Multiple Phase Screen Calculation of Wide Bandwidth Propagation

Dennis L. Knepp
L. J. Nickisch

NorthWest Research Associates
Monterey, California

2008 URSI General Assembly
Chicago, August 2008

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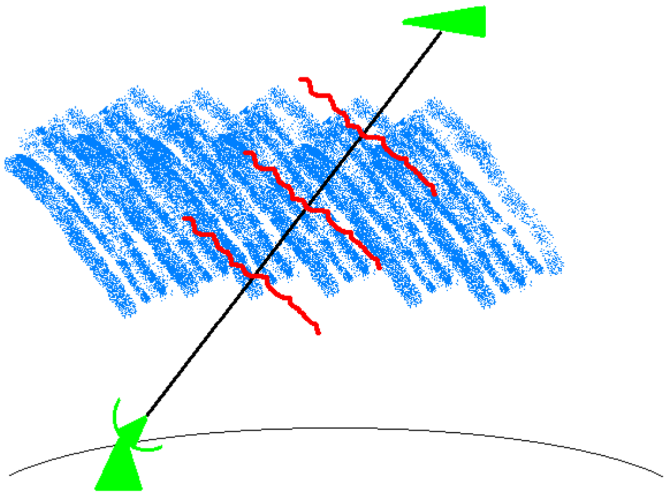
Outline

- Summary of MPS technique
- Examples of scattering by a large cloud of structured ionization
- Comparison of a calculation of the average generalized power spectrum using the MPS technique and Nickisch's phase-screen diffraction method (PDM)

Introduction

- **Multiple phase screen (MPS) calculations have long been useful for calculation of ionospheric scintillation**
  - **Study the effects of turbulence on EM propagation**
  - **Help understand ionospheric sensor measurements and how they relate to ionospheric physical properties**
  - **Provide realizations of received signals for receiver design**
- **We utilize an updated version of an MPS code first written in the 1980s**
  - **New code has better handling of time domain results**
  - **Several useful animations of transfer function and impulse response function**

MPS Solution Technique



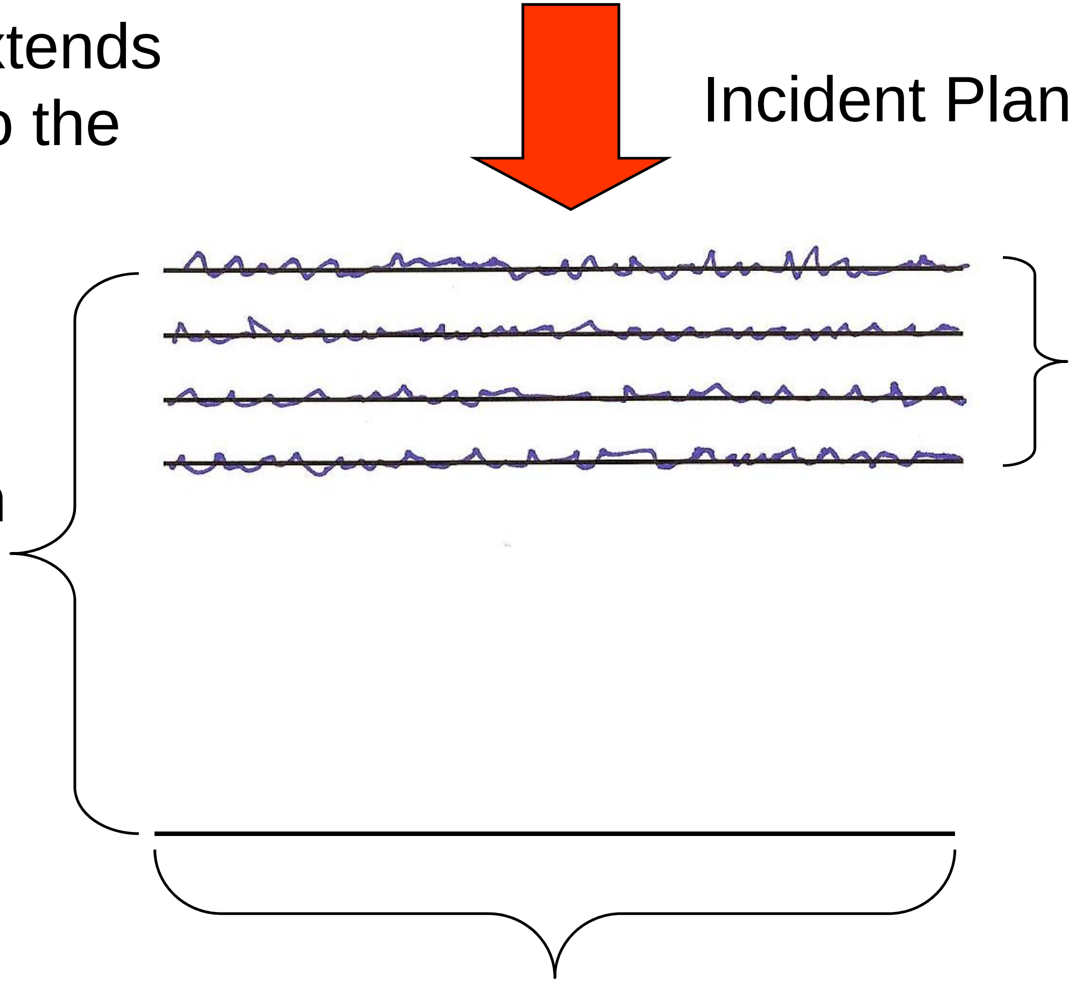
- Parabolic Wave Equation for E-field

$$-2ik\frac{\partial U(\vec{r})}{\partial x} = \underbrace{\nabla_{\perp}^{2} U(\vec{r})}_{\text{Diffraction term}} + \underbrace{k^{2}\varepsilon_{1}(\vec{r})U(\vec{r})}_{\text{Source term}}$$

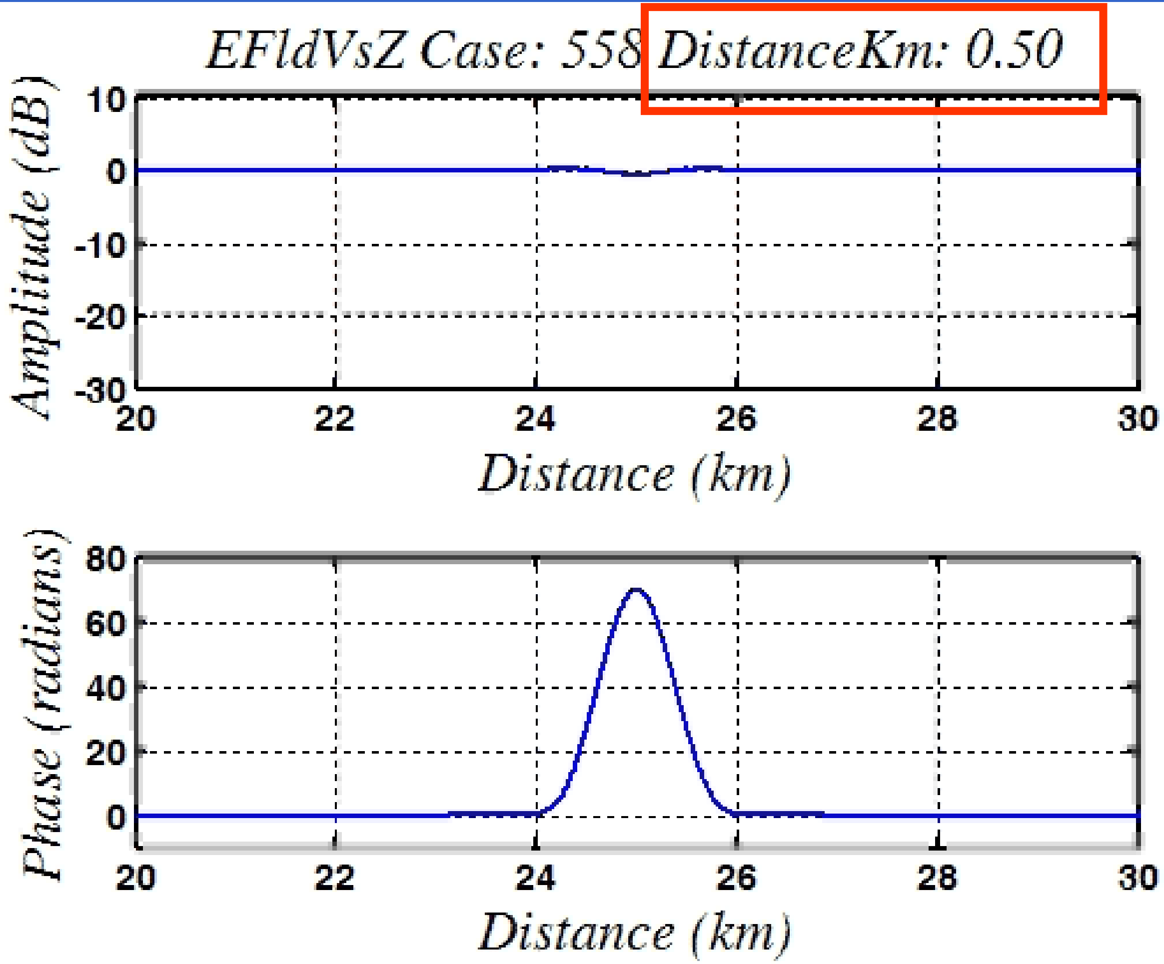
Solution Method:

- Collapse ionospheric structure into multiple thin phase-changing screens with free-space between
- At phase screen, neglect diffraction term
- Between screens, the parabolic wave equation is source free, solve by Fourier Transform method
- Perform MPS calculation for all frequency components in the signal bandwidth

MPS Propagation Geometry



Scattering From a Strong Gaussian Defocusing Lens



Propagation distance

- **Defocusing lens is a Gaussian with 500 m scale size**
- **2^{19} = 524,288 points along MPS grid, no small scale structure**

