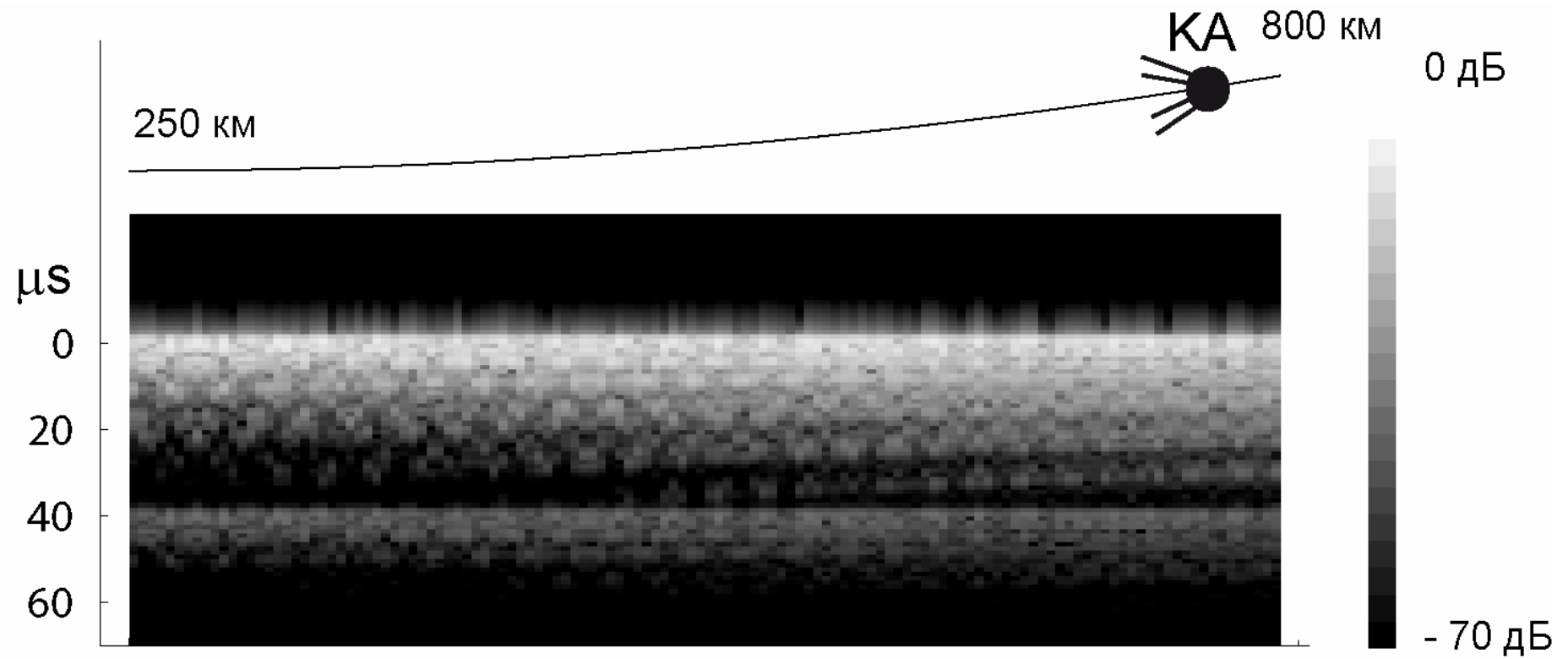


Aperture synthesis (optimal aperture length)

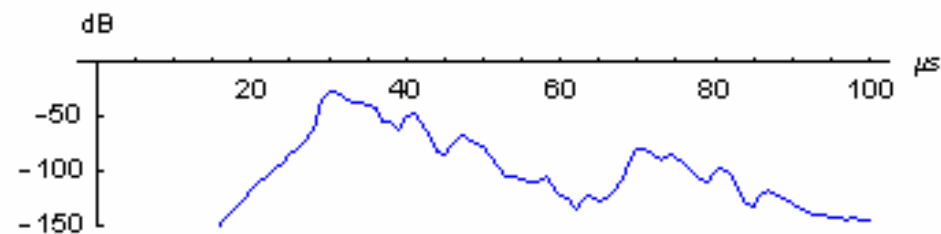
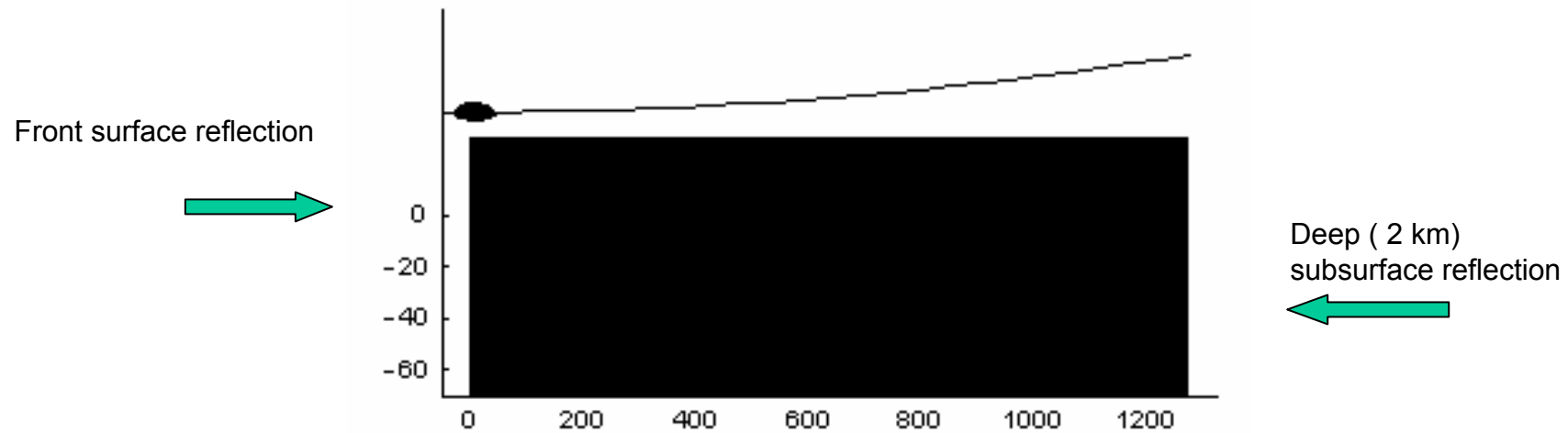


When stochastic phase fluctuations in the ionosphere are of moderate strength (r.m.s. phase deviation does not exceed one whole period), synthetic aperture technique allows to effectively suppress diffracted signals coming from side directions. When the phase fluctuations are stronger than 2π r.m.s., the effect of the aperture synthesis rapidly vanishes.

Subsurface radargram profile: numerical simulation.



Subsurface radargram profile: numerical simulation.



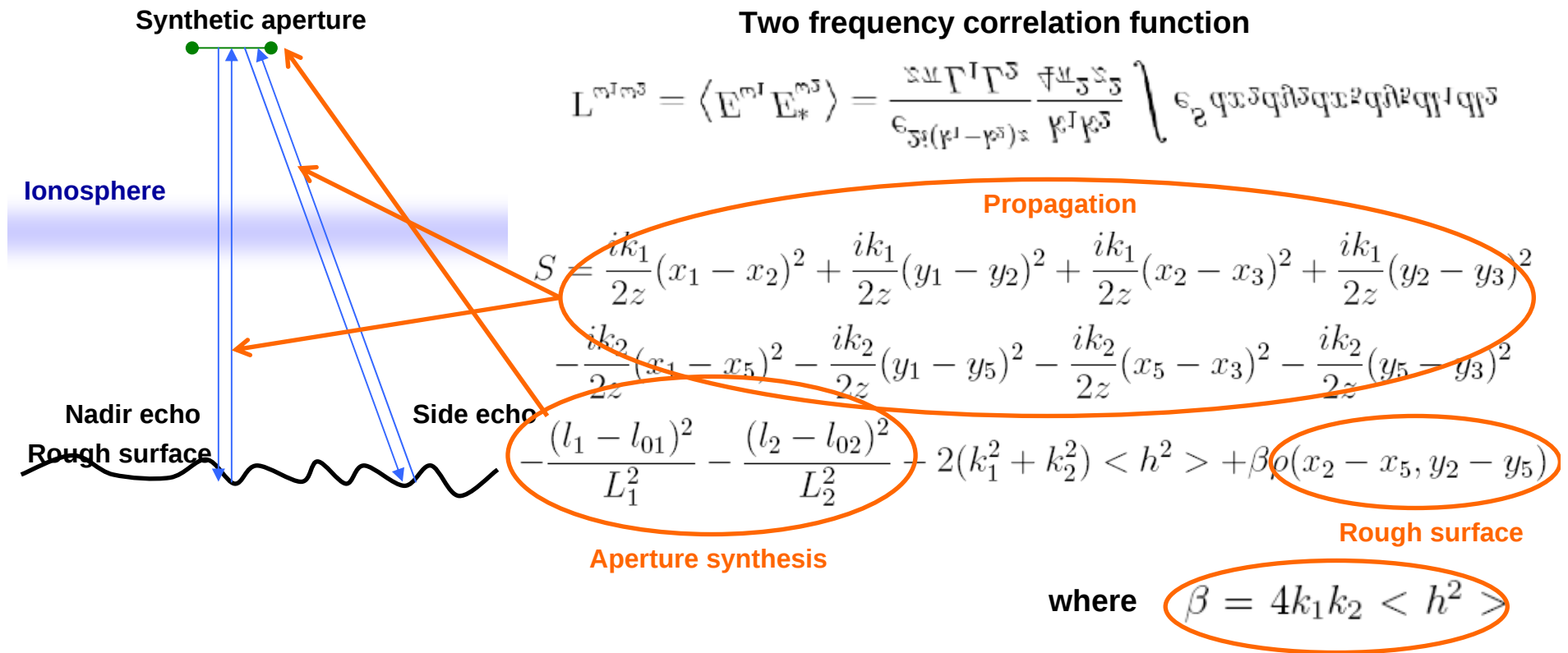
Front surface reflection



Deep subsurface reflection

Surface Clutter

(Side Reflections Coming From the Rough Surface)



Gaussian height correlation function

$$\rho(\delta \vec{r}) = \langle h(\vec{r}) h(\vec{r} + \delta \vec{r}) \rangle = \langle h^2 \rangle \exp \left(-\frac{\delta x^2}{\sigma_x^2} - \frac{\delta y^2}{\sigma_y^2} \right)$$

Exponential height correlation function

$$\rho(\delta \vec{r}) = \langle h(\vec{r}) h(\vec{r} + \delta \vec{r}) \rangle = \langle h^2 \rangle \exp \left(-\frac{|\delta \vec{r}|}{r_0} \right)$$

Side clutter.

