Amplitude domain inversion of narrow radar targets

Juha Vierinen, Markku Lehtinen

Sodankylä Geophysical Observatory, Finland

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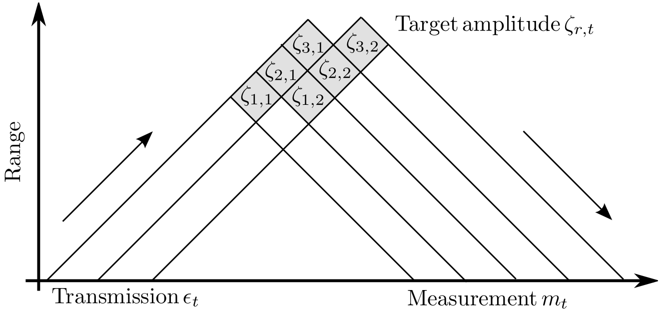
Overview

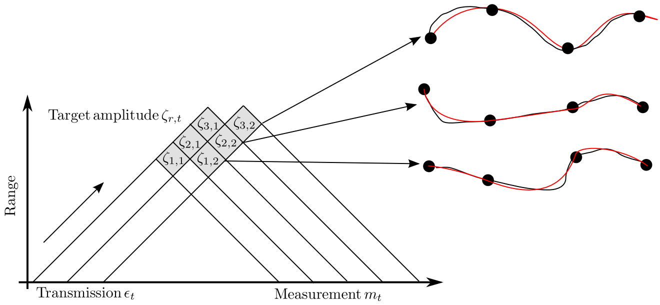
1. Narrow range and Doppler spread target
   - For example: Ion line and plasma line overshoot echos
   - Narrow range and Doppler spread
2. Meteor head echos
   - Strong, nearly point-like, fast-moving
   - Very high resolution results are possible, even for experiments with long bauds.
3. Transmission code optimality
   - Sub-baud range resolution
   - Non-uniform baud-lengths improve estimation accuracy

Range and Doppler spread target model



$$m_t = \sum_r \epsilon_{t-r}\,\zeta_{r,t-\frac{1}{2}r} + \xi_t. \tag{1}$$

Range and Doppler spread target model $\zeta_{r,t}$



- Assume that target backscatter $\zeta_{r,t}$ is band-limited.
- Linear models: B-Spline, Fourier series, ...

$$\hat{\zeta}_{r,t} = S_r^k(t) \tag{2}$$

Solution strategy

- The model $\mathcal{M}_B$ is a linear inverse problem.

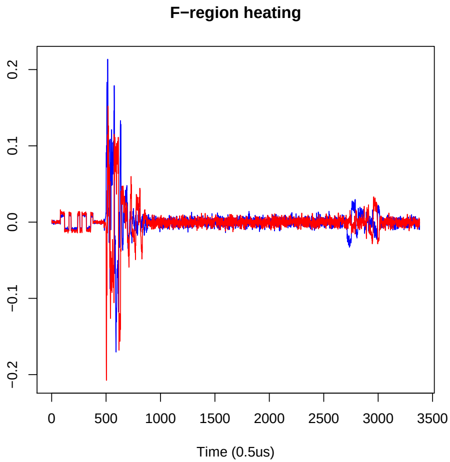
$$\mathbf{m} = \mathbf{A}\boldsymbol{\theta} + \boldsymbol{\xi} \tag{3}$$

- $\mathbf{A}$ Theory matrix
- $\boldsymbol{\theta}$ Model parameter vector
- $\mathbf{m}$ Measurements
- $\boldsymbol{\xi}$ Error
- Efficient solution using linear algebra:

$$\boldsymbol{\theta}_{\mathrm{MAP}} = \left(\overline{\mathbf{A}}^{\mathrm{T}}\mathbf{A}\right)^{-1}\overline{\mathbf{A}}^{\mathrm{T}}\mathbf{m} \tag{4}$$