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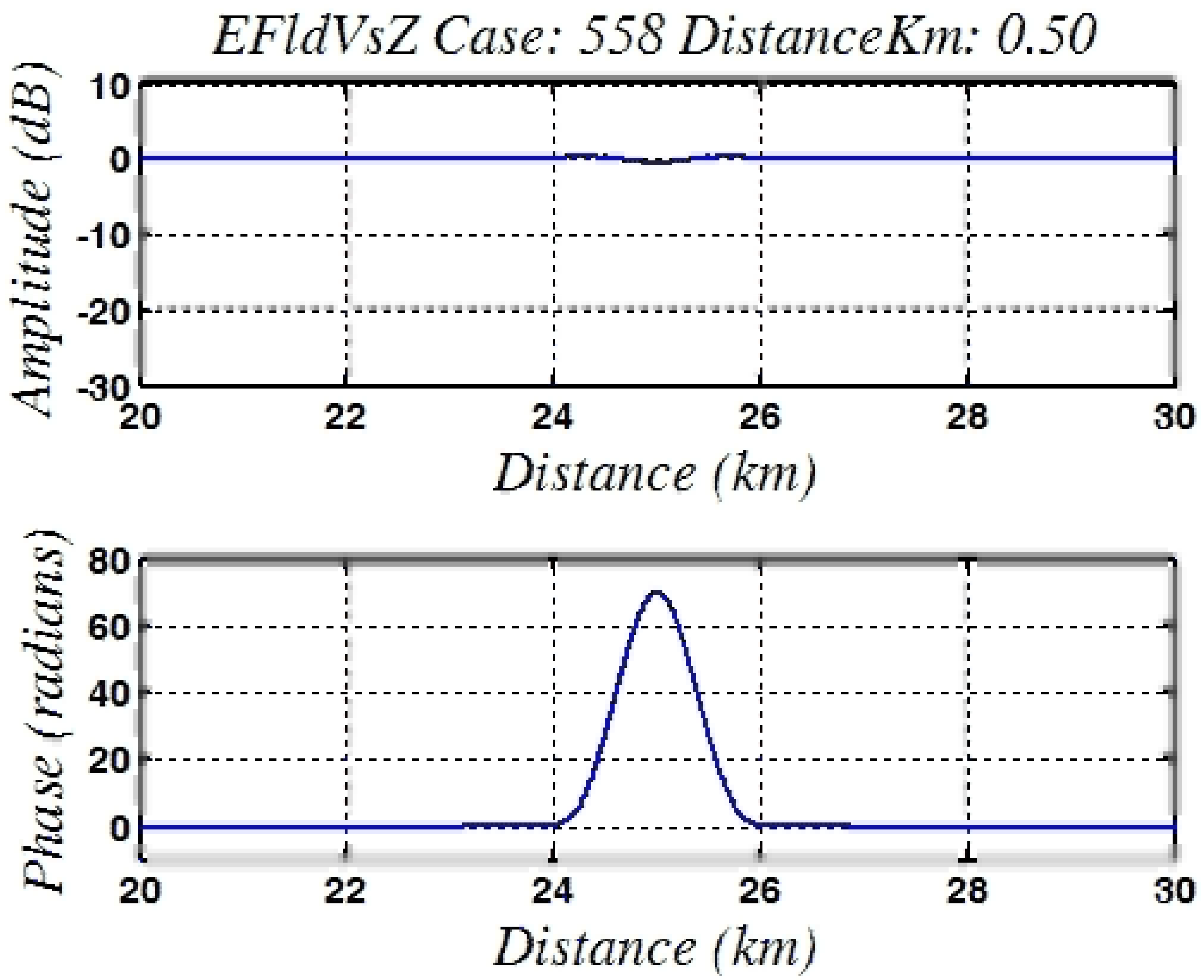
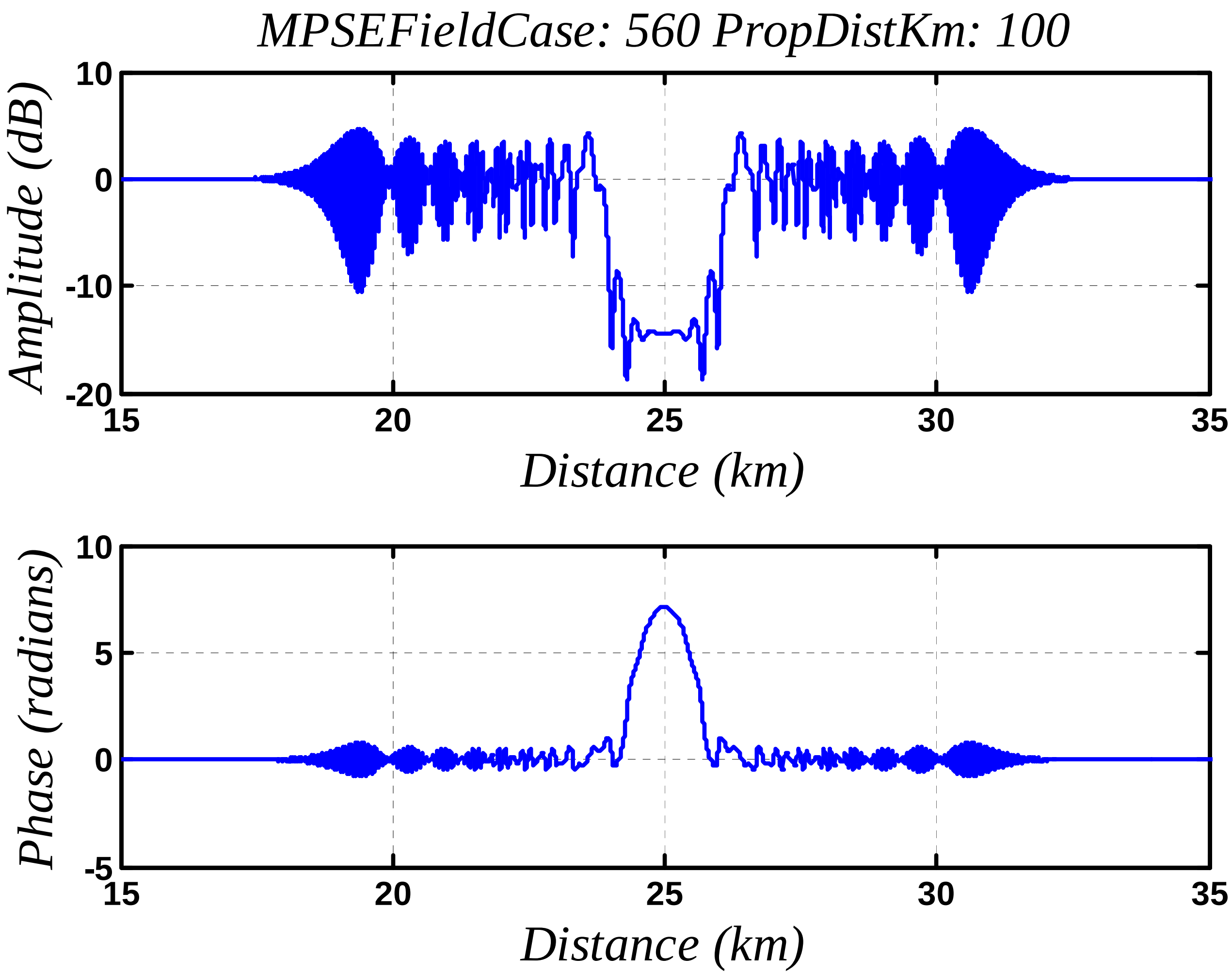
Strong Gaussian Defocusing Lens
Amplitude and Phase vs Propagation Distance



Strong Gaussian Defocusing Lens
After Propagating 100 km



- **Close up of last frame of previous movie**

Time Domain Calculation

Triangular time domain waveform:

$$p(\tau) = \left\{ \begin{array}{ll} 1 - \tau/T_c & -T_c \leq \tau \leq 0 \\ 1 + \tau/T_c & 0 \leq \tau \leq T_c \end{array} \right\}$$

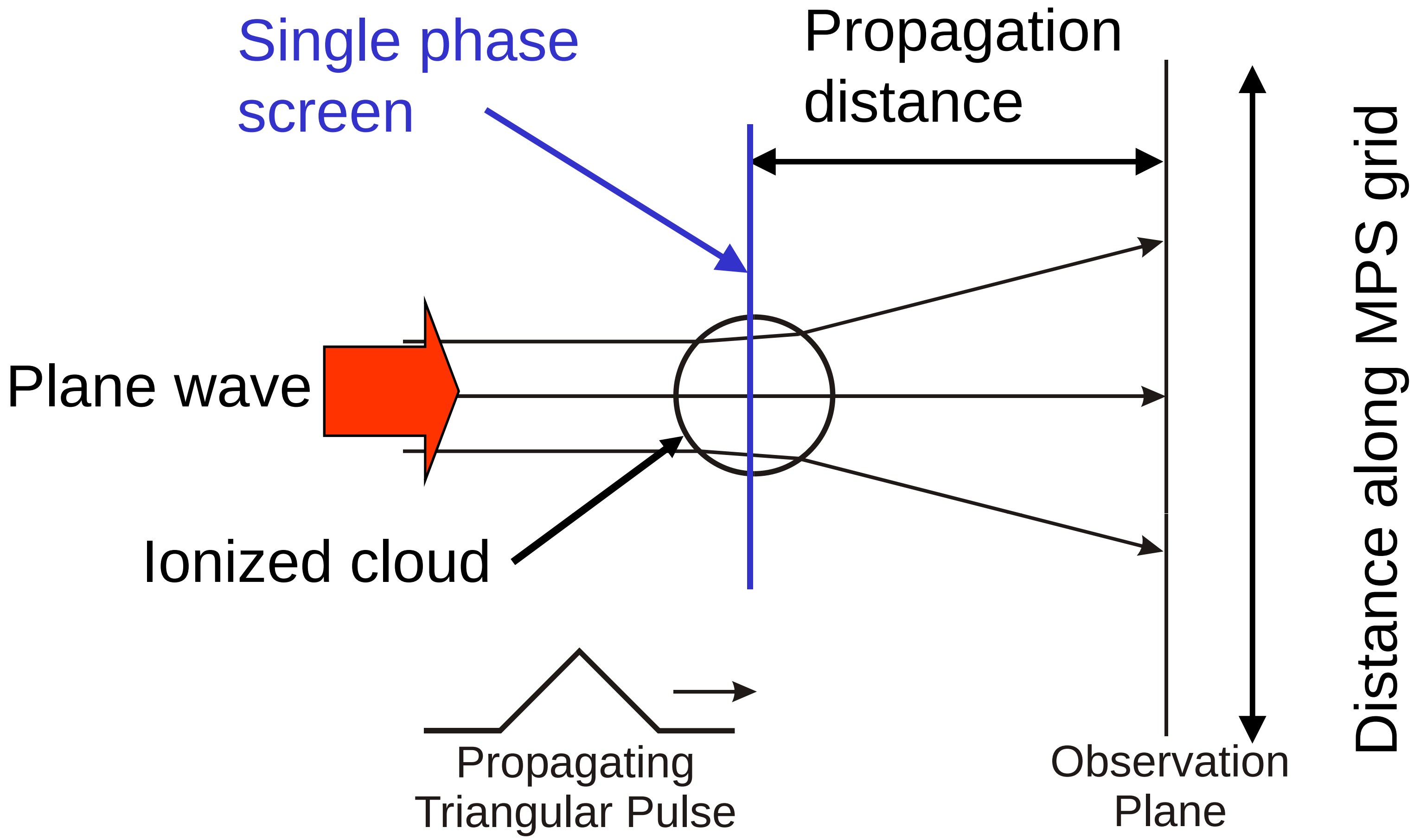
Fourier transform:

$$\begin{aligned} P(f) &= \int p(\tau) \exp(-i2\pi f \tau) d\tau \\ &= T_c \frac{\sin^2(\pi f T_c)}{(\pi f T_c)^2} \end{aligned}$$