Two frequency correlation function evaluation

$$\int \exp(-A_{ij}x_i x_j + B_i x_i + C) d^n x = \sqrt{\frac{\pi^n}{\det A_{ij}}} \exp\left(\frac{B^T A_{ij}^{-1} B}{4} + C\right)$$

$$A_{ij} = \begin{pmatrix} \frac{n}{\sigma_x^2} - \frac{ik_1}{z} & 0 & -\frac{n}{\sigma_x^2} & 0 & \frac{ik_1 \cos \phi}{z} & 0 \\ 0 & \frac{n}{\sigma_y^2} - \frac{ik_1}{z} & 0 & -\frac{n}{\sigma_y^2} & \frac{ik_1 \sin \phi}{z} & 0 \\ -\frac{n}{\sigma_x^2} & 0 & \frac{ik_2}{z} + \frac{n}{\sigma_x^2} & 0 & 0 & -\frac{ik_2 \cos \phi}{z} \\ 0 & -\frac{n}{\sigma_y^2} & 0 & \frac{ik_2}{z} + \frac{n}{\sigma_y^2} & 0 & -\frac{ik_2 \sin \phi}{z} \\ \frac{ik_1 \cos \phi}{z} & \frac{ik_1 \sin \phi}{z} & 0 & 0 & \frac{1}{L_1^2} - \frac{ik_1}{z} & 0 \\ 0 & 0 & -\frac{ik_2 \cos \phi}{z} & -\frac{ik_2 \sin \phi}{z} & 0 & \frac{ik_2}{z} + \frac{1}{L_2^2} \end{pmatrix}$$

$$B_i = \left\{ -\frac{i\delta l_1 k_1 \cos \phi}{z}, -\frac{i\delta l_1 k_1 \sin \phi}{z}, \frac{i\delta l_2 k_2 \cos \phi}{z}, \frac{i\delta l_2 k_2 \sin \phi}{z}, \frac{i\delta l_1 k_1}{z} + \frac{2l_0}{L_1^2}, -\frac{i\delta l_2 k_2}{z} \right\}$$

$$C = \frac{i(\delta l_1^2 k_1 - \delta l_2^2 k_2)}{2z} - \frac{l_0^2}{L_1^2}.$$

Side clutter.

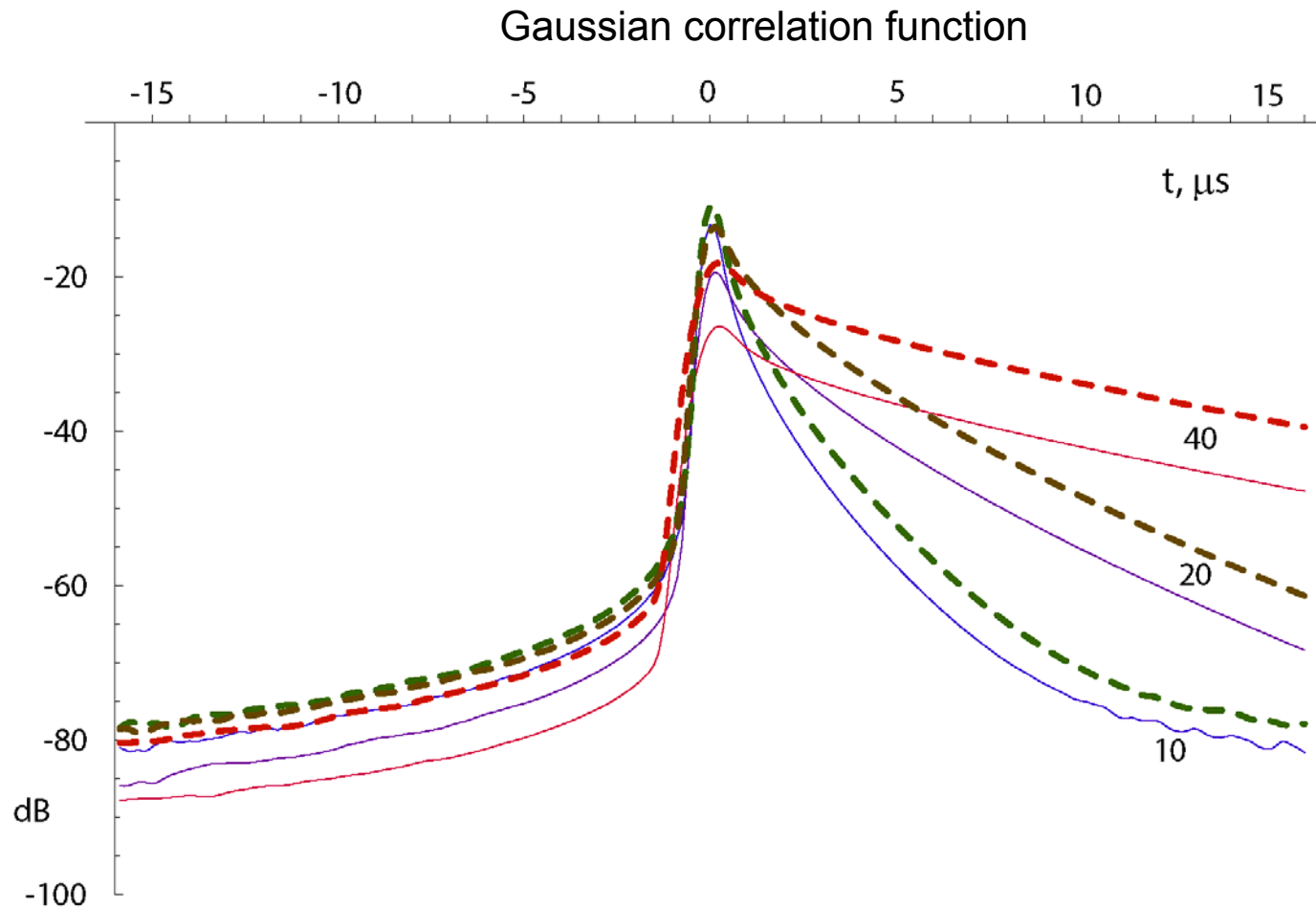
Two frequency correlation function evaluation

$$\langle E_{\omega_1} E_{\omega_2}^* \rangle = \sum_{n=0}^{\infty} \frac{(\beta^n / n!) k_1 k_2 \sigma_x \sigma_y \exp \left(- \frac{k_1 k_2 n l_0^2}{(k_1 k_2 (n(L_1^2 + L_2^2) + \sigma_x^2) - i(k_1 - k_2)nz)} \right)}{\sqrt{(k_1 k_2 \sigma_y^2 - i(k_1 - k_2)nz) (k_1 k_2 (n(L_1^2 + L_2^2) + \sigma_x^2) - i(k_1 - k_2)nz)}}$$

The synthetic aperture lengths can be different at different frequencies and vary with the position of the spacecraft