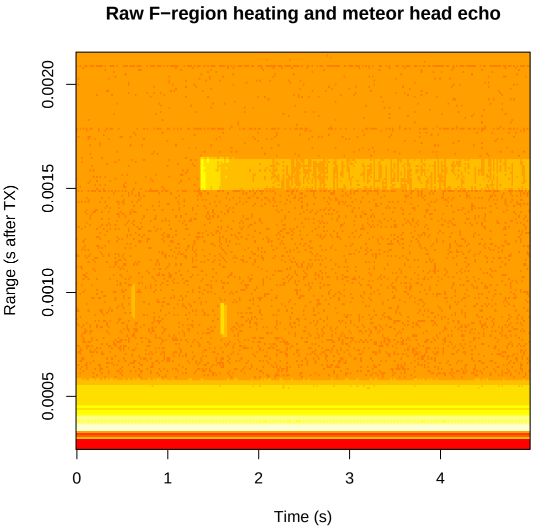
F-region ion-line overshoot

TX, ground clutter and ionospheric heating induced echo from the F-region.



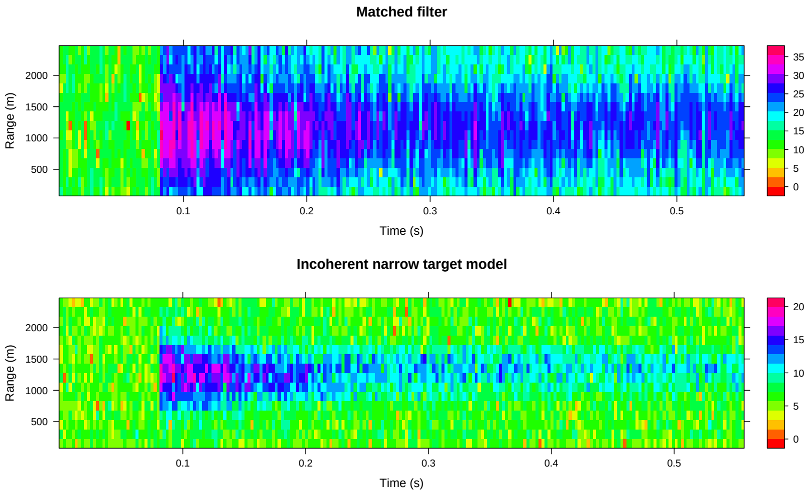
F-region ion-line overshoot

Surface plot of consecutive echos. F-region heating effect and two meteor head echos.



F-region ion-line overshoot

Comparison of target power estimates (dB scale)

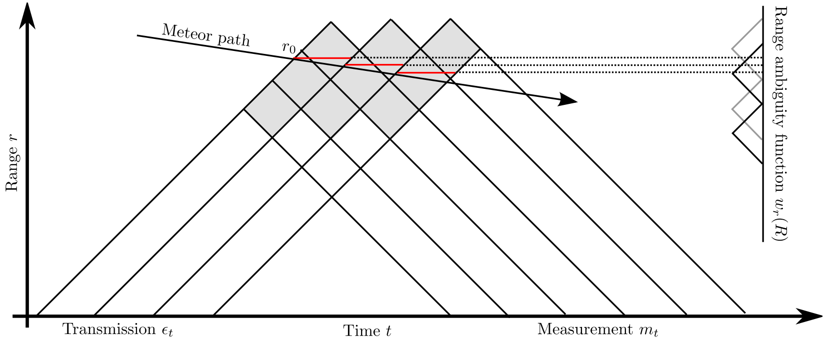


Meteor head echo

- How accurately can a meteor range and Doppler shift be estimated?
- Range ambiguity corrected moving point model.
- In principle, better range resolution than sampling rate possible.

Range ambiguity corrected moving point model

Model the spreading of the target when it moves down.



Range ambiguity corrected moving point measurement

- One "point" travelling at velocity $v \in \mathbb{R}$, starting from range $r_0 \in \mathbb{R}^+$, following radial trajectory $R = vts^{-1} + r_0$. Sample rate s. Doppler shift is $\omega = vf/c$.
- Range ambiguity function $w_r(R)$ gives contribution of target for each measurement sample m_t. True range: $R \in \mathbb{R}^+$, range gate $r \in \mathbb{N}$.

$$m_t = \underbrace{\sum_r w_r(vts^{-1} + r_0)\epsilon_{t-r} c_r \exp(i\omega ts^{-1})}_{f_t(\theta)} + \xi_t \tag{5}$$

Solution method

- Examine the *a posteriori* probability distribution, the probability of model parameters given data:

$$p(\theta \mid D) = \frac{p(D \mid \theta)p(\theta)}{\int d\theta p(D \mid \theta)p(\theta)}$$

- The probability distributions solved using Markov chain Monte-Carlo (Hastings 1970).
- Other faster methods also possible for finding the peak of the probability distribution.