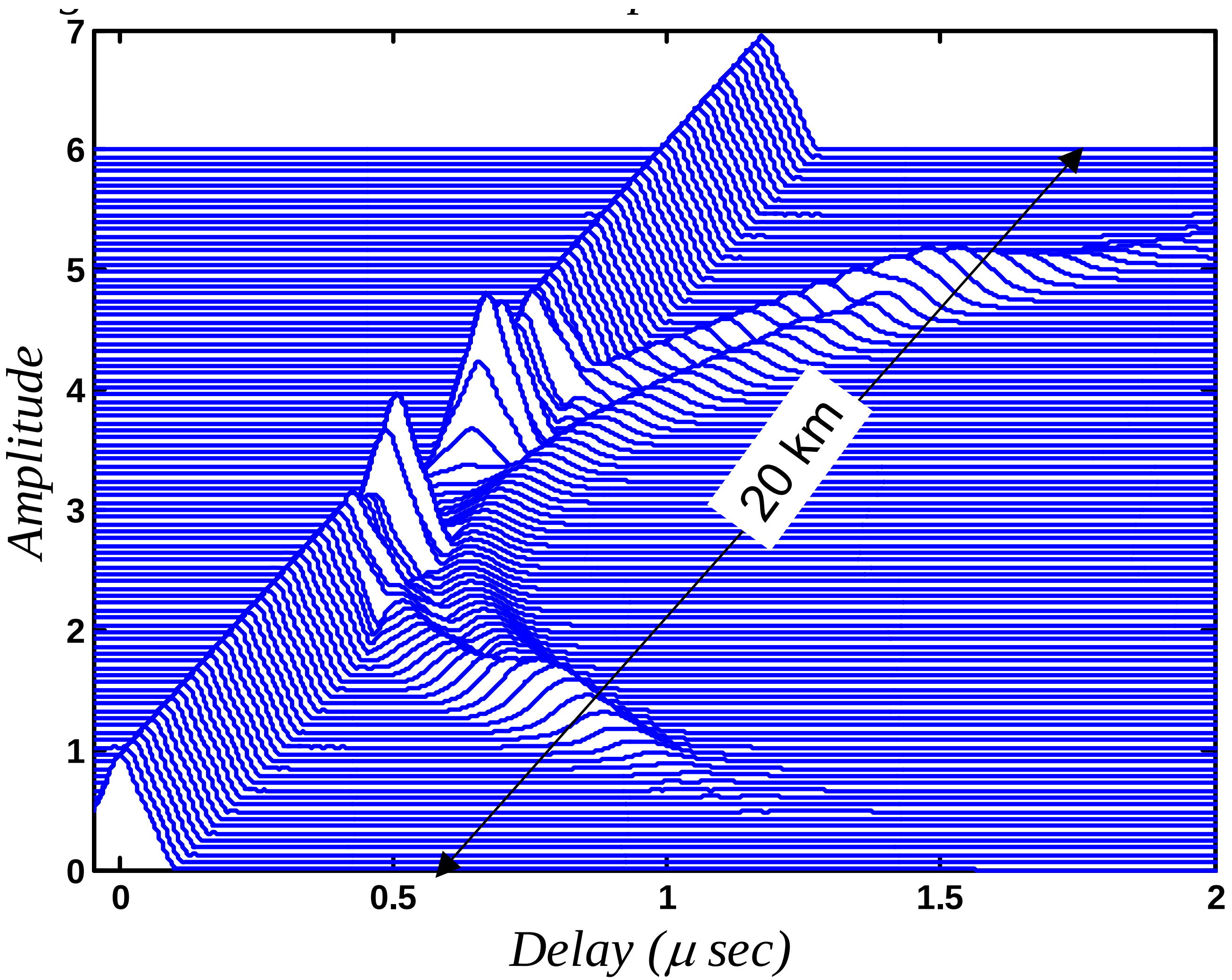
MPS calculation:

T_c = 100 nsec
Bandwidth = 20 MHz
No of frequency components = 64
Carrier frequency = 100 MHz

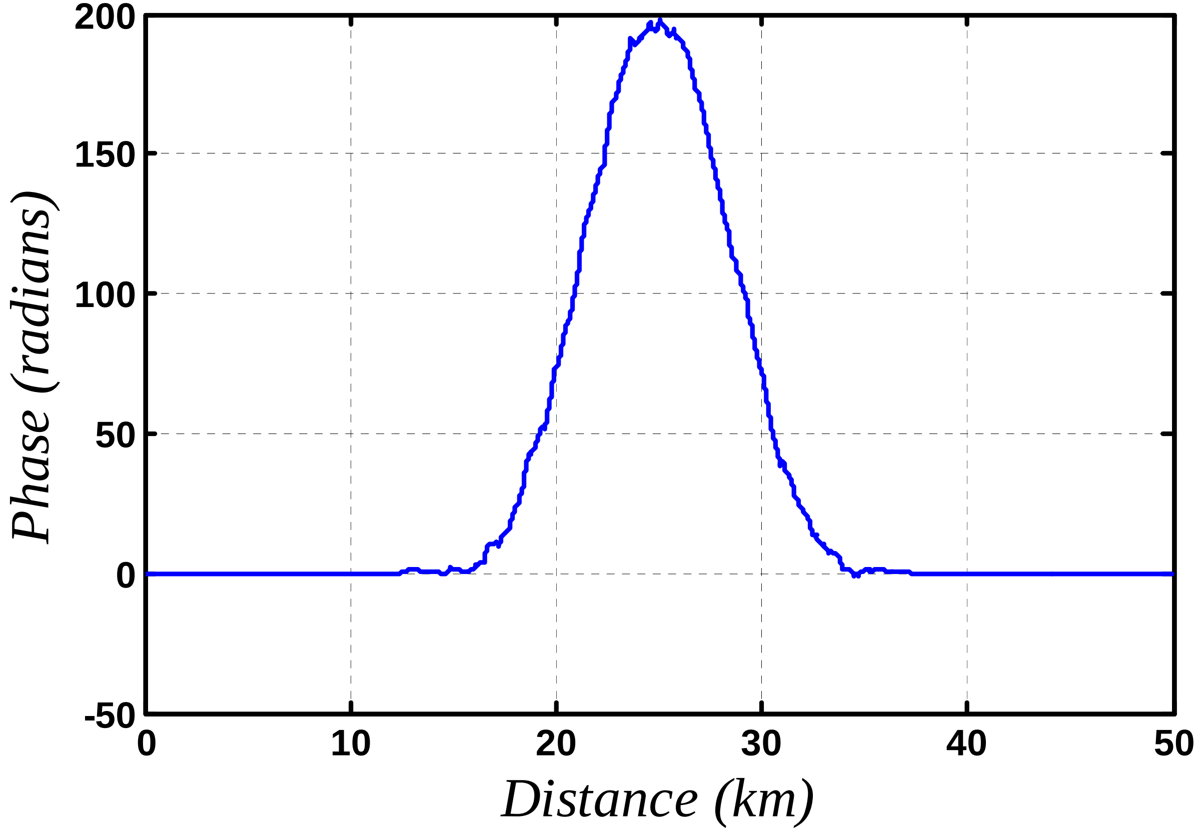
Geometry of Ionized Cloud



Time Domain Signal After Propagating 100 km Past the Strong Gaussian Lens



Single Phase-Screen Representation of a Large Structured Ionized Cloud



Cloud is a large smooth Gaussian with structure added
Structure has a q^{-3} power-law power spectral density
Outer scale = 390 m, Inner scale = 10 m, 2^{19} points
Phase standard deviation = 4 radians

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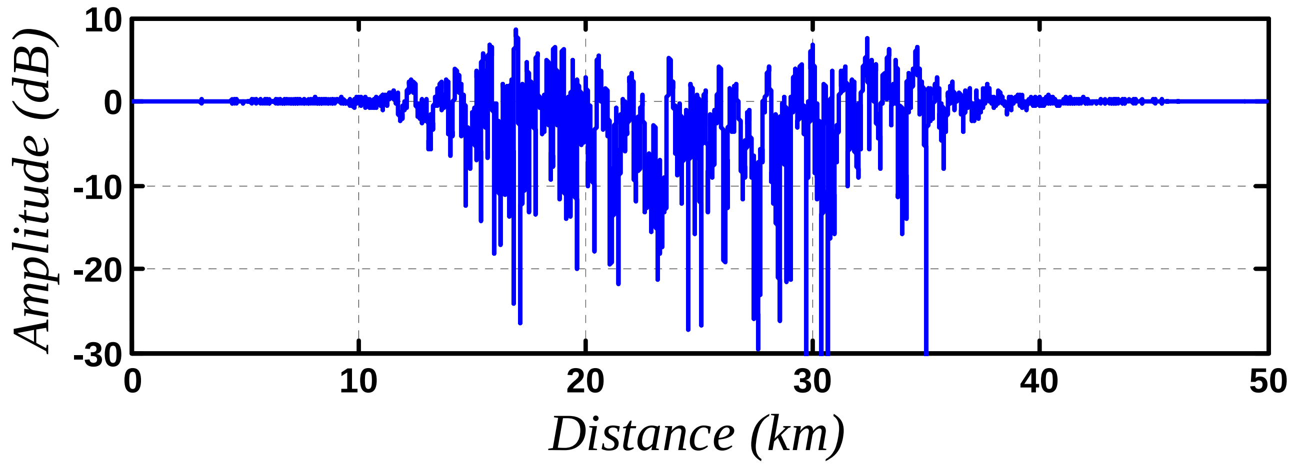
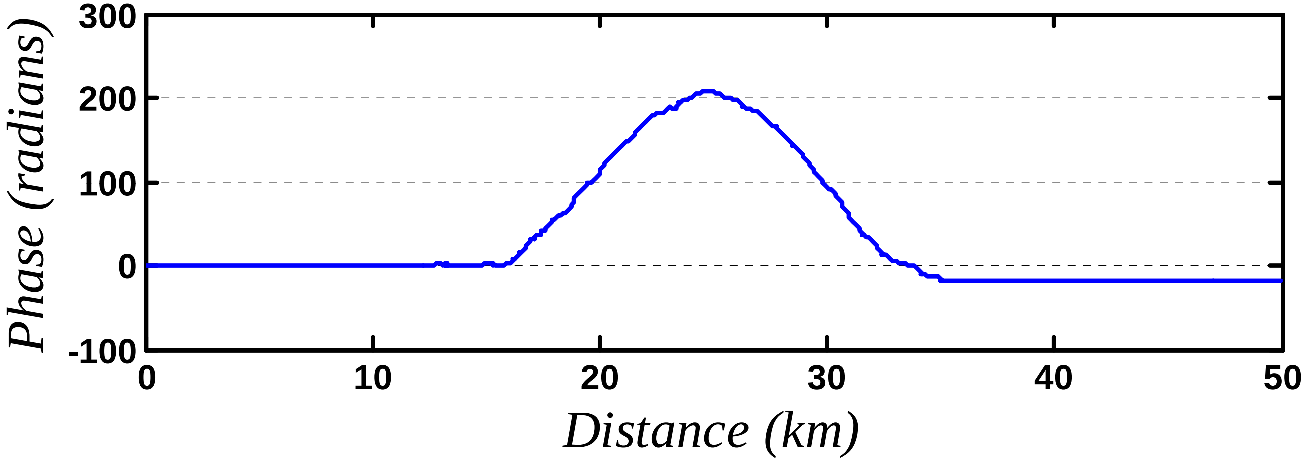
Amplitude and Phase at the Carrier Frequency after Propagating 100 km – Stuctured Cloud