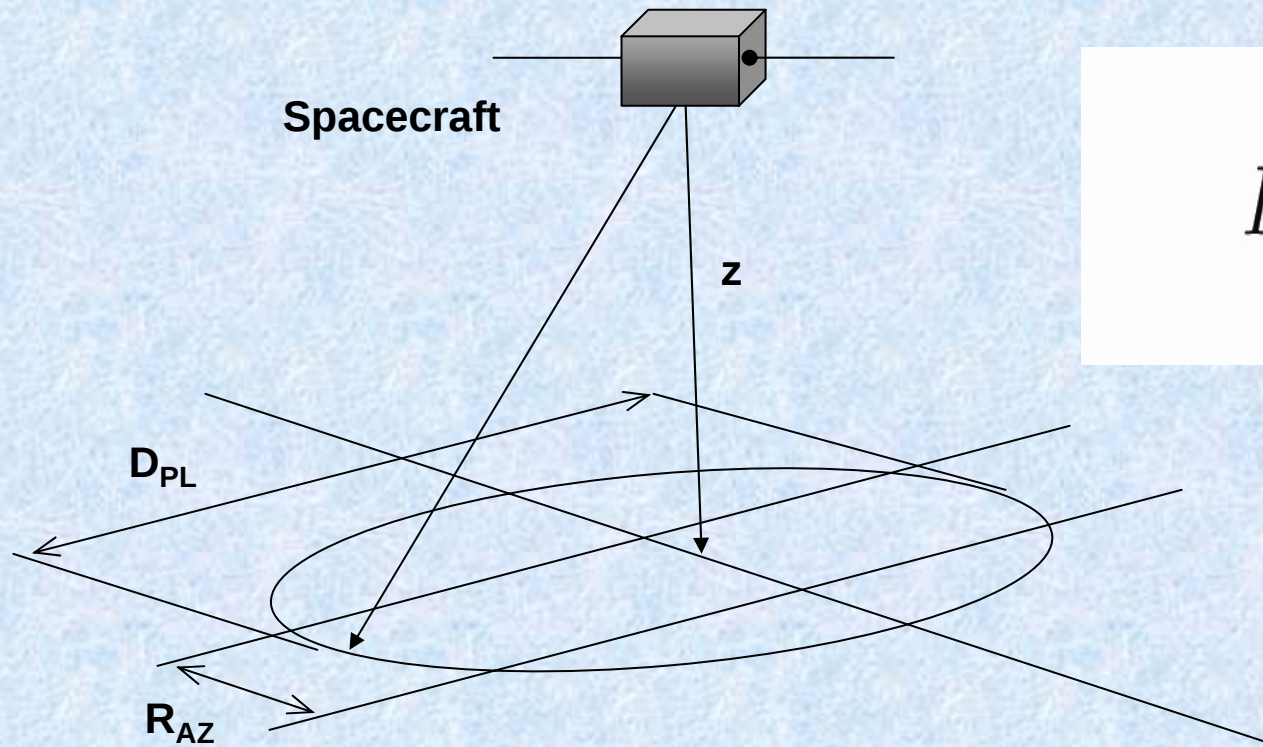
Spatial displacement between synthetic aperture centers at two frequencies. For the step frequency radar (SFR) must be taken into account

Anisotropic surface roughness height correlation function



Compressed UWB LFM signals coming from rough front surface. Solid curves correspond to $\sigma_x=1000$ μm , dashed curves - $\sigma_x=10000$ μm . For all signals $\sigma_y=1000$ μm . R.m.s. roughness height deviation shown by numbers near each curve.

Rough surface reflection from the planet: radar equation approximation



$$P = P_0 \frac{g^2 \lambda^2 \sigma_0 A}{(4\pi)^3 z^4},$$

Radar equation

$$A = R_{AZ} D_{PL}$$

Diffuse scattering area

$$D_{PL} \approx 2\sqrt{zC\tau}$$

Radar pulse length limited
diameter of the scattering
area

$$R_{AZ} = \lambda z / 2L_s$$

Azimuth resolution of the radar

Hagfors' law: reflection from the rough surface

Exponential surface roughness height correlation function

$$\rho(\delta\vec{r}) = \langle h(\vec{r})h(\vec{r} + \delta\vec{r}) \rangle = \langle h^2 \rangle \exp\left(-\left|\frac{\delta\vec{r}}{r_0}\right|\right)$$

Hagfors' roughness parameter

$$C = \frac{\lambda^2 r_0^2}{16\pi^2 \langle h^2 \rangle^2}$$

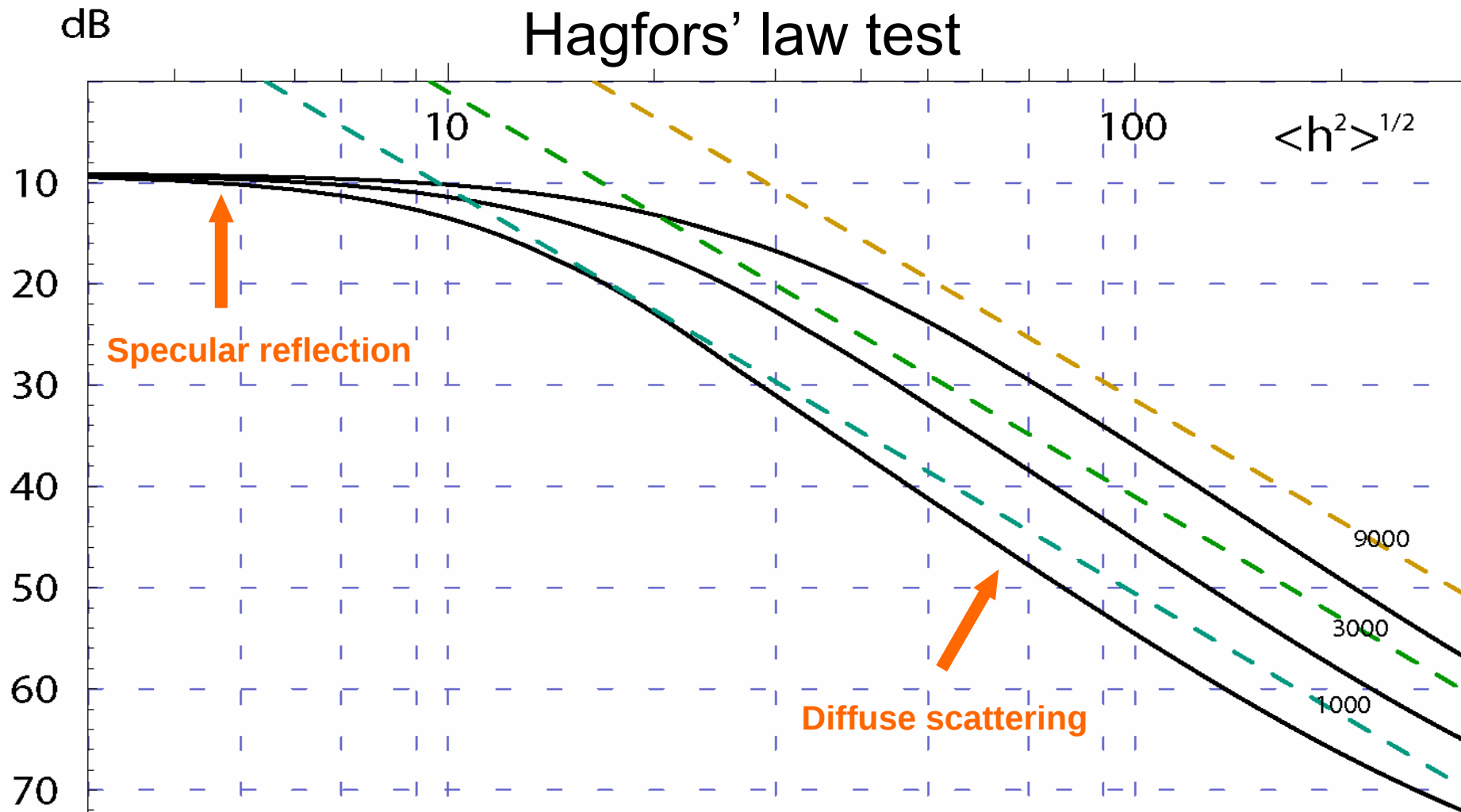
Scattering cross section of the unit area

$$\sigma_H(\vartheta) = \frac{R}{2} \frac{C}{(\cos^4 \vartheta + C \sin^2 \vartheta)^{3/2}}$$

Normalized diffuse reflection power from the nadir

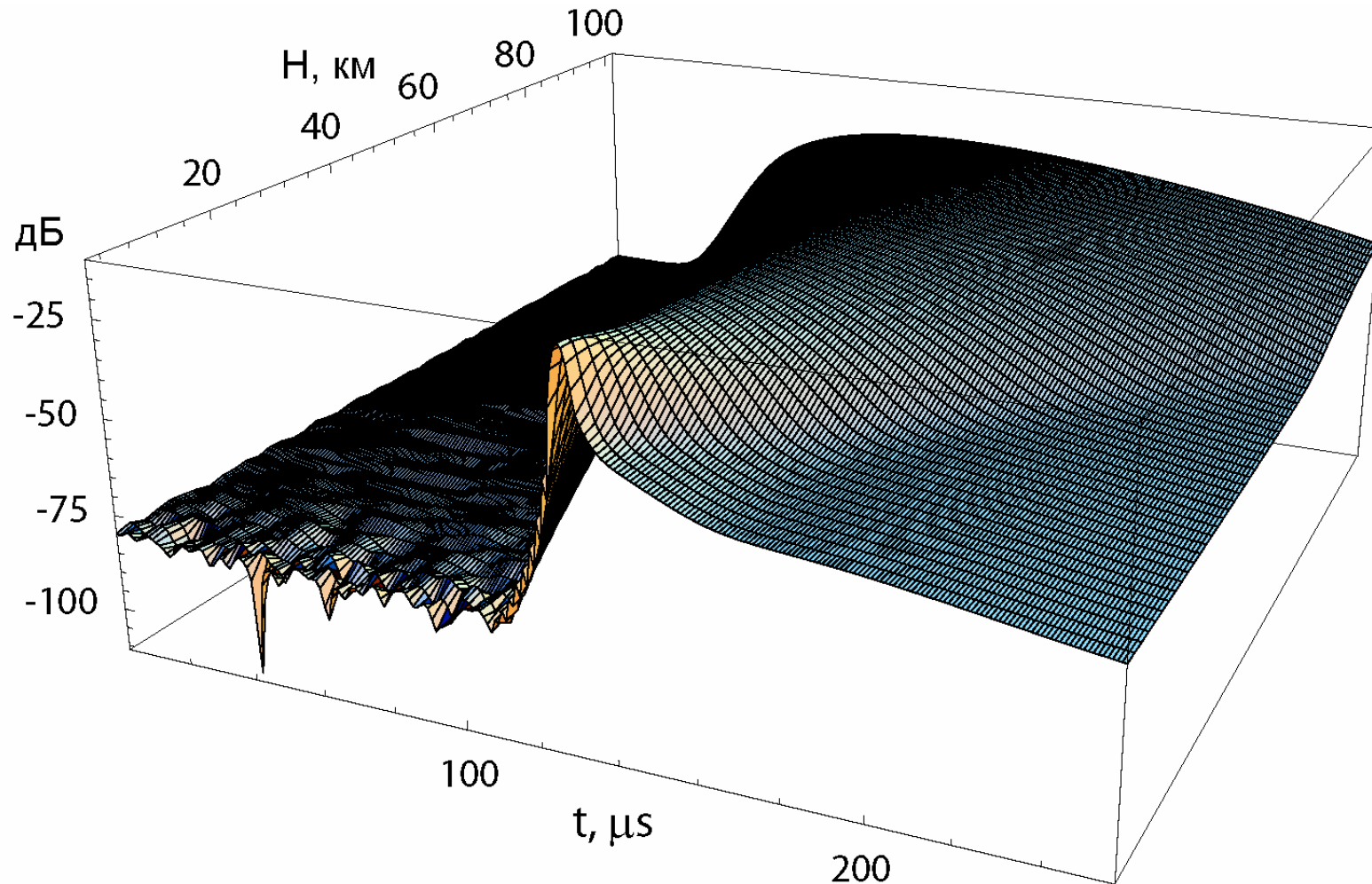
$$\frac{\sigma_0 z}{\pi z^2} = \frac{1}{2} \left(\frac{\sigma \lambda}{4\pi \langle h^2 \rangle} \right)^2 \frac{\lambda z^{3/2} (c\tau)^{1/2}}{L_s}.$$