Likelihood function $p(D|\theta)$ and priors $p(\theta)$

- Measurement $D = (m_1, ..., m_N) \subset \mathbb{C}^N$
- Point-target parameters: $\theta = (\sigma, c, r_0, \omega) \subset \mathbb{R} \times \mathbb{C} \times \mathbb{R} \times \mathbb{R}$

Likelihood function, the probability of data given parameters:

$$p(D \mid \theta) = \prod_{t \in R} \frac{1}{\pi \sigma^2} \exp \left\{ -\frac{|m_t - z_t(\theta)|^2}{\sigma^2} \right\}$$

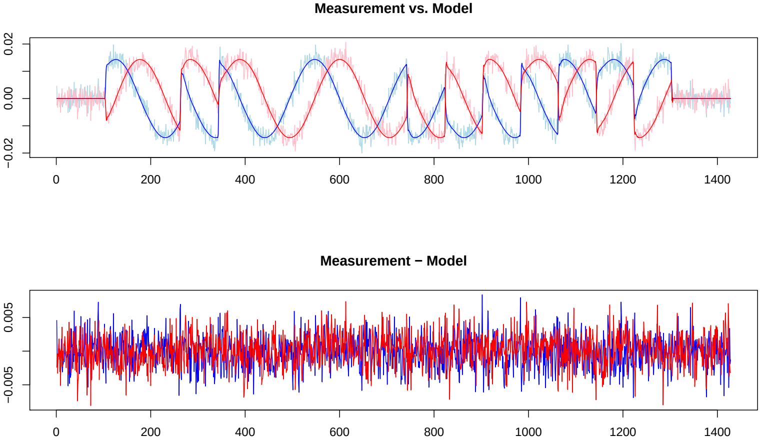
Priors, the probability distribution of model parameters:

- Measurement noise close to known system noise power $P = kTB$.

$$\sigma^2 \sim N_T(P, 0.1P)$$

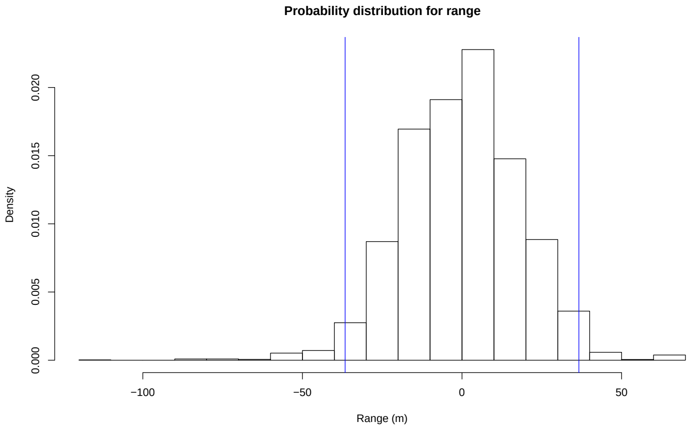
- Other parameters c_r, v, and r_0 uniformly distributed.