Propagation distance = 100 km
Transmission frequency = 100 MHz

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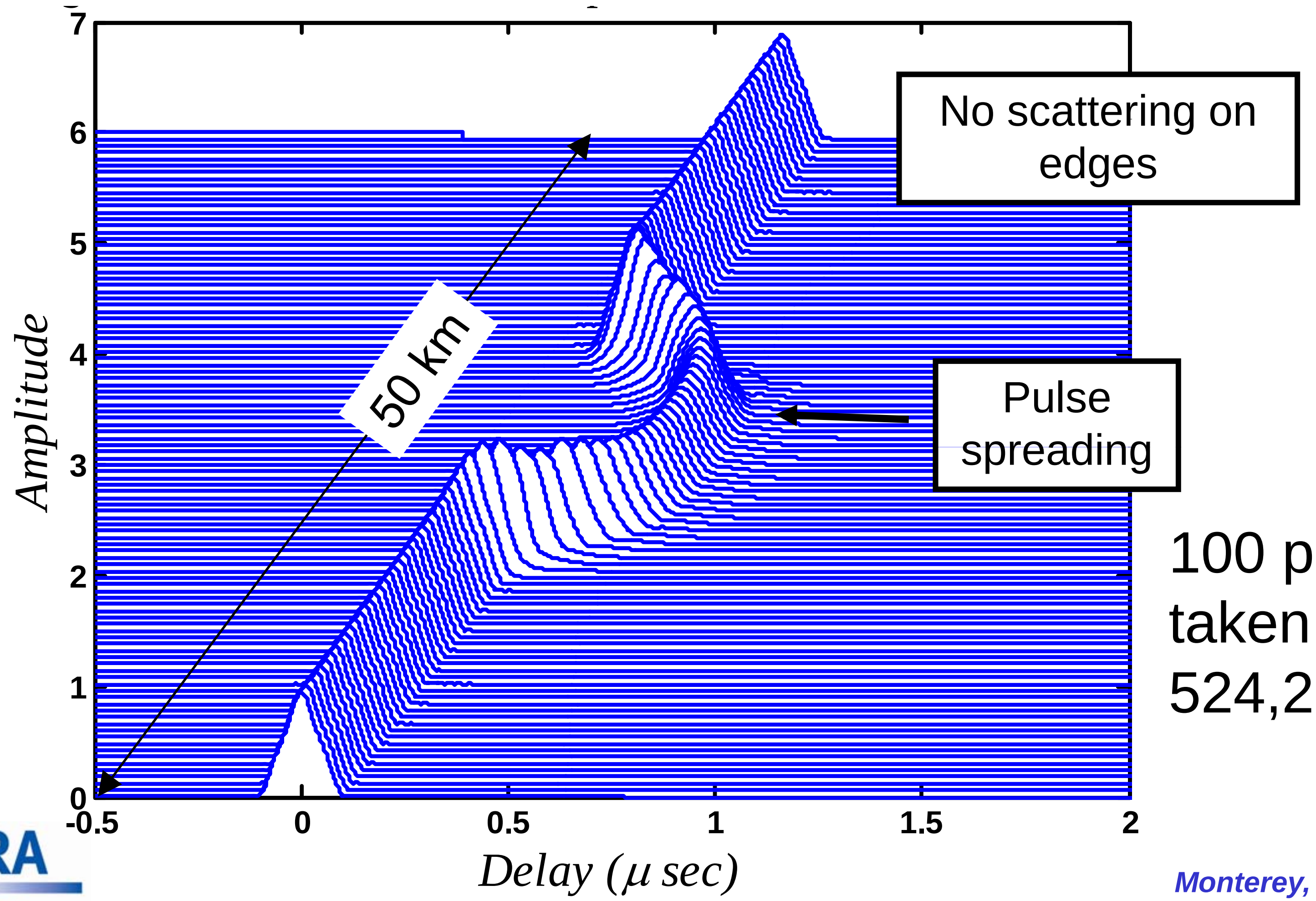
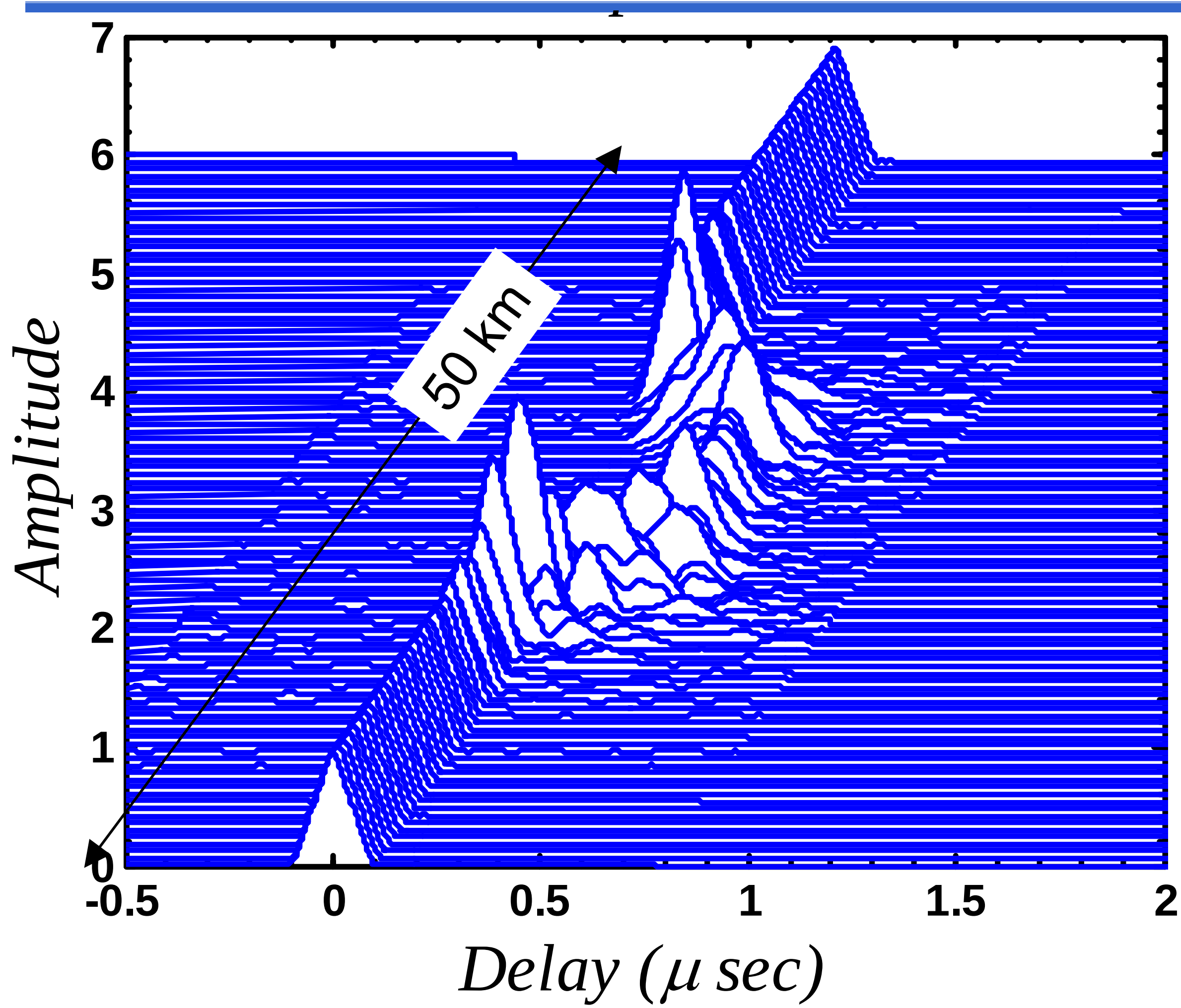
Scattering from the Large Gaussian Cloud With No Structure

Signal experiences delay, focusing, and dispersion after propagating a distance of 100 km past the screen



100 points taken from 524,288

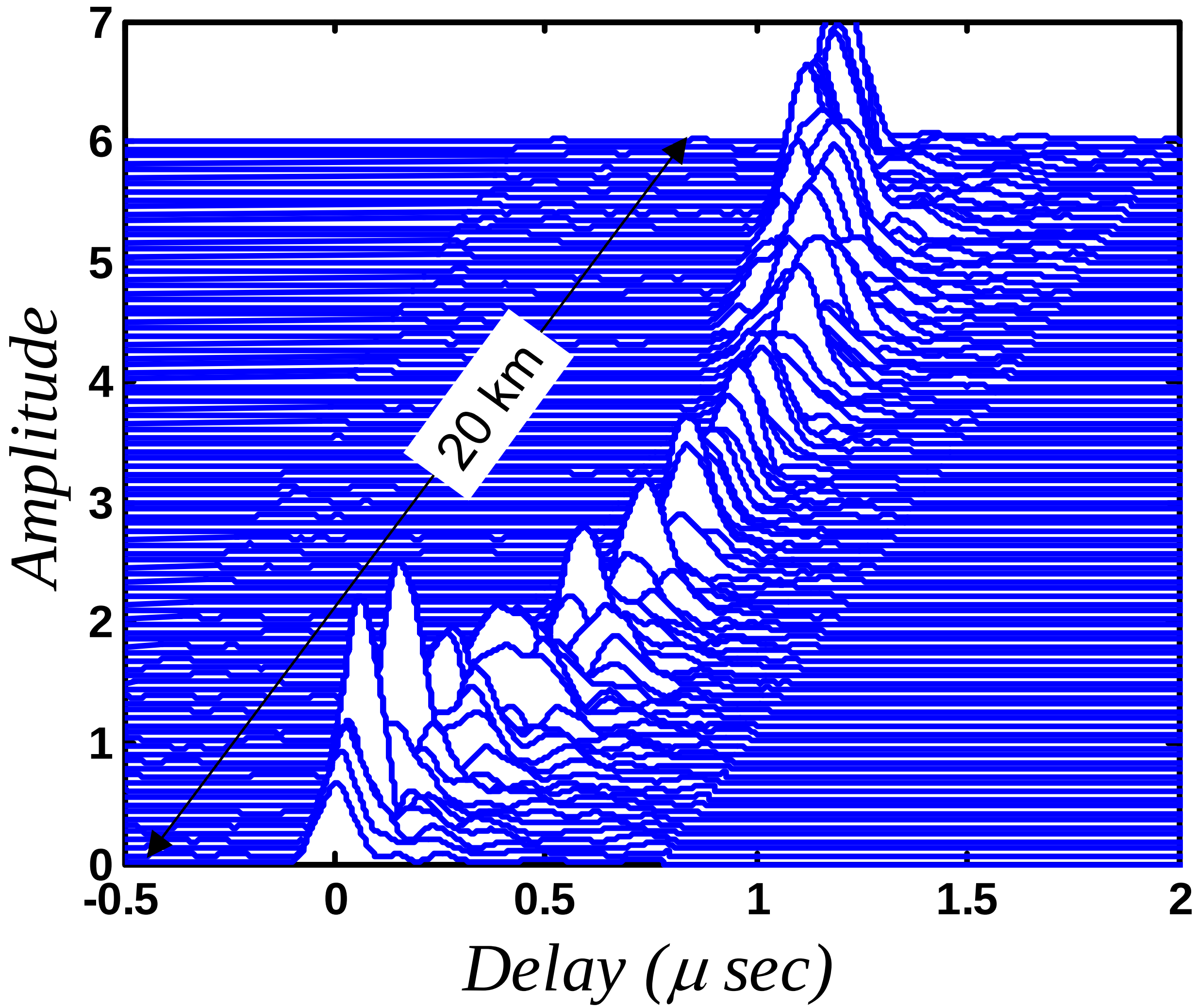
Time Domain Signal After Propagating a Distance of 100 km – Structured Cloud