Exponential height correlation function: Hagfors' law test



Peak amplitudes of the compressed UWB LFM signals vs. r.m.s. height of the roughness. Solid **black** curves – amplitude calculation through the two frequency correlation function, dashed **colored** curves – approximate estimation by the unit area scattering cross section (radar equation). Height correlation functions are isotropic, correlation scales are shown near each pair of the curves by numbers.

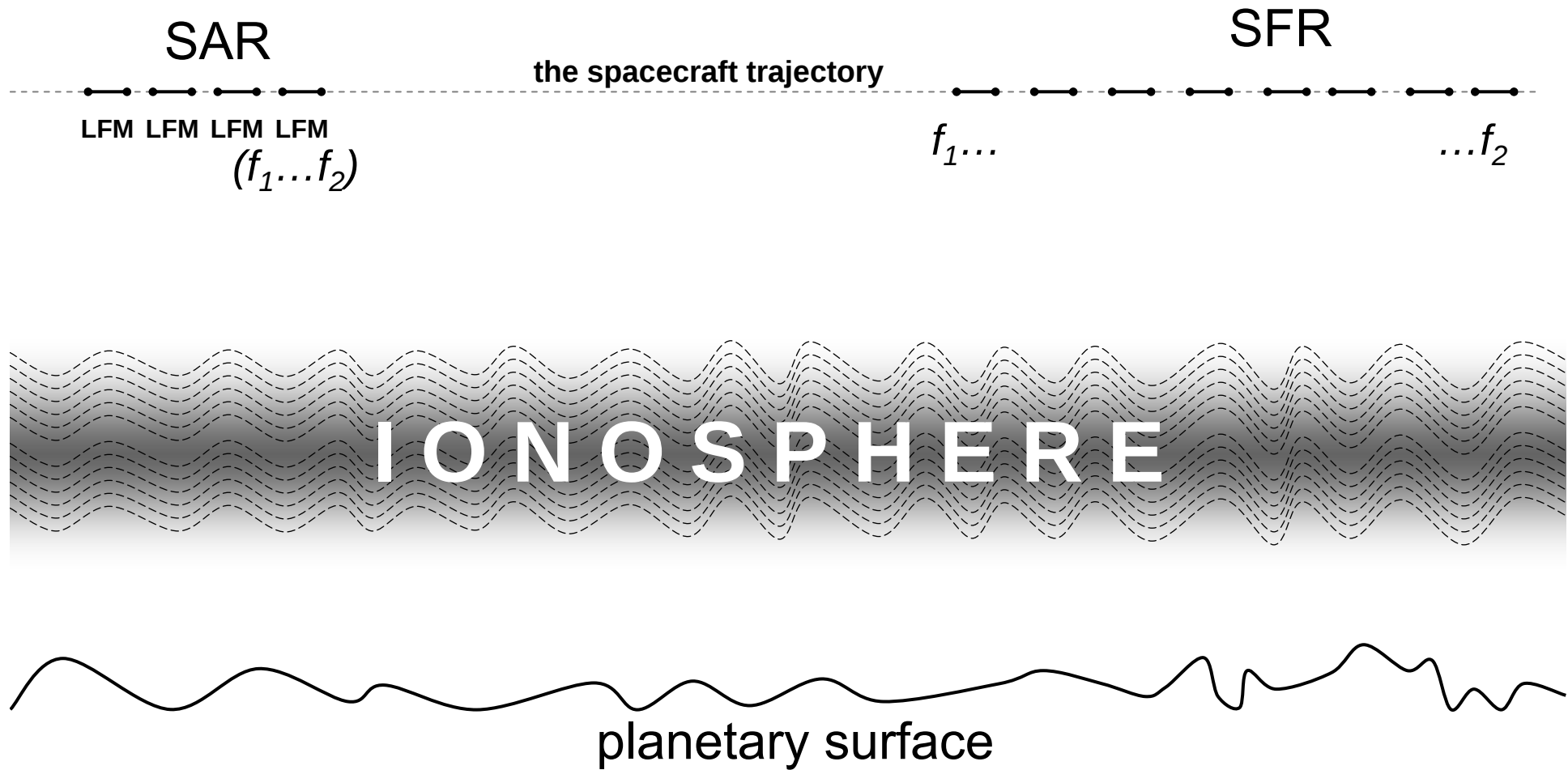
Surface clutter and systematic ionospheric dispersion working together



Compressed UWB LFM pulses vs. thickness of the uncompensated ionospheric plasma layer are shown in the figure. The plasma critical frequency 1 MHz, LFM bandwidth 1 MHz, central frequency of the LFM frequency band 3 MHz.