Point-target example (EISCAT VHF)



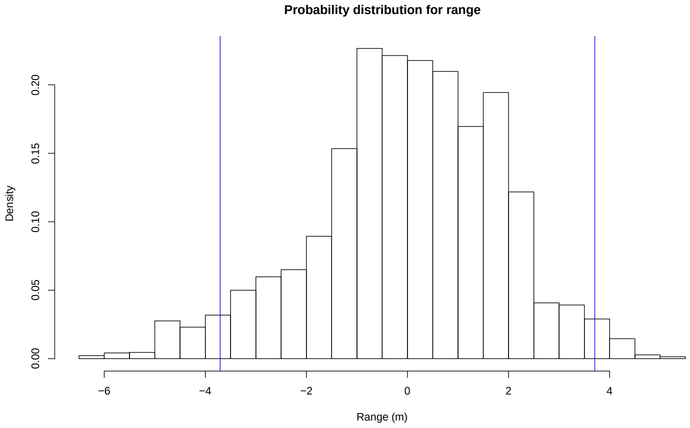
Marginal range distribution $p(r_0|D)$

Weak echo

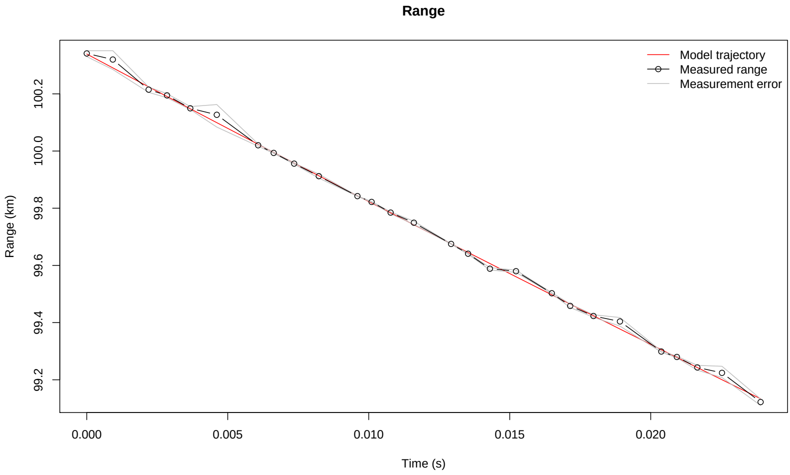


Marginal range distribution $p(r_0|D)$

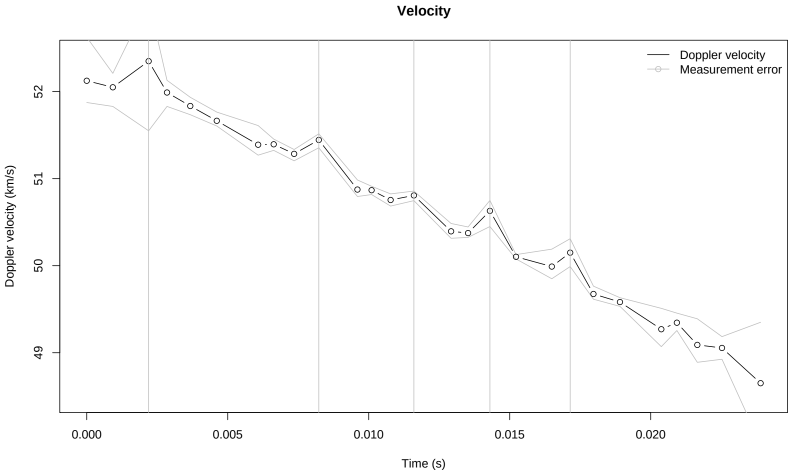
Strong echo



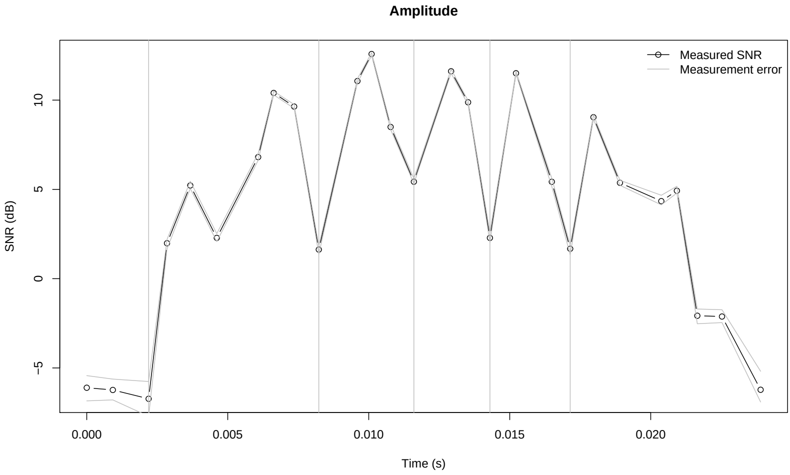
Point-target trajectory (Range)