100 points taken from 524,288

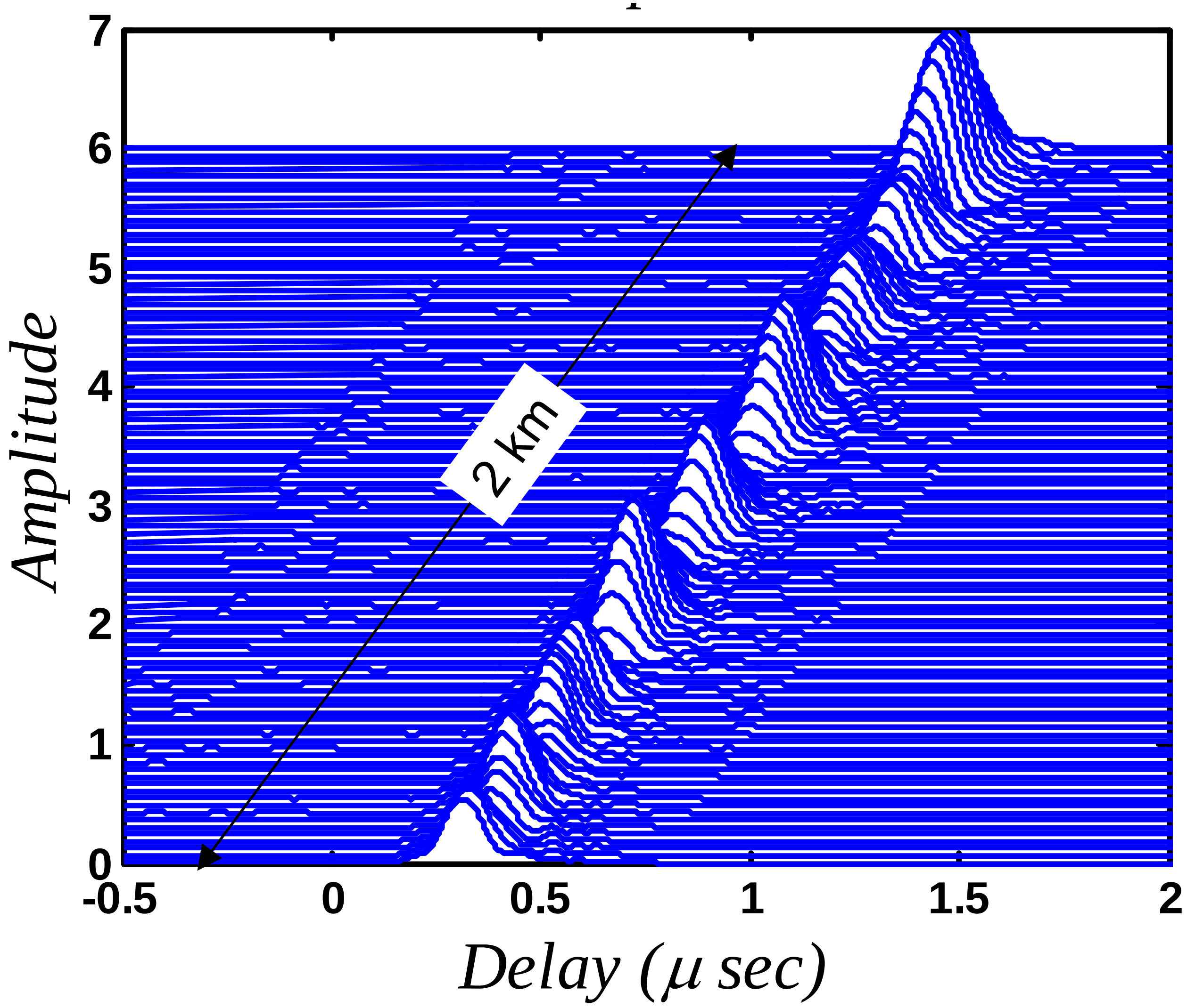
Ionization structure "adds" amplitude and phase scintillation to the previous figure

Close-Up of Center of the Previous Figure



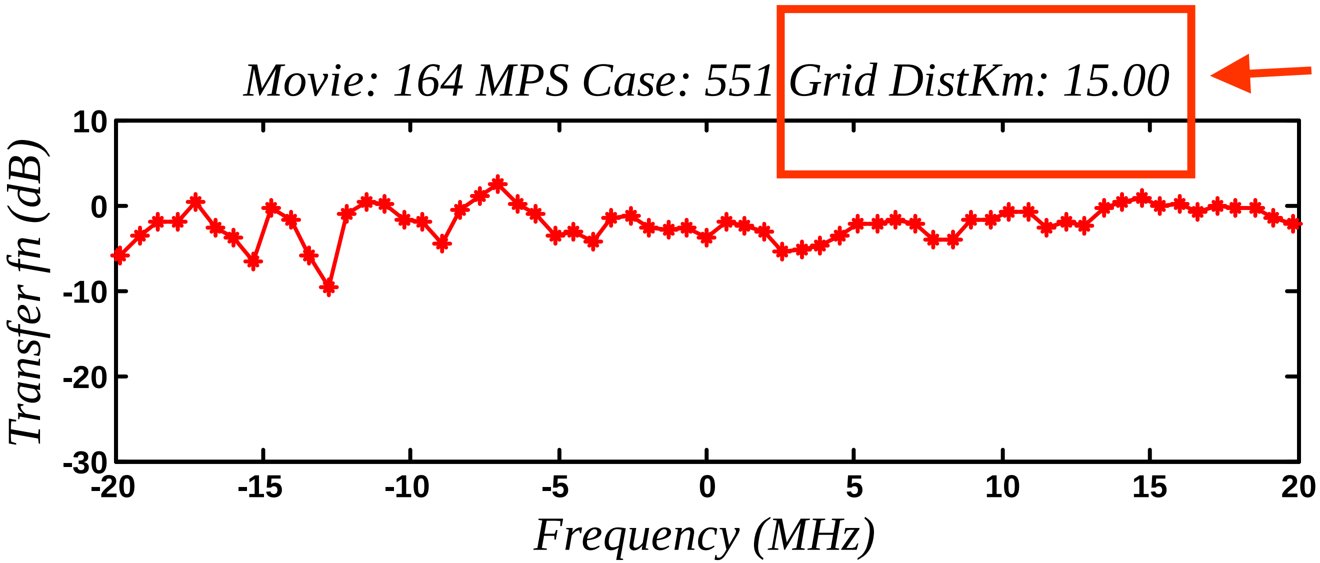
100 points taken from 524,288

Close-up of Center of Previous Figure



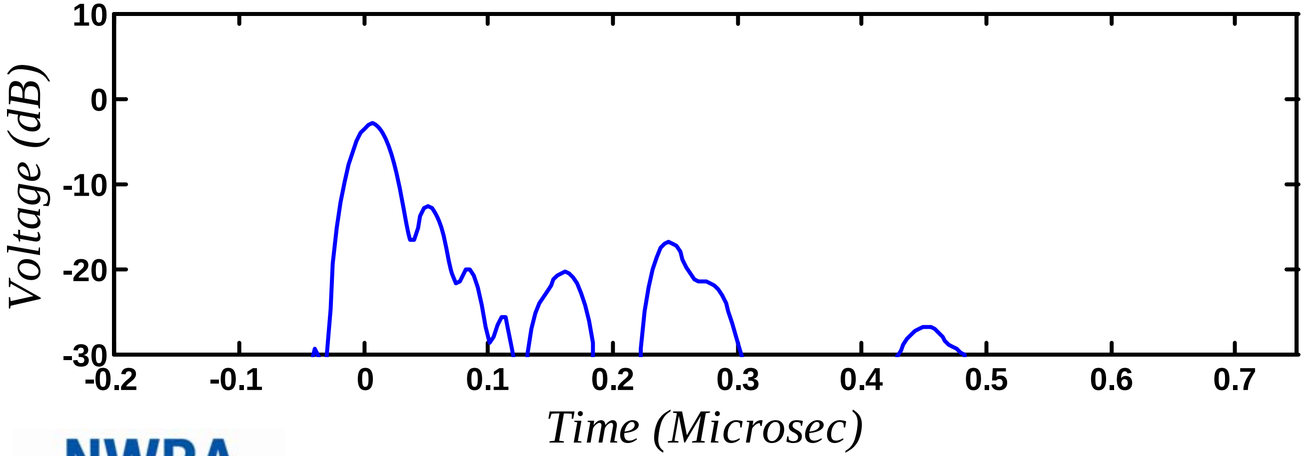
100 points taken from 524,288

First Frame of MPS Animation



Distance along the grid

Amplitude of transfer function



Amplitude of impulse response function

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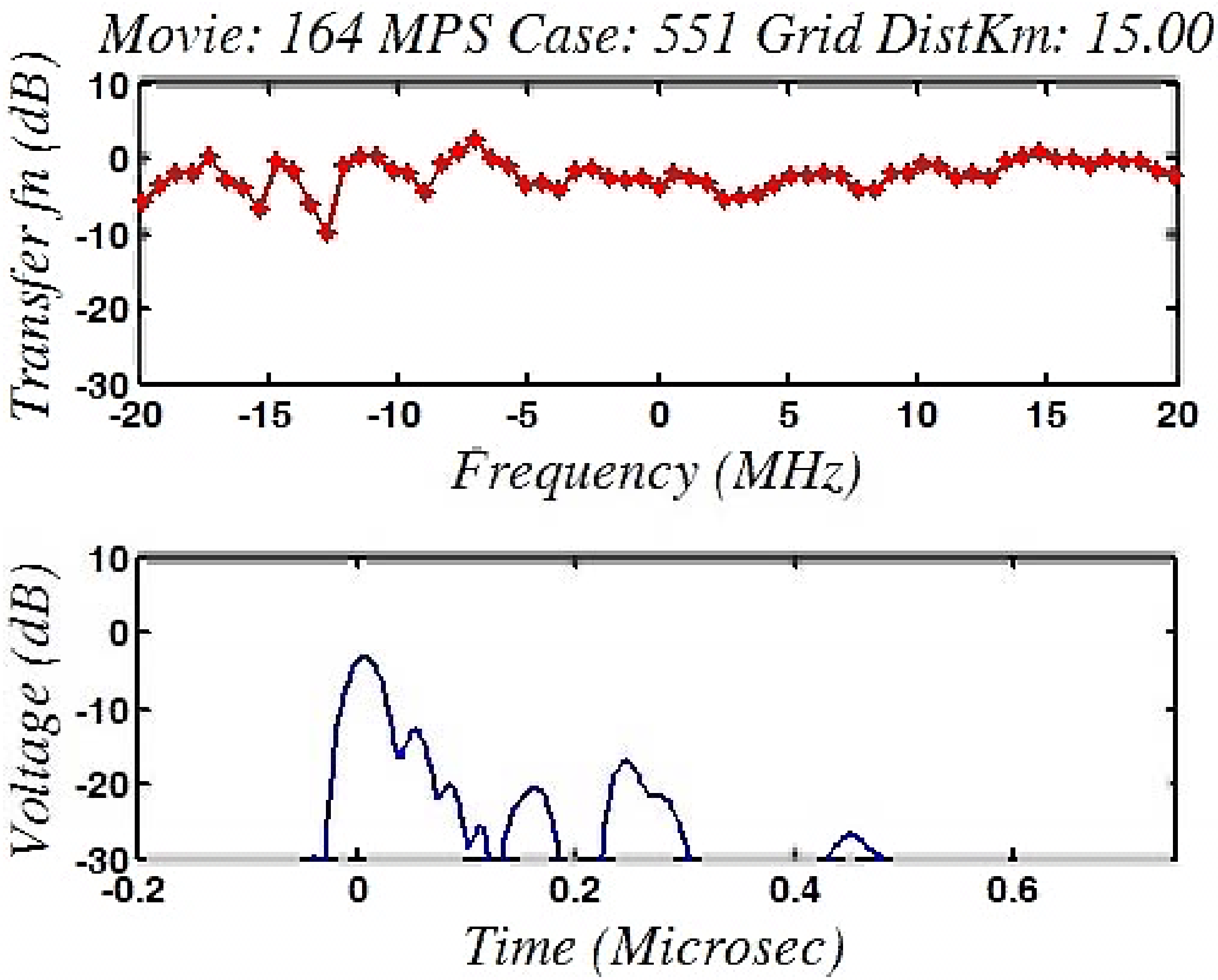
Variation of Transfer Function and Impulse Response Function Along the MPS Grid