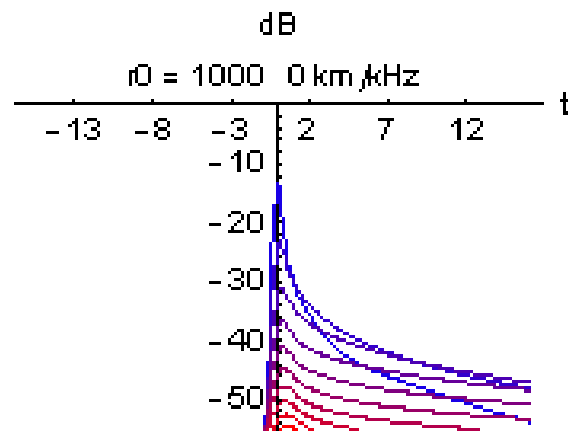
SAR vs. SFR

Synthetic Aperture Radar vs. Step Frequency Radar

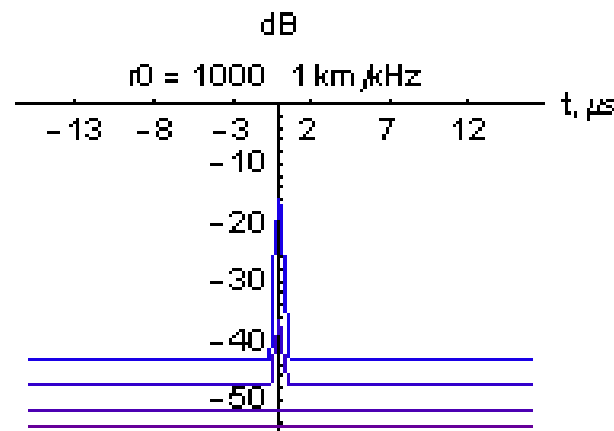


Synthetic Aperture Radar Vs. Step Frequency Radar

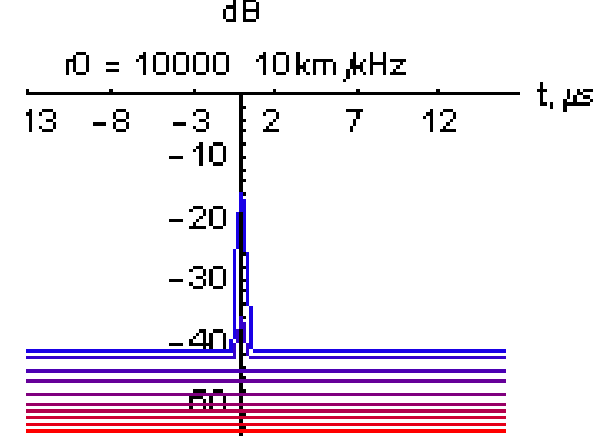
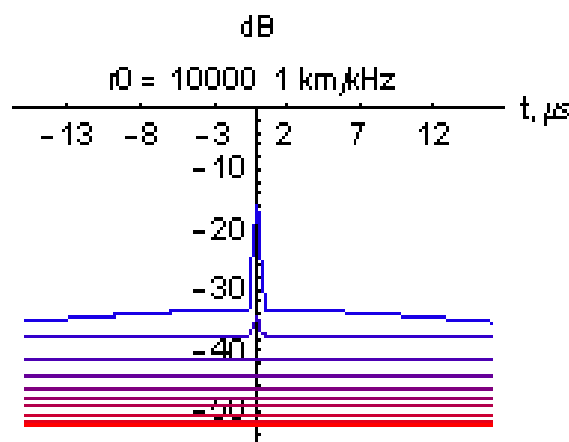
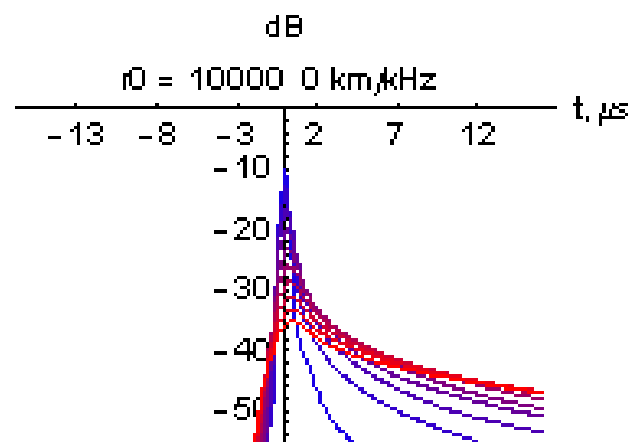
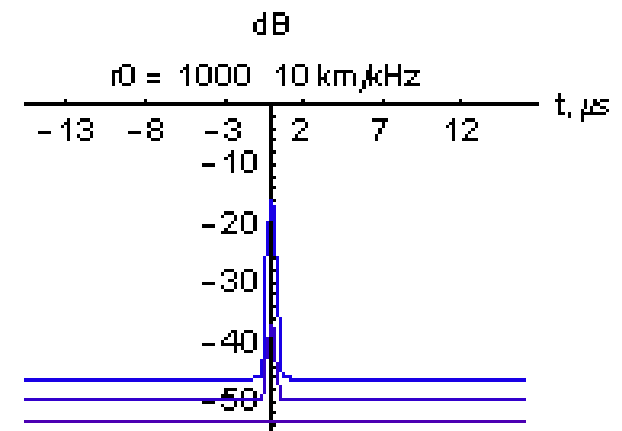
SAR (0 km/kHz)



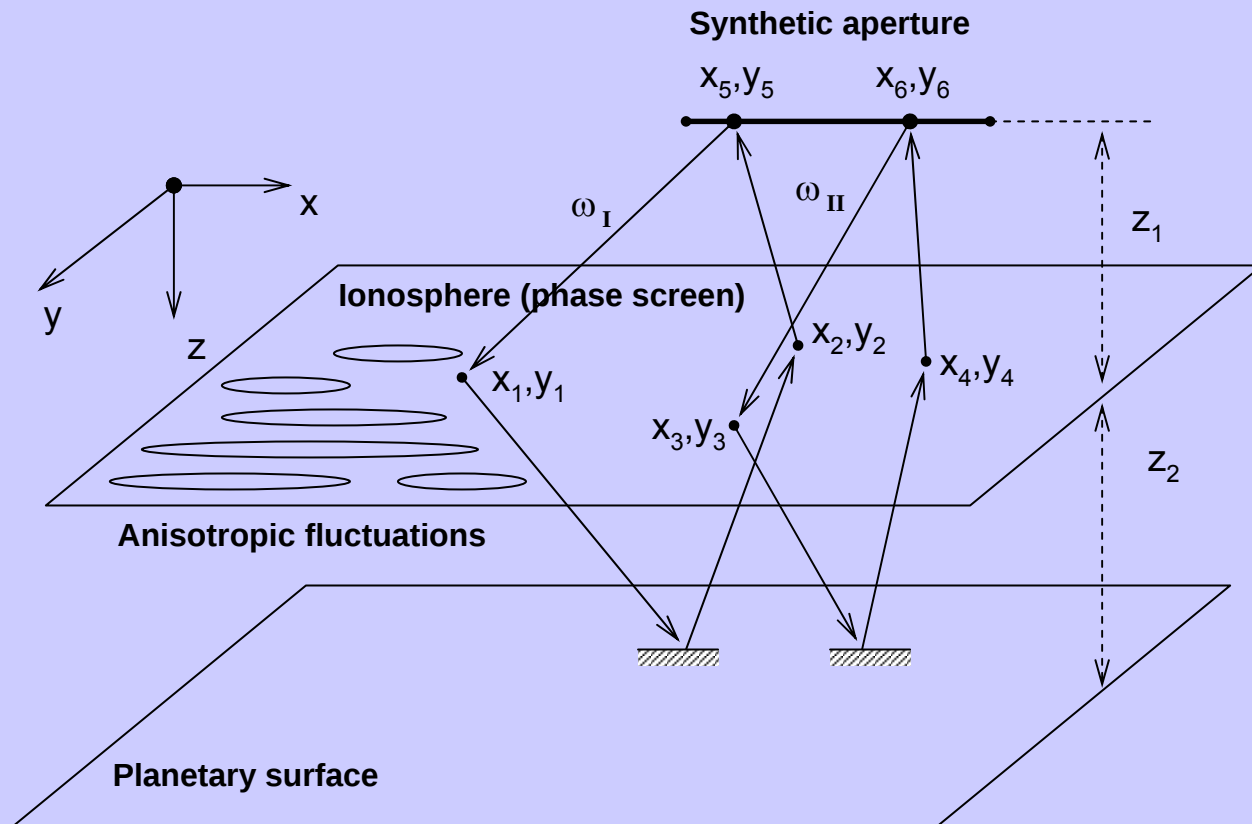
SFR (1 km/kHz)



SFR (10 km/kHz)



Anisotropic correlation function of the ionospheric plasma fluctuations



Anisotropic correlation function of the ionospheric plasma fluctuations: coherency function $\Gamma(\omega_I, \omega_{II})$

Random phases
must be averaged all together

$$\Gamma(\omega_I, \omega_{II}) = \frac{1}{z_1^2} \frac{k_I}{2\pi i 2z_2} \frac{k_I}{2\pi i z_1} \frac{1}{\sqrt{\pi} L} \frac{k_{II}}{2\pi i 2z_2} \frac{k_{II}}{2\pi i z_1} \frac{1}{\sqrt{\pi} L} \int dx_1 dy_1 dx_2 dy_2 dx_3 dy_3 dx_4 dy_4 dx_6$$

$$\exp \left(ik_I z_1 + \frac{ik_I(x_5 - x_1)^2 + ik_I(y_5 - y_1)^2}{2z_1} + 2ik_I z_2 + \frac{ik_I(x_1 - x_2)^2 + ik_I(y_1 - y_2)^2}{4z_2} + i\phi(x_1, y_1) \right.$$

$$\left. + ik_I z_1 + \frac{ik_I(x_5 - x_2)^2 + ik_I(y_5 - y_2)^2}{2z_1} + i\phi(x_2, y_2) - \frac{(x_5 - x_0)^2}{L^2} + i \frac{\pi \nu (x_5 - x_0)}{L} \right.$$

$$\left. - ik_{II} z_1 - \frac{ik_{II}(x_6 - x_3)^2 - ik_{II}(y_5 - y_3)^2}{2z_1} - 2ik_{II} z_2 - \frac{ik_{II}(x_3 - x_4)^2 - ik_{II}(y_3 - y_4)^2}{4z_2} - i\phi(x_3, y_3) \right.$$

$$\left. - ik_{II} z_1 - \frac{ik_{II}(x_6 - x_4)^2 - ik_{II}(y_5 - y_4)^2}{2z_1} - i\phi(x_4, y_4) - \frac{(x_6 - x_0)^2}{L^2} - i \frac{\pi \nu (x_6 - x_0)}{L} \right)$$

Synthetic aperture terms

Ionospheric phase fluctuations: effective phase screen model

Correlation function of the
dielectric permittivity ε

$$B_\varepsilon(\vec{r}_1, \vec{r}_2) = \langle \varepsilon_1(\vec{r}_1, \omega_1) \varepsilon_1(\vec{r}_2, \omega_2) \rangle$$

$$= \frac{\omega_{p1}^4 / (\omega_1 \omega_2)^2}{(1 - \omega_{p0}^2 / \omega_1^2)(1 - \omega_{p0}^2 / \omega_2^2)} \exp\left(-\frac{(x_1 - x_2)^2}{\sigma_x^2} - \frac{(y_1 - y_2)^2}{\sigma_y^2} - \frac{(z_1 - z_2)^2}{\sigma_z^2}\right).$$

$$B_\varepsilon(\vec{\rho}, z) \approx A_{\omega_1, \omega_2}(\vec{\rho}) \delta z,$$

Integrated correlation function

$$A_{\omega_1, \omega_2}(\vec{\rho}) = \int_{-\infty}^{+\infty} B_\varepsilon(\vec{\rho}, z) dz = A_{\omega_1, \omega_2}(0) \exp\left(-\frac{(x_1 - x_2)^2}{\sigma_x^2} - \frac{(y_1 - y_2)^2}{\sigma_y^2}\right),$$

$$A_{\omega_1, \omega_2}(0) = \frac{\omega_{p1}^4 / (\omega_1 \omega_2)^2}{(1 - \omega_{p0}^2 / \omega_1^2)(1 - \omega_{p0}^2 / \omega_2^2)} \sqrt{\pi} \sigma_z,$$

Random phase shift correlation coefficients

$$\langle \phi_i \phi_j \rangle = \frac{H}{4} k_i k_j A_{\omega_i, \omega_j}(\vec{\rho})$$