Point-target trajectory (Doppler)

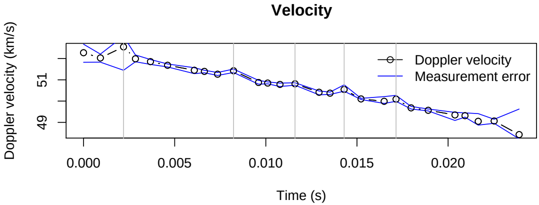


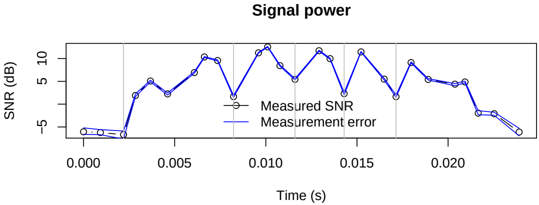
Point-target trajectory (Amplitude)



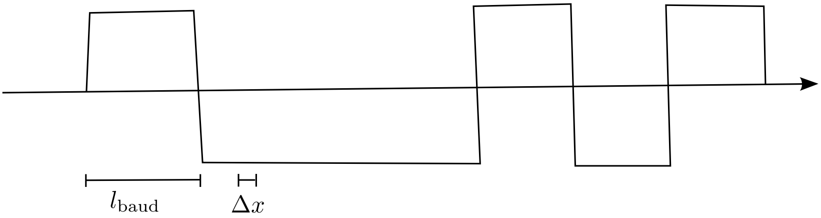
Forward expanding plasma?

- The order of 10^{-2} m forward expansion of sufficient to explain Doppler shift.
- Is the length scale consistent with interference?





Transmission code optimality



- Radar transmission bandwidth is limited.
  - How do you optimize sub-baud range resolution?
  - This can be determined by looking at the code-dependent estimation covariance $\Sigma_p(\epsilon_t) \propto (\overline{\mathbf{A}(\epsilon_t)}^{\mathrm{T}} \mathbf{A}(\epsilon_t))^{-1}$.
- Coding with **non-uniform baud-lengths**

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Fractional baud-length coding

J. Vierinen

Sodankylä geophysical observatory, Sodankylä, Finland

We present a novel approach for timing radar transmission envelopes in order to improve target range and Doppler resolution. This is achieved by using non-uniform baud lengths. With this method, it is possible to significantly increase sub-baud range-resolution of radar measurements while maintaining a narrow bandwidth. We first derive target estimation accuracy in terms of a covariance matrix for arbitrary targets when estimating backscatter in amplitude domain. We define target optimality and discuss different search strategies that can be used to find well performing transmission envelopes. We give several examples and compare the results to conventional uniform baud length transmission codes.

1. Introduction

We have previously described a method for estimating range and Doppler spread radar targets in amplitude domain at sub baud-length range-resolution using linear statical inversion [*Vierinen et al.*, 2007b]. However, we did not use codes optimized for the targets that we analyzed. Also, we only briefly discussed code optimality. In this paper we will focus on optimal transmission codes for a target range resolution that is smaller than the minimum allowed baud-length. We will introduce so called *fractional baud-length codes* that are optimal for range and possibly Doppler spread targets, with a better resolution than the minimum allowed radar transmission envelope baud-length.

In radar systems, there is a limit to the smallest baud length, which arises from available bandwidth

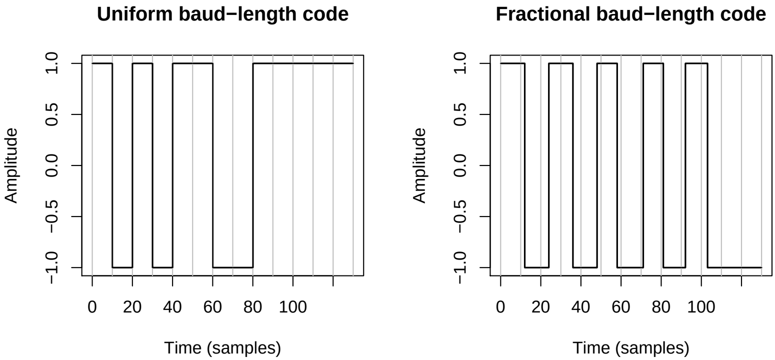
form baud-length radar transmission code with baud lengths that are integer multiples of 1 μs. The reason is that the uniform baud-length will cause a singular or near-singular covariance matrix when analyzing experiments with sub-baud range-resolution.