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Comparison of Averaged MPS Results to Strong-Scatter Theory

Phase Screen Diffraction Method (PDM)

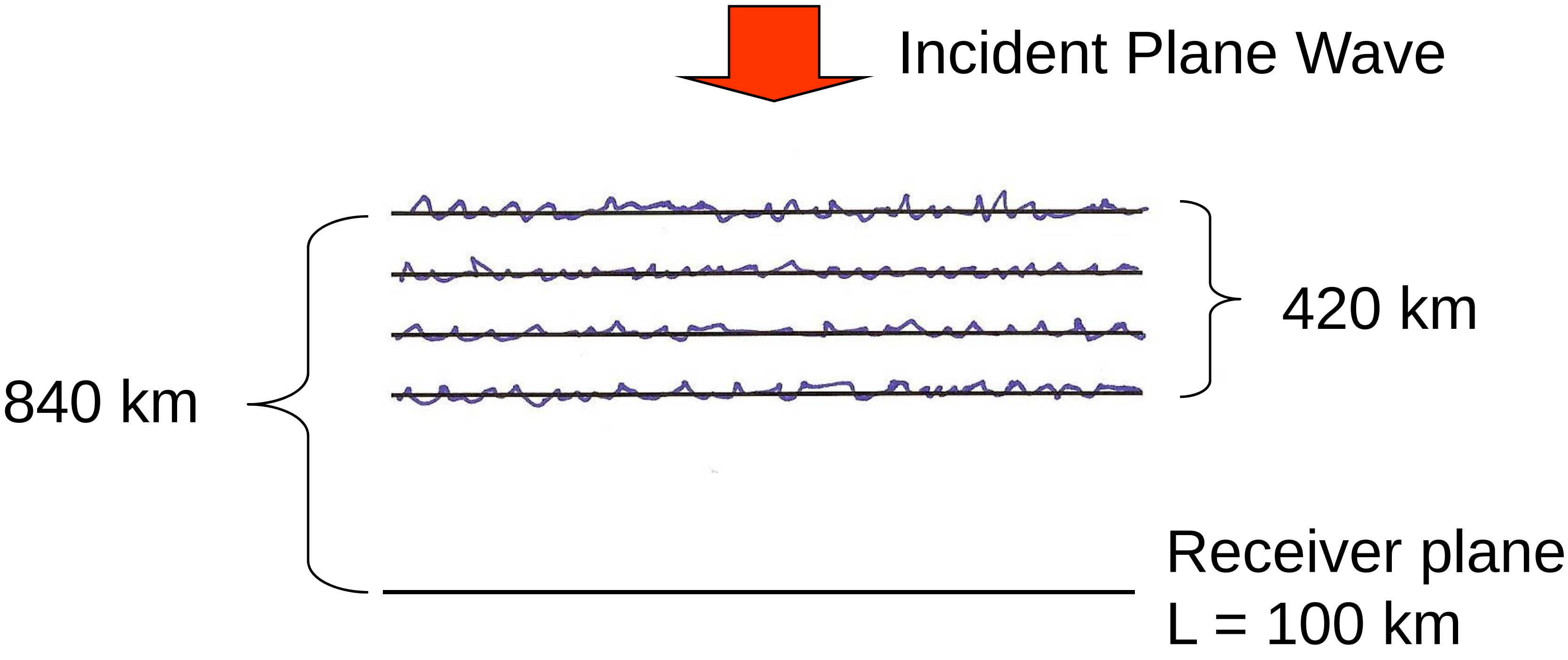
$$\frac{\partial \Gamma}{\partial z} - \frac{i}{2}\frac{k_d}{k^2}\Delta_d^2\Gamma + \frac{1}{z}\left[x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right]\Gamma + A(\omega_d, x, y, t; z)\Gamma = 0$$

Where

ω_d, x, y are difference variables
1/z term applies to spherical wave propagation
A is the quadratic expansion of the phase structure function
PDM is valid for strong scattering

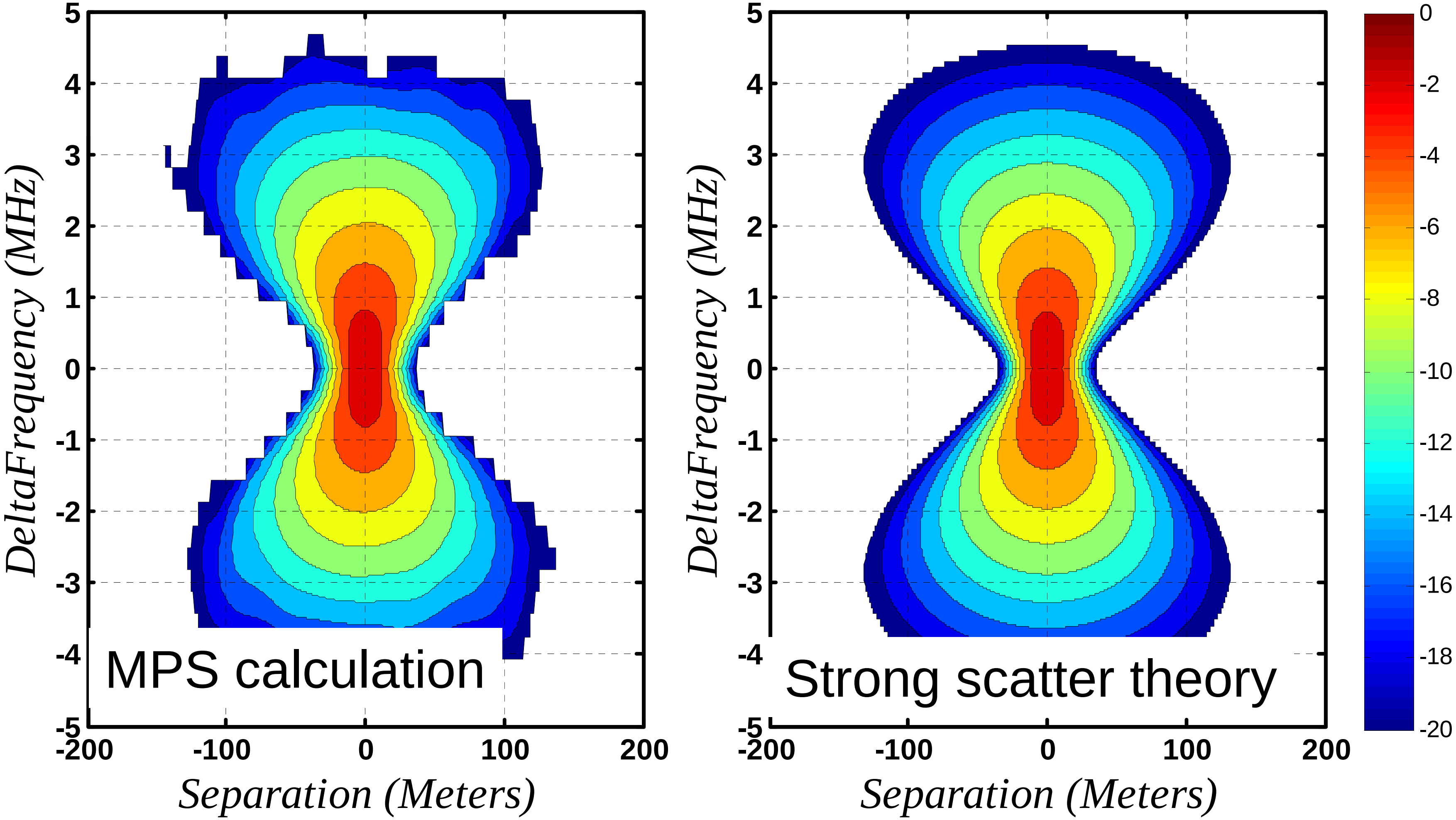
Solution employs a split-step Fourier transform to march from phase screen to phase screen

Thick-Layer Propagation Geometry



Structure has a q^{-3} power-law power spectral density
Outer scale = 5 km, Inner scale = 10 m, 2^{17} points
Phase standard deviation = 170 radians
Carrier = 300 MHz, BW = 40 MHz, 128 discrete freqs
Based on unpublished measurements (March 2001)

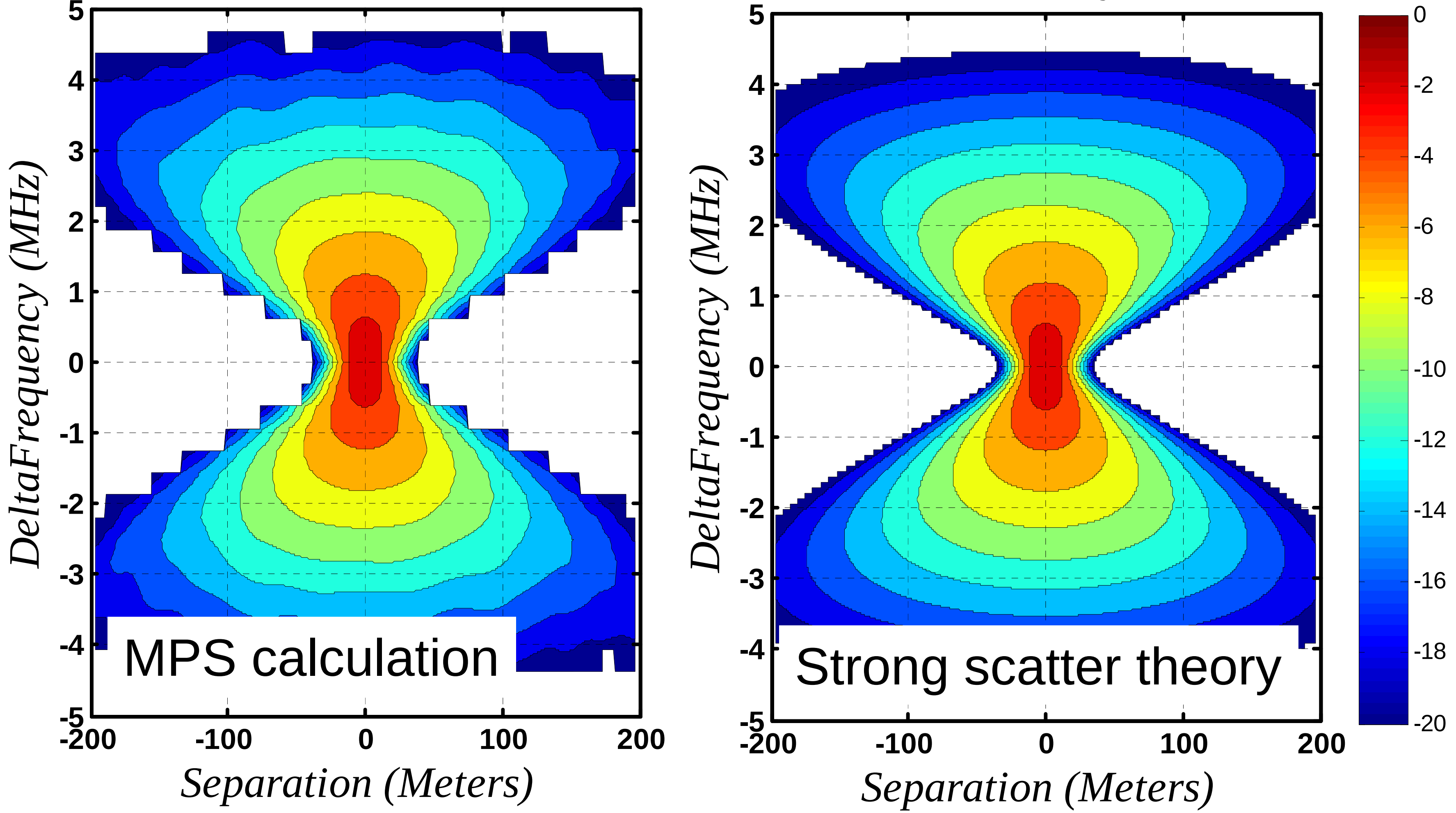
$\mathrm{Log}_{10}(|\Gamma(x,f)|)$ – Thick Layer (420 km)