Phase shift characteristic function
(averaged exponent of all the random phase shifts) is

$$M(\phi_1, \phi_2, \phi_3, \phi_4) = \langle \exp(i\phi_1 + i\phi_2 - i\phi_3 - i\phi_4) \rangle = \exp\left(-\frac{1}{2} \sum \lambda_{ij}\right)$$
$$M(\phi_1, \phi_2, \phi_3, \phi_4) = \sum_{\{n_{ij}\}} \prod_{i,j} \frac{\beta_{ij}^{n_{ij}}}{n_{ij}!} \exp\left(-\frac{n_{ij}(x_i - x_j)^2}{\sigma_x^2} - \frac{n_{ij}(y_i - y_j)^2}{\sigma_y^2} - \frac{n_{ij}(t_i - t_j)^2}{\tau^2}\right)$$

where

$$\beta_{ij} = -\frac{k_i k_j H}{4} A_{\omega_i \omega_j}(0)$$

(we perform the Taylor series expansion in the β_{ij})

Two frequency correlation function

$$\Gamma(\omega_I, \omega_{II}) = \left(\frac{k_I k_{II}}{z_1^2 2\pi 2z_2 \sqrt{\pi} L} \right)^2 \exp(-\beta_{22} - \beta_{44}) \sum_{\{n\}} \frac{\beta_{12}^{n_{12}}}{n_{12}!} \frac{\beta_{34}^{n_{12}}}{n_{34}!} \frac{\beta_{13}^{n_{13}}}{n_{13}!} \frac{\beta_{14}^{n_{14}}}{n_{14}!} \frac{\beta_{23}^{n_{23}}}{n_{23}!} \frac{\beta_{24}^{n_{24}}}{n_{24}!} \\ \int \exp \left(-A_{ij}^{(x)} x_i x_j \right) dx_1 \dots dx_6 \int \exp \left(-A_{ij}^{(y)} y_i y_j \right) dy_1 \dots dy_4$$

where

$$\int \exp \left(-A_{ij} x_i x_j \right) d^n x = \sqrt{\frac{\pi^n}{\det A_{ij}}}$$

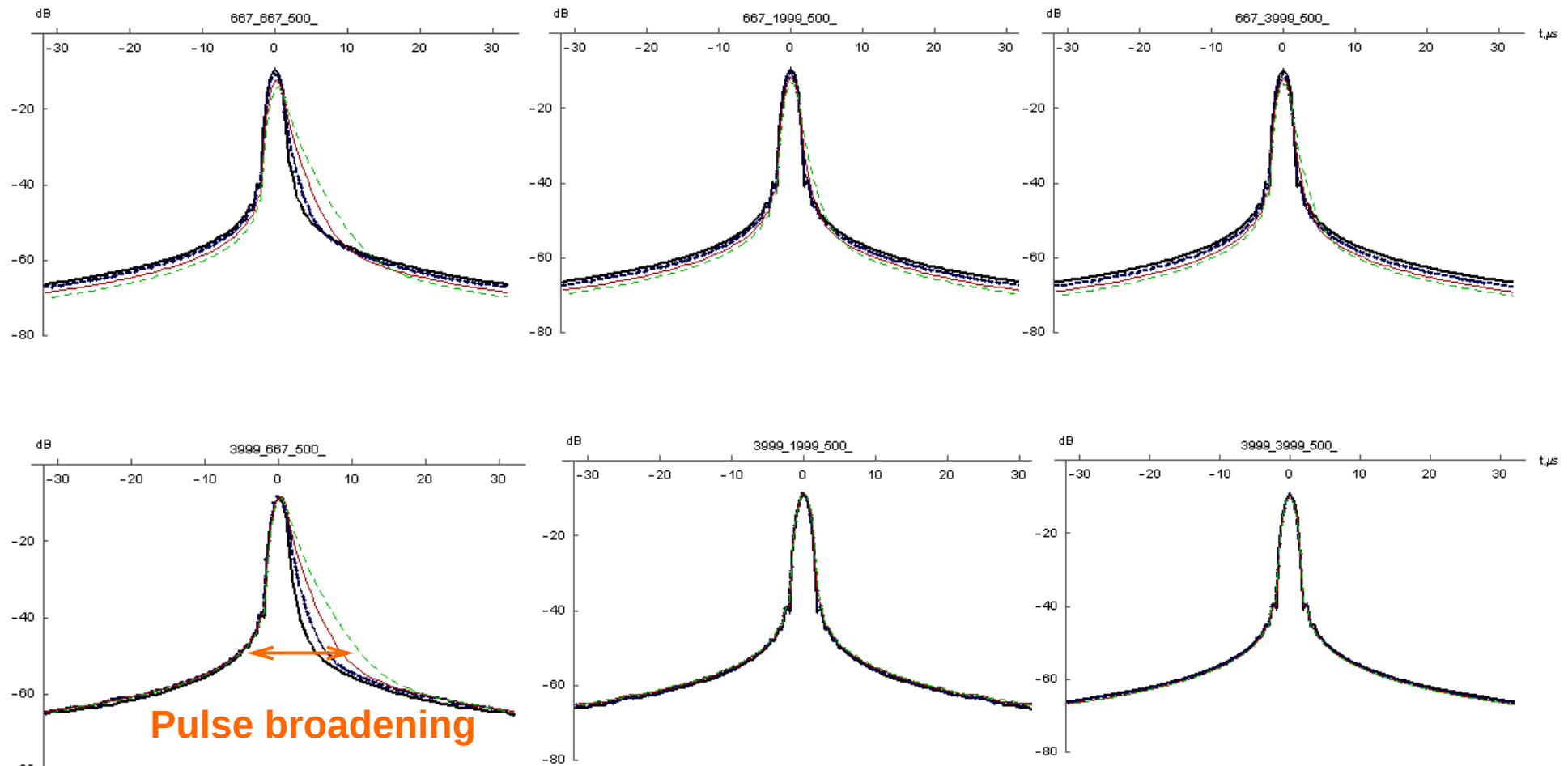
Matrices of the Gaussian integrals

$$A_{ij}^{(x)} = \begin{pmatrix} \frac{n_{12}+n_{13}+n_{14}}{\sigma_x^2} - \frac{ik_1(z_1+2z_2)}{4z_1z_2} & \frac{ik_1}{4z_2} - \frac{n_{12}}{\sigma_x^2} & -\frac{n_{13}}{\sigma_x^2} & -\frac{n_{14}}{\sigma_x^2} & \frac{ik_1}{2z_1} & 0 \\ \frac{ik_1}{4z_2} - \frac{n_{12}}{\sigma_x^2} & \frac{n_{12}+n_{23}+n_{24}}{\sigma_x^2} - \frac{ik_1(z_1+2z_2)}{4z_1z_2} & -\frac{n_{23}}{\sigma_x^2} & -\frac{n_{24}}{\sigma_x^2} & \frac{ik_1}{2z_1} & 0 \\ -\frac{n_{13}}{\sigma_x^2} & -\frac{n_{23}}{\sigma_x^2} & \frac{n_{13}+n_{23}+n_{34}}{\sigma_x^2} + \frac{ik_2(z_1+2z_2)}{4z_1z_2} & -\frac{ik_2}{4z_2} - \frac{n_{34}}{\sigma_x^2} & 0 & -\frac{ik_2}{2z_1} \\ -\frac{n_{14}}{\sigma_x^2} & -\frac{n_{24}}{\sigma_x^2} & -\frac{ik_2}{4z_2} - \frac{n_{34}}{\sigma_x^2} & \frac{n_{14}+n_{24}+n_{34}}{\sigma_x^2} + \frac{ik_2(z_1+2z_2)}{4z_1z_2} & 0 & -\frac{ik_2}{2z_1} \\ \frac{ik_1}{2z_1} & \frac{ik_1}{2z_1} & 0 & 0 & \frac{1}{L^2} - \frac{ik_1}{z_1} & 0 \\ 0 & 0 & -\frac{ik_2}{2z_1} & -\frac{ik_2}{2z_1} & 0 & \frac{ik_2}{z_1} + \frac{1}{L^2} \end{pmatrix}$$