In this paper, we first derive the target parameter estimation covariance for range and Doppler spread radar targets when estimating target parameters in amplitude domain. Then we define transmission code optimality for a given target. After this, we then present two search strategies which can be used to find optimal transmission codes: an exhaustive search algorithm, and an optimization search algorithm. As an example, we study code optimality in the case of a range spread coherent target, and a range and Doppler spread target.

2. Transmission code

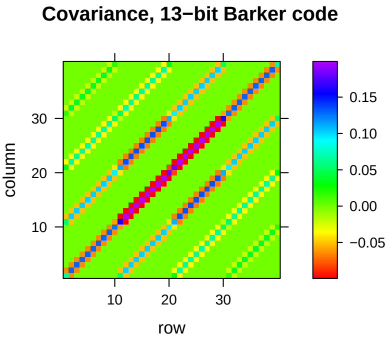
Fractional baud-length code

Baud lengths timed very accurately, but no baud is not shorter than the minimum allowed length.



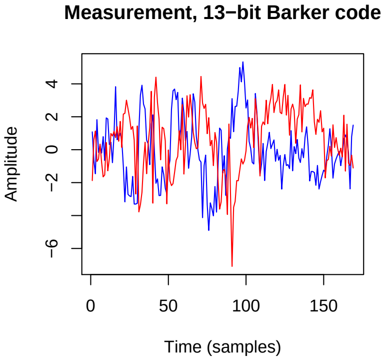
Error covariance matrix

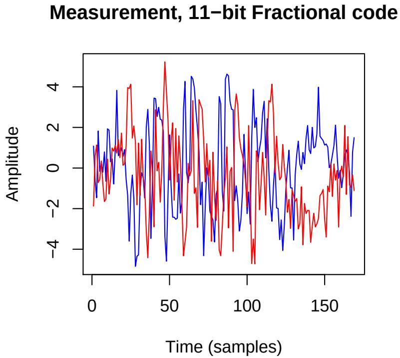
Uniform baud-length code has close to singular covariance.
Fractional baud-length code has smaller variance and very low off-diagonal elements.



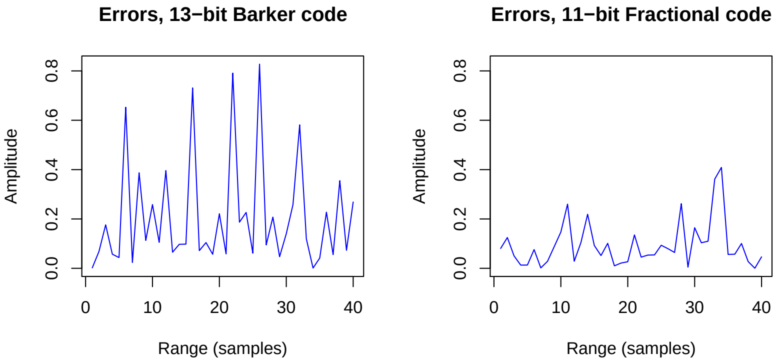
Simulated measurement

SNR $\approx$ -2 dB. Target range extent 20 samples. Code length 130 samples, with 10 sample bauds.





Simulation Errors



Conclusions

- Heating related strong range and Doppler spread echos can be analyzed in amplitude domain on a single echo basis if they are narrow enough (in range and Doppler spread)
- Meteor head echo parameters can be determined very accurately even for low-bandwidth transmissions
- *Fractional baud-length* coding improves sub-baud range resolution estimation accuracy.
- Future work will focus on amplitude domain inversion of overspread weak incoherent backscatter.