Two-position, two-frequency MCF

Average of 50 realizations

$Log_{10}(|\Gamma(x,f)|)$ - Single Phase Screen



Two-position, two-frequency MCF

Average of 50 realizations

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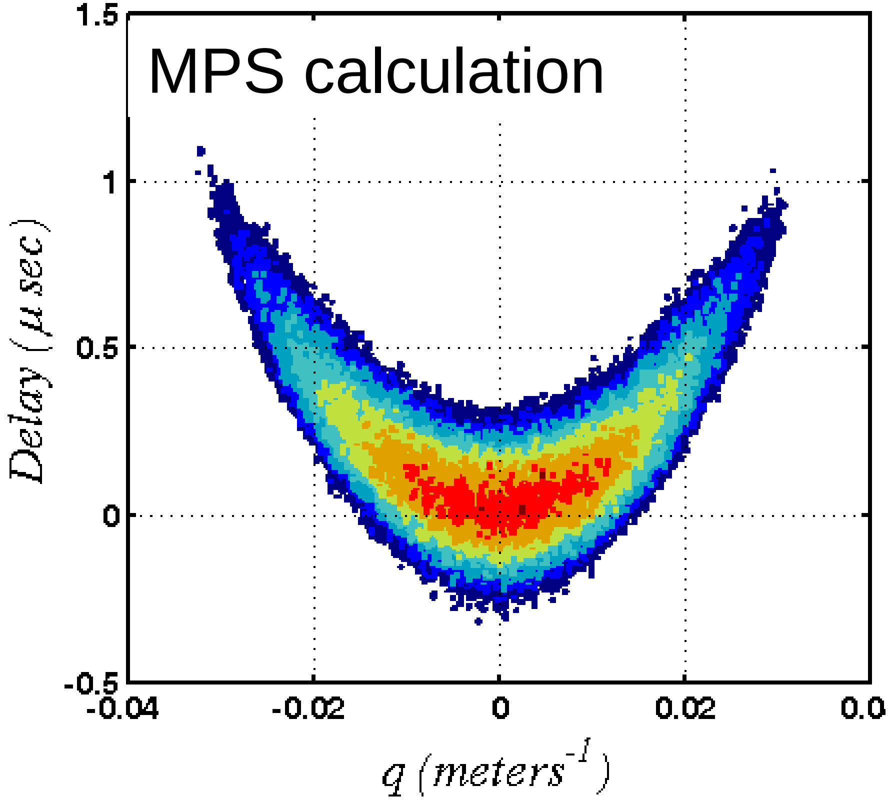
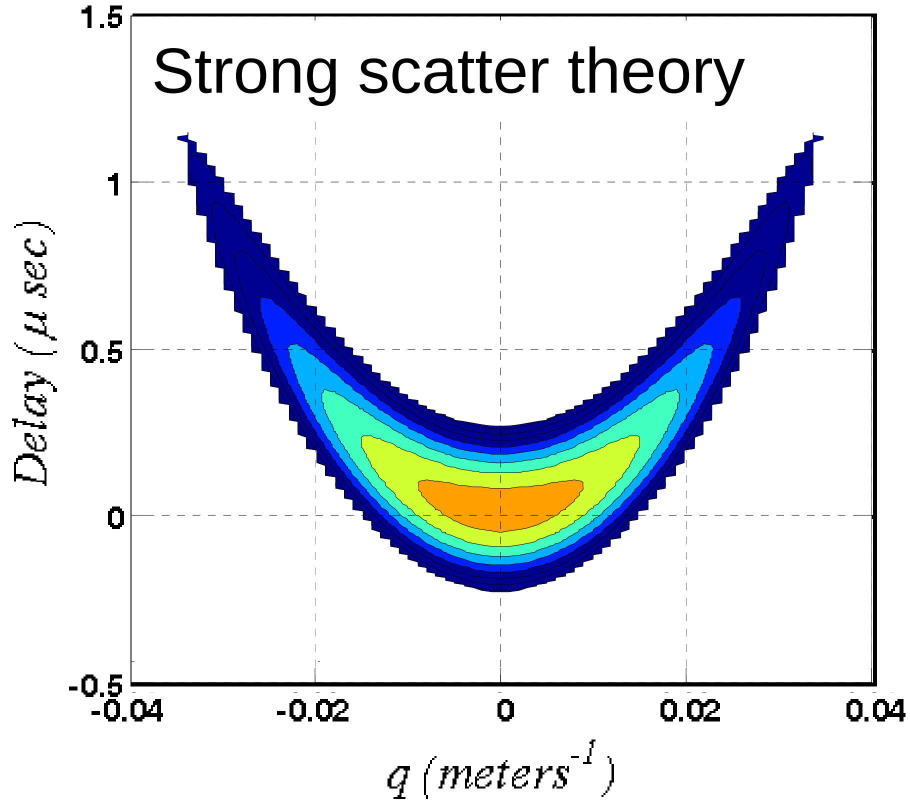
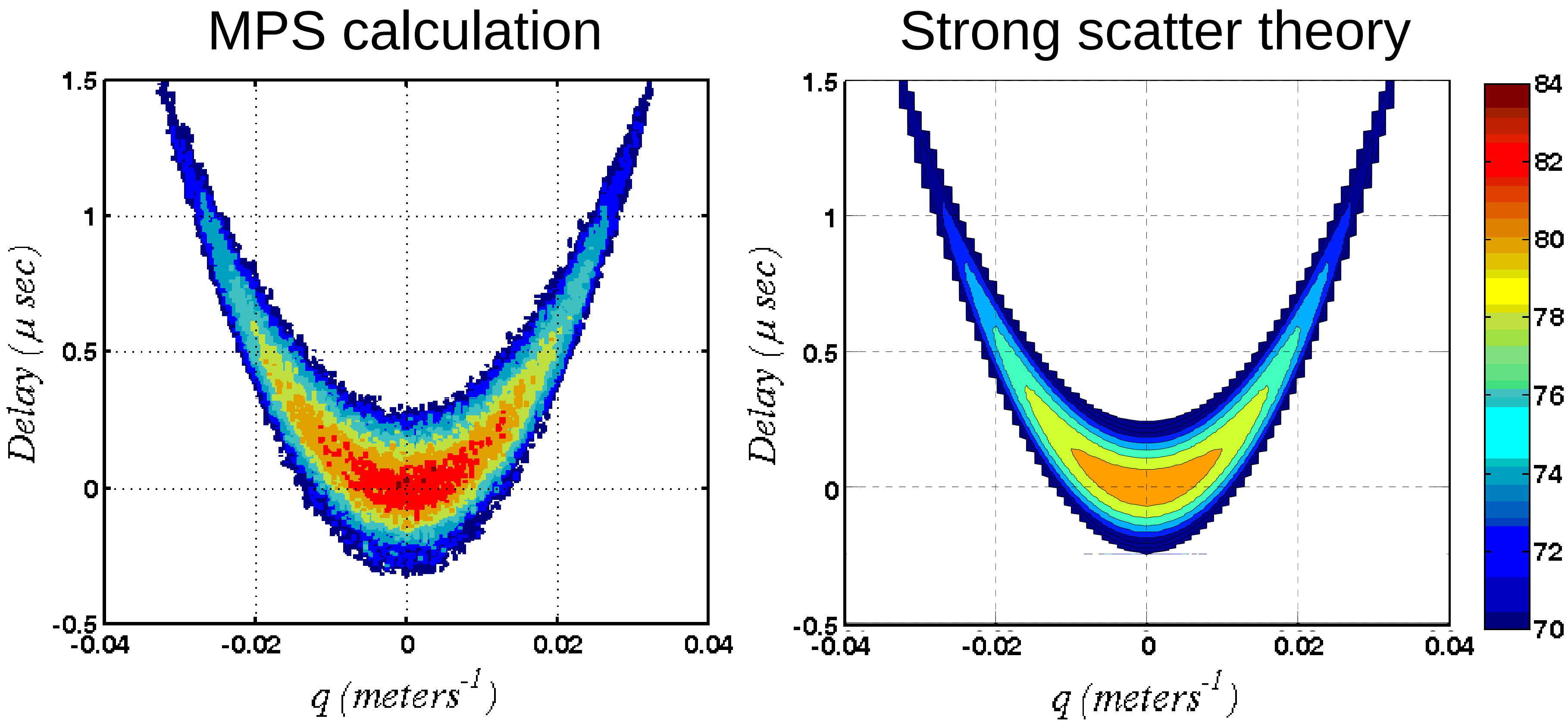
Delay-Angle Power Spectrum – Layer Thickness = 420 km





For frozen-in turbulence, this becomes the Delay-Doppler power spectrum

Delay-Angle Power Spectrum – Layer Thickness = 0 km



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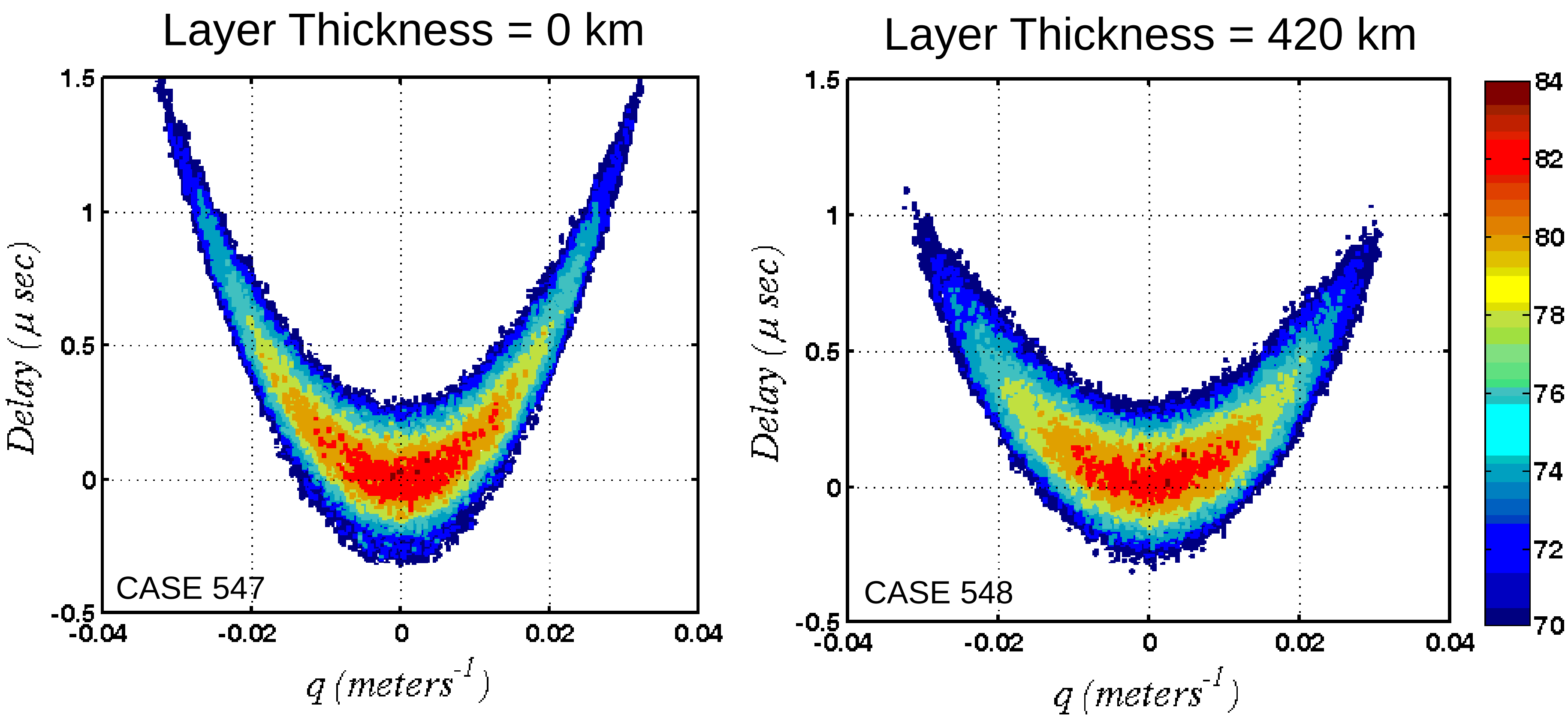
Comparison of Thick Layer and Single Thin Phase Screen