Matrices of the Gaussian integrals

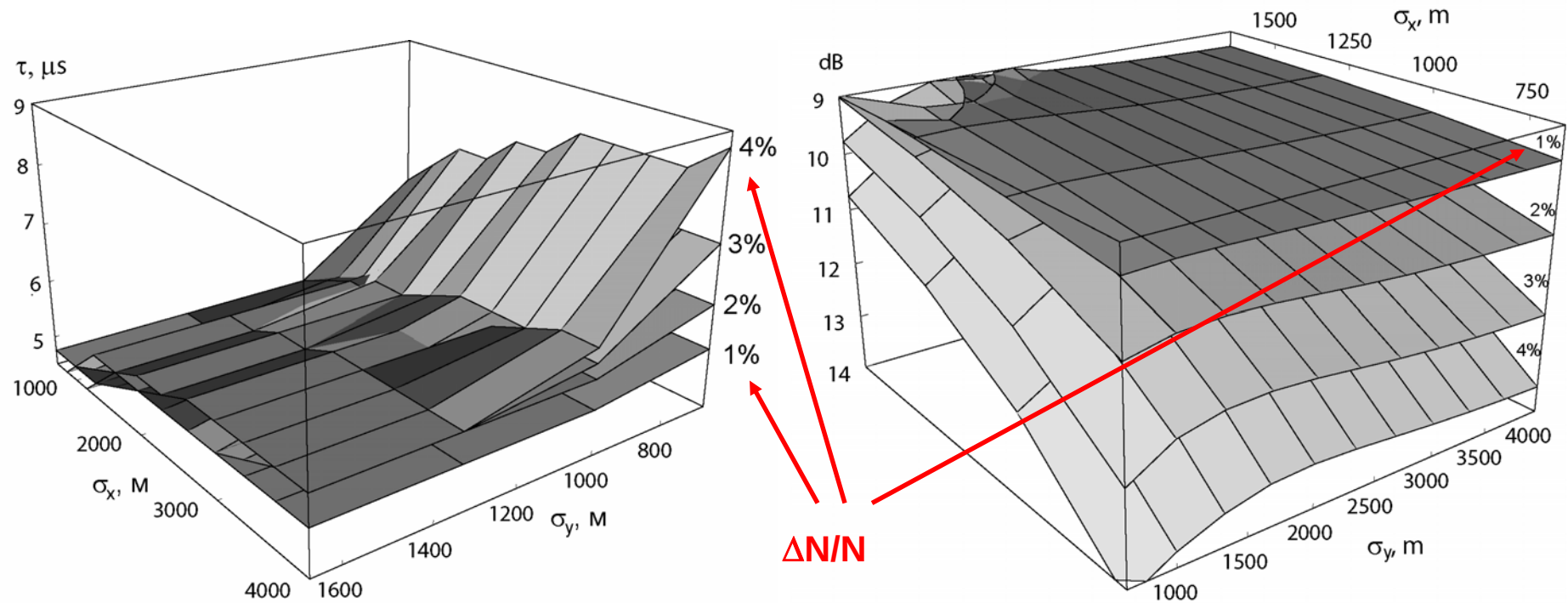
$$A_{ij}^{(y)} = \begin{pmatrix} \frac{n_{12}+n_{13}+n_{14}}{\sigma_y^2} & \frac{ik_1}{4z_2} - \frac{n_{12}}{\sigma_y^2} & -\frac{n_{13}}{\sigma_y^2} & -\frac{n_{14}}{\sigma_y^2} \\ -\frac{ik_1(z_1+2z_2)}{4z_1z_2} & \frac{n_{12}+n_{23}+n_{24}}{\sigma_y^2} & -\frac{n_{23}}{\sigma_y^2} & -\frac{n_{24}}{\sigma_y^2} \\ \frac{ik_1}{4z_2} - \frac{n_{12}}{\sigma_y^2} & -\frac{ik_1(z_1+2z_2)}{4z_1z_2} & -\frac{n_{23}}{\sigma_y^2} & -\frac{n_{24}}{\sigma_y^2} \\ -\frac{n_{13}}{\sigma_y^2} & -\frac{n_{23}}{\sigma_y^2} & \frac{n_{13}+n_{23}+n_{34}}{\sigma_y^2} + \frac{ik_2(z_1+2z_2)}{4z_1z_2} & -\frac{ik_2}{4z_2} - \frac{n_{34}}{\sigma_y^2} \\ -\frac{n_{14}}{\sigma_y^2} & -\frac{n_{24}}{\sigma_y^2} & -\frac{ik_2}{4z_2} - \frac{n_{34}}{\sigma_y^2} & \frac{n_{14}+n_{24}+n_{34}}{\sigma_y^2} + \frac{ik_2(z_1+2z_2)}{4z_1z_2} \end{pmatrix}$$

Anisotropic ionospheric fluctuations: degradation and broadening of compressed UWB LFM signals



Black, blue, red and green color correspond to plasma density fluctuation levels $\Delta N/N$ 1%, 2%, 3% and 4% respectively

Anisotropic ionospheric fluctuations: degradation and broadening of compressed UWB LFM signals



Broadening of the compressed
UWB LFM signals' peaks

Degradation of the amplitude of the
compressed UWB LFM signals

Non-stationary ionospheric fluctuations (scintillations)

$$B_{\varepsilon}(\vec{r}_1, \vec{r}_2) = \langle \varepsilon_1(\vec{r}_1, \omega_1) \varepsilon_1(\vec{r}_2, \omega_2) \rangle$$

$$= \frac{\omega_{p1}^4 / (\omega_1 \omega_2)^2}{(1 - \omega_{p0}^2 / \omega_1^2)(1 - \omega_{p0}^2 / \omega_2^2)} \exp\left(-\frac{(x_1 - x_2)^2}{\sigma_x^2} - \frac{(y_1 - y_2)^2}{\sigma_y^2} - \frac{(z_1 - z_2)^2}{\sigma_z^2} - \frac{(t_1 - t_2)^2}{\tau^2}\right)$$

$$B_{\varepsilon}(\vec{\rho}, z) \approx A_{\omega_1, \omega_2}(\vec{\rho}) \delta z$$

$$A_{\omega_1, \omega_2}(\vec{\rho}) = \int_{-\infty}^{+\infty} B_{\varepsilon}(\vec{\rho}, z) dz = A_{\omega_1, \omega_2}(0) \exp\left(-\frac{(x_1 - x_2)^2}{\sigma_x^2} - \frac{(y_1 - y_2)^2}{\sigma_y^2} - \frac{(t_1 - t_2)^2}{\tau^2}\right)$$

$$A_{\omega_1, \omega_2}(0) = \frac{\omega_{p1}^4 / (\omega_1 \omega_2)^2}{(1 - \omega_{p0}^2 / \omega_1^2)(1 - \omega_{p0}^2 / \omega_2^2)} \sqrt{\pi} \sigma_z$$

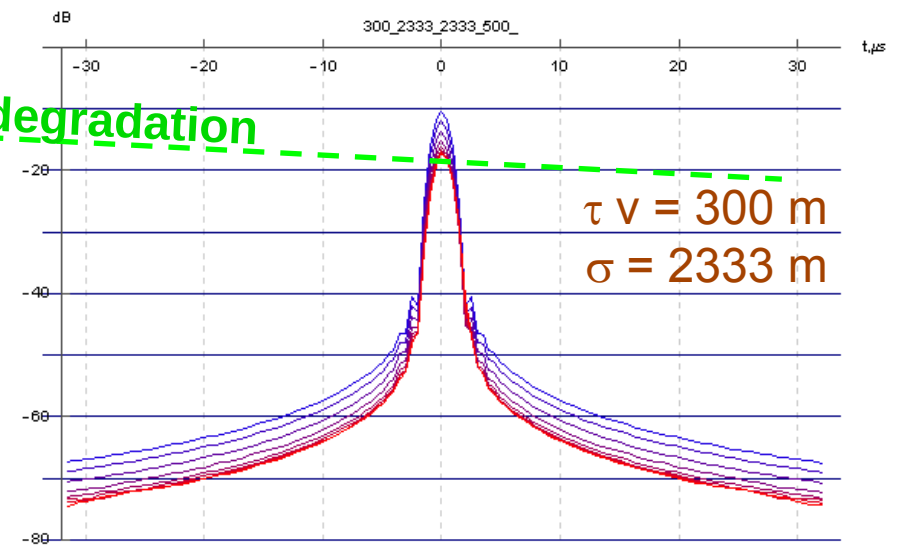
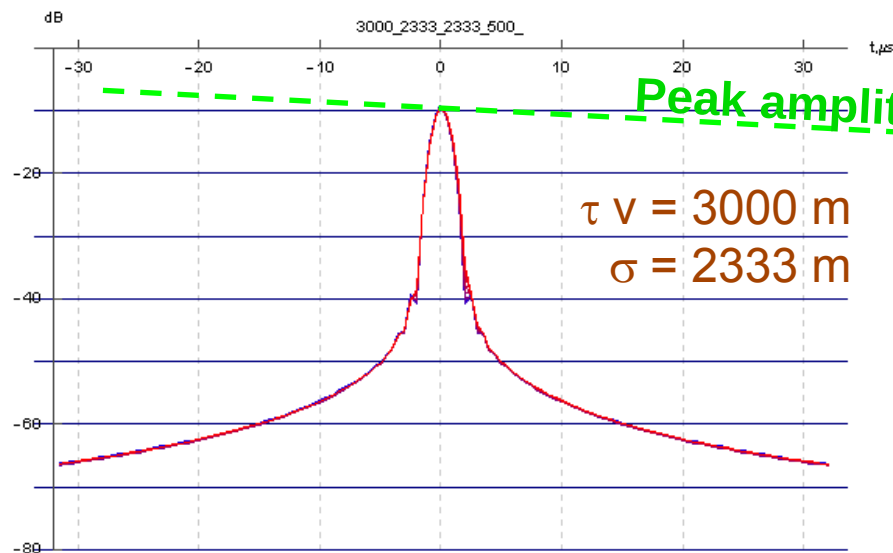
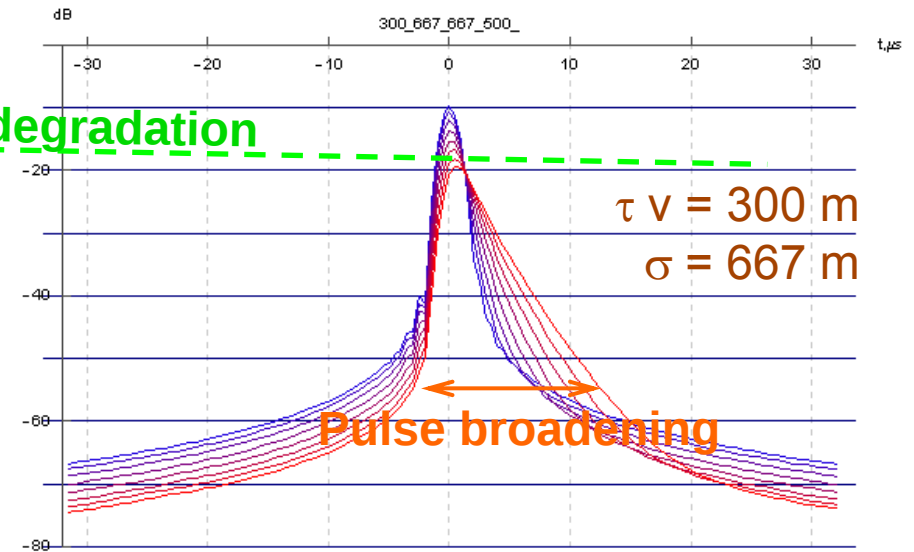
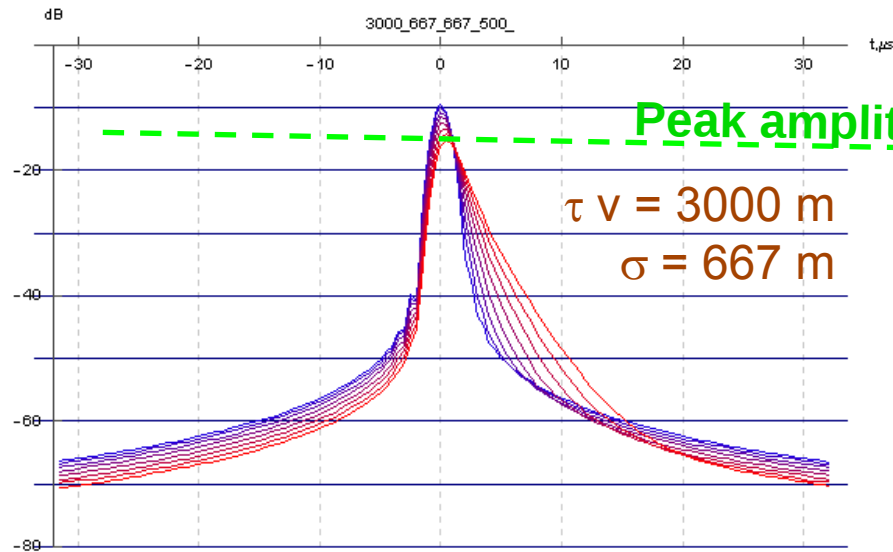
Non-stationary fluctuations