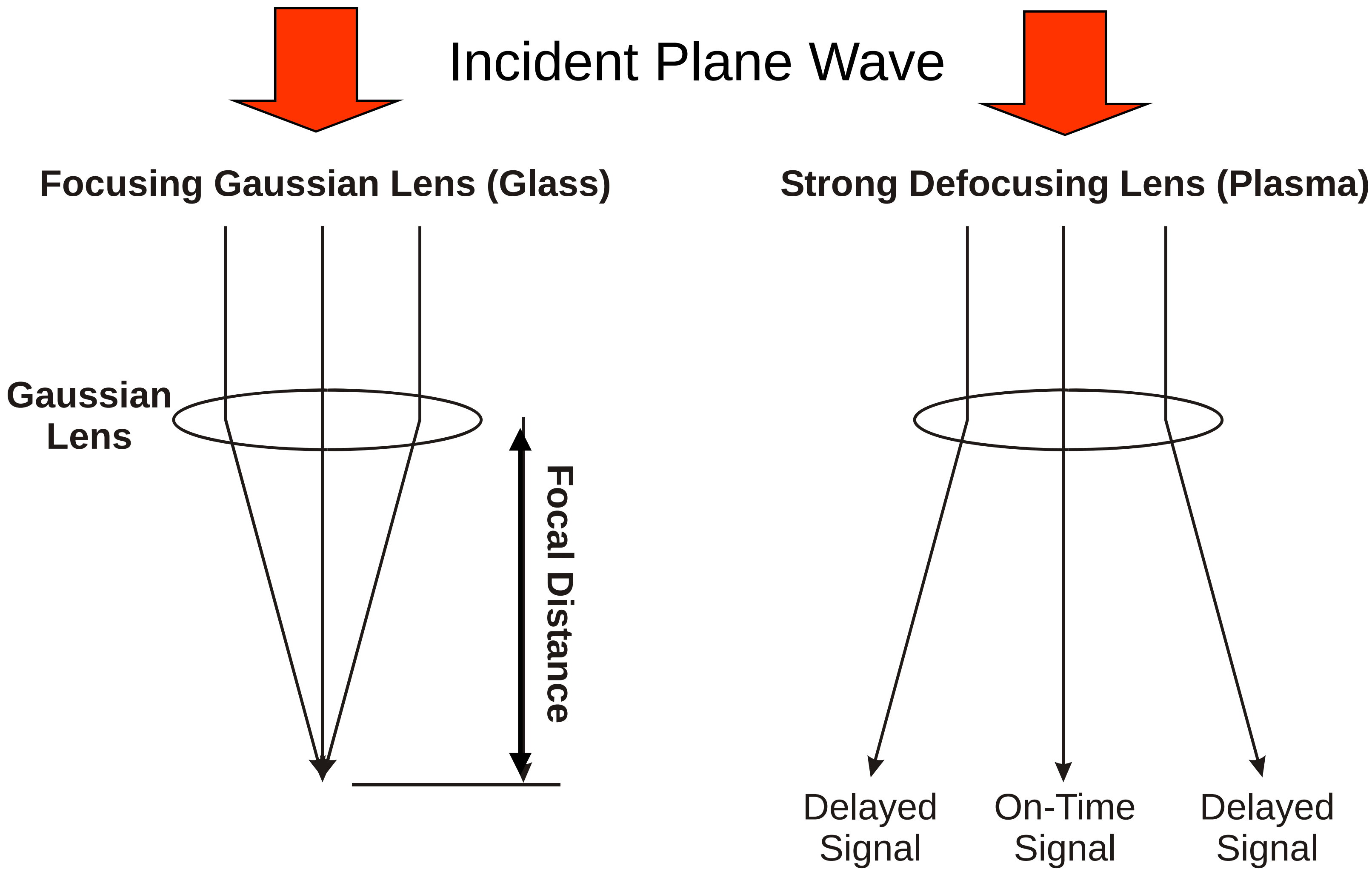
Delay-angle power spectrum
Average of 50 realizations

Conclusions

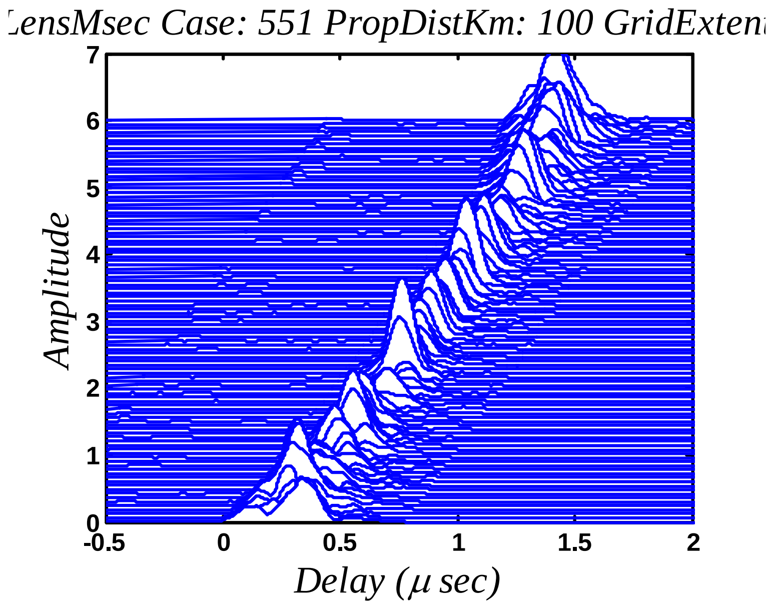
- **MPS codes are very useful to analyze wide bandwidth signals**
  - **Include diffraction and scintillation**
  - **Include the effects of ionospheric dispersion**
  - **Allow for non-homogeneous media**
  - **Time delay results are obtained by FFTs**
- **Animations of the combined transfer function and impulse response function are useful to understand wideband scintillation and develop mitigation**
- **MPS and PDM agree for our example of a wideband signal**

- **BACKUPS**

Focusing and Defocusing



20 to 30 km



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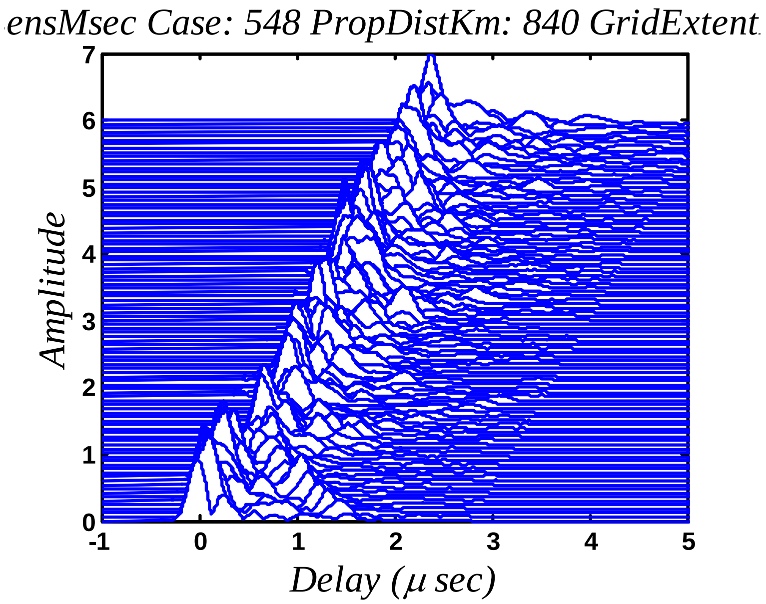
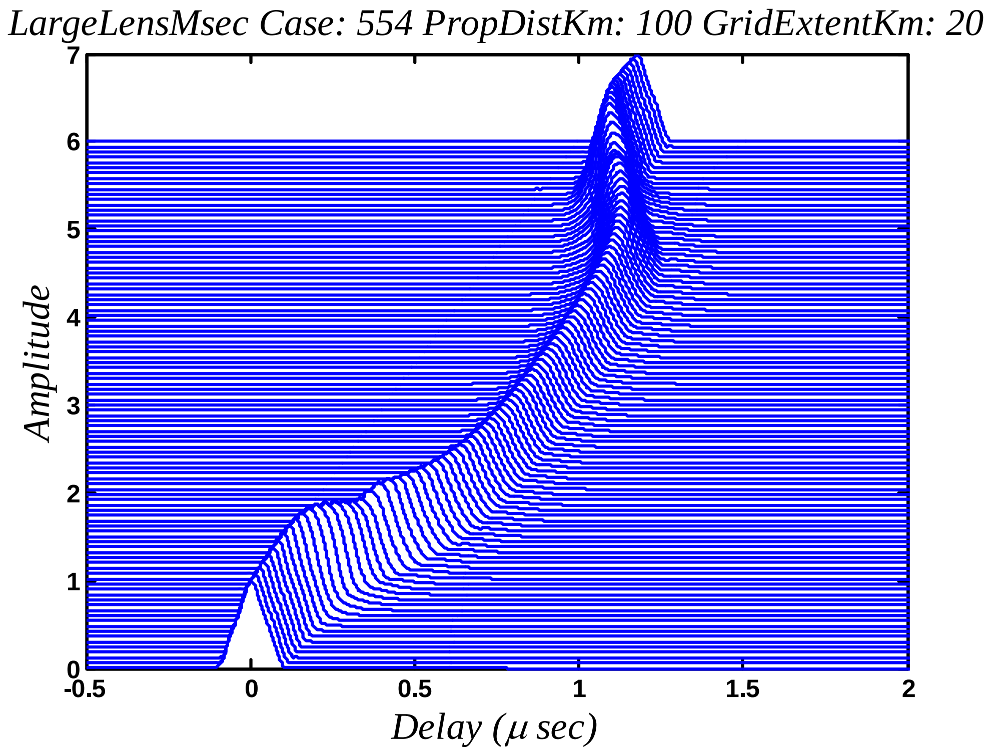
Entire 100 km grid





LargeLensMsec Case: 554 PropDistKm: 100 GridExtentKm: 20
Amplitude
0
1
2
3
4
5
6
7
-0.5
0
0.5
1
1.5
2
Delay (μ sec)