Non-stationary ionospheric fluctuations: Gaussian integrals matrices

$$A_{ij}^{(x)} = \begin{pmatrix} \frac{n_{12}+n_{13}+n_{14}}{\sigma_x^2} - \frac{ik_1(z_1+2z_2)}{4z_1z_2} & \frac{ik_1}{4z_2} - \frac{n_{12}}{\sigma_x^2} & -\frac{n_{13}}{\sigma_x^2} & -\frac{n_{14}}{\sigma_x^2} & \frac{ik_1}{2z_1} & 0 \\ \frac{ik_1}{4z_2} - \frac{n_{12}}{\sigma_x^2} & \frac{n_{12}+n_{23}+n_{24}}{\sigma_x^2} - \frac{ik_1(z_1+2z_2)}{4z_1z_2} & -\frac{n_{23}}{\sigma_x^2} & -\frac{n_{24}}{\sigma_x^2} & \frac{ik_1}{2z_1} & 0 \\ -\frac{n_{13}}{\sigma_x^2} & -\frac{n_{23}}{\sigma_x^2} & \frac{n_{13}+n_{23}+n_{34}}{\sigma_x^2} + \frac{ik_2(z_1+2z_2)}{4z_1z_2} & -\frac{ik_2}{4z_2} - \frac{n_{34}}{\sigma_x^2} & 0 & -\frac{ik_2}{2z_1} \\ -\frac{n_{14}}{\sigma_x^2} & -\frac{n_{24}}{\sigma_x^2} & -\frac{ik_2}{4z_2} - \frac{n_{34}}{\sigma_x^2} & \frac{n_{14}+n_{24}+n_{34}}{\sigma_x^2} + \frac{ik_2(z_1+2z_2)}{4z_1z_2} & 0 & -\frac{ik_2}{2z_1} \\ \frac{ik_1}{2z_1} & \frac{ik_1}{2z_1} & 0 & 0 & \frac{1}{L^2} - \frac{ik_1}{z_1} + \frac{n_{13}+n_{14}}{\tau^2 v^2} + \frac{n_{23}+n_{24}}{\tau^2 v^2} & -\frac{n_{13}+n_{14}}{\tau^2 v^2} - \frac{n_{23}+n_{24}}{\tau^2 v^2} \\ 0 & 0 & -\frac{ik_2}{2z_1} & -\frac{ik_2}{2z_1} & -\frac{n_{13}+n_{14}}{\tau^2 v^2} - \frac{n_{23}+n_{24}}{\tau^2 v^2} & \frac{ik_2}{z_1} + \frac{1}{L^2} + \frac{n_{13}+n_{14}}{\tau^2 v^2} + \frac{n_{23}+n_{24}}{\tau^2 v^2} \end{pmatrix}$$

Additional terms due to non-stationary effects

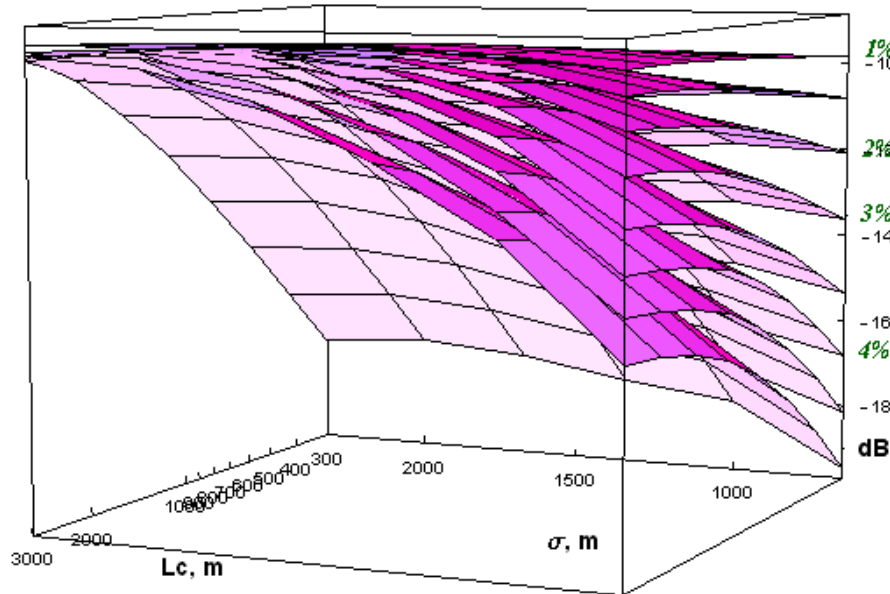
Degradation of the compressed LFM UWB signals due to non-stationary ionospheric scintillations



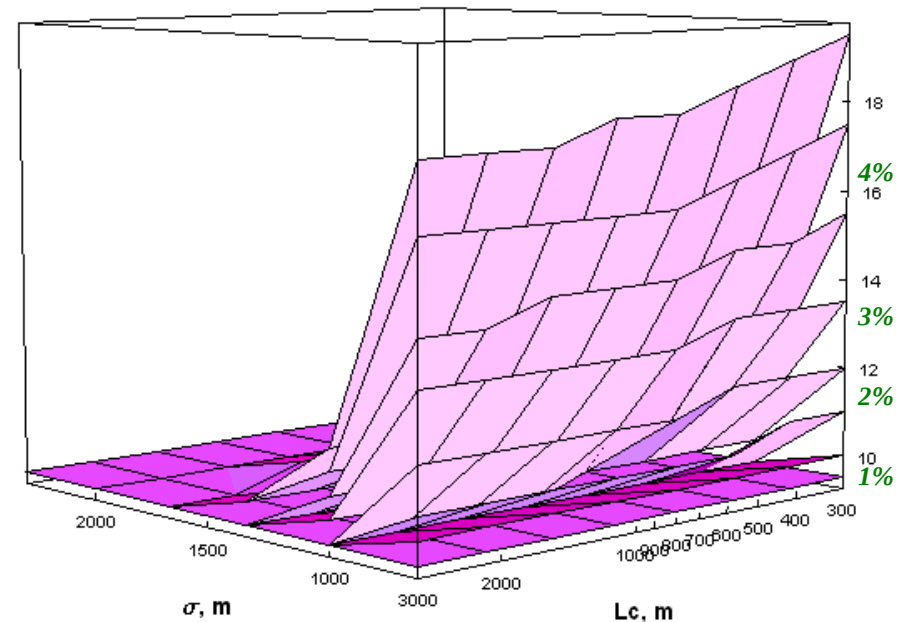
Colors varying from blue to red correspond to increasing plasma fluctuation level $\Delta N/N = 1\% \dots 4\%$

Non-stationary ionospheric scintillations: degradation and broadening of compressed LFM signals

Correlation function of the plasma inhomogeneities is assumed to be isotropic ($\sigma_x = \sigma_y = \sigma$). Fluctuation levels $\Delta N/N$ are marked by green labels. The peak amplitude is affected both by σ and L_c while only σ is responsible for the peak broadening.



Peak amplitude degradation



Pulse broadening at ~ -50 dB
(correction for the amplitude
degradation is applied)

$L_c = \tau v$ - non-stationary correlation length (distance traveled by the spacecraft during the characteristic period of the scintillations)

Conclusions and remarks

- The impact of the stochastic small-scale irregular structure of the ionosphere on the performance of the orbital ground-penetrating synthetic aperture radar (SAR) instrument is considered.
- Several numerical models for the computer simulations of the orbital ground-penetrating SAR experiment have been implemented, tested and exploited.
- Different effects, caused by the plasma irregularities and surface roughness, have been revealed and estimated numerically.
- Applicability of the results to the GPR sounding data validation and to the experimental radar studies of the ionospheric irregularities has been discussed.

Thank you for your attention!



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