Multi-dimensional representation of the electron density from satellite data

M. Schmidt[1], D. Bilitza[2], C.K. Shum[3], C. Zeilhofer[1], D. Dettmering[1], J. Zhang[1], L.-C. Tsai[4]

[1]Deutsches Geodätisches Forschungsinstitut (DGFI), Munich, Germany
[2]Heliospheric Physics Laboratory/GMU, NASA Goddard Space Flight Center, Greenbelt, Maryland, USA
[3]Geodetic Science, School of Earth Sciences,The Ohio State University, Columbus, Ohio, USA
[4]Center for Space and Remote Sensing Research, National Central University, Taiwan

Email: schmidt@dgfi.badw.de

Introduction

- In the last decade **satellite missions** have become a promising tool for **measuring** ionospheric parameters such as the **electron density** or the **vertical total electron content** (VTEC) .
- **Dual-frequency GPS observations** can be used to determine the **slant total electron content** (STEC), i.e. the integral of the electron density along the ray-path of the signal.
- The insensitivity of ground-based observations to the **radial geometry** can be overcome by considering measurements from **space-borne receivers** flying on low-Earth-orbiting (LEO) satellites, e.g., the **FORMOSAT-3/COSMIC** 6-satellite constellation, **CHAMP** or **GRACE.**
- In the following, we present a general **multi-dimensional approach** for modeling **spatio-temporal variations** of the ionosphere.

Procedure (multi-dimensional input options)

- Our **multi-dimensional approach** allows the following input strategies:

2D modeling	**3D modeling**	**4D modeling**
$VTEC(\lambda,\varphi)$	$VTEC(\lambda,\varphi,t)$	$N(\lambda,\varphi,h,t)$
	$N(\lambda,\varphi,h)$	

↓

Subtraction of the corresponding values from the **reference model**;
here: **International Reference Ionosphere** (IRI-2000)

↓

Residual input data (observations)
Area: global, regional, local

↓

Parameterization:
B-spline-only expansions, combined expansions: B-splines/EOFs, trig. B-splines/B-splines, B-splines/Chapman function, SHs/EOFs, etc.)

- Other **ionospheric parameters** like the maximum value of the electron density N_mF2 can be modeled in the same manner.

GNSS - observation equation

- The observation equation of the **geometry-free linear combination** reads

$$\boxed{\begin{aligned}\phi_4(t) &= \phi_1(t) - \phi_2(t) \\ &= \alpha \cdot STEC(t) + \beta_R + \beta_S + \beta_{S,R} + e(t)\end{aligned}}$$

$$\begin{aligned}\beta_R, \beta_S, \beta_{S,R} &= \text{inter-frequency delays in receiver } R \text{ and satellite } S\text{, ambiguity term },\\ \alpha, e(t) &= \text{constant, measurement error },\\ \phi_1(t), \phi_2(t) &= \text{phase observations on frequencies } f_1 = 1.5 \text{ GHz and } f_2 = 1.2 \text{ GHz },\\ STEC(t) &= \textbf{Slant Total Electron Content} .\end{aligned}$$

- The ambiguity term $\beta_{S,R}$ can be determined in a **pre-processing step**.

- The inter-frequency delays β_R and β_S can be determined in a **common adjustment** together with the parameters of the electron density approach we will present in the following.

STEC modeling

- The STEC is defined as the integral of the space- and time-dependent four-dimensional **electron density** $N(\boldsymbol{x}, t)$ along the ray-path between the satellite S and the receiver R, i.e.

$$\boxed{STEC(t) = \int_R^S N(\boldsymbol{x}, t)\, ds \,,} \tag{1}$$

wherein

$$\boldsymbol{x} = r \left[\, \cos\varphi \cos\lambda \,,\ \cos\varphi \sin\lambda \,,\ \sin\varphi \,\right]^T = \text{geocentric position vector}$$

$$\lambda, \varphi, r = \text{longitude, latitude, radial distance} \,, \quad t = \text{time}$$

$$r = \rho + h \,, \quad \rho, h = \text{mean Earth radius, height} \,.$$

- We decompose the electron density into a **given reference model** $N_{ref}(\boldsymbol{x}, t)$, computed from IRI, and an **unknown correction term** $\Delta N(\boldsymbol{x}, t)$, i.e.

$$N(\boldsymbol{x}, t) = N_{ref}(\boldsymbol{x}, t) + \Delta N(\boldsymbol{x}, t) \,. \tag{2}$$

STEC modeling — 2

- Inserting Eq. (2) into Eq. (1) yields

$$\boxed{STEC(t) = STEC_{ref}(t) + \int_R^S \Delta N(\boldsymbol{x}, t)\, ds.}$$

- **Approach:** **3D B-splines**, i.e., a series expansion

$$\Delta N(\boldsymbol{x}, t) = \sum_{k_1=0}^{K_1-1} \sum_{k_2=0}^{K_2-1} \sum_{k_3=0}^{K_3-1} d_{k_1,k_2,k_3}(t)\, \phi_{k_1}^{J_1}(\lambda)\, \phi_{k_2}^{J_2}(\varphi)\, \phi_{k_3}^{J_3}(h)$$

in 3D products of 1D **scaling functions** $\phi_k^J(\cdot)$ with $J =$ level (scale) and $k =$ shift.

GNSS - observation equation — 2

- The substitution of the series expansion for $\Delta N(\boldsymbol{x}, t)$ into the "STEC" equation yields

$$STEC(t) = STEC_{ref}(t) + \sum_{k_1=0}^{K_1-1} \sum_{k_2=0}^{K_2-1} \sum_{k_3=0}^{K_3-1} d_{k_1,k_2,k_3}(t)\; k_{k_1,k_2,k_3}(R,S)\;,$$

wherein

$$k_{k_1,k_2,k_3}(R,S) = \int_R^S \phi_{k_1}^{J_1}(\lambda)\, \phi_{k_2}^{J_2}(\varphi)\, \phi_{k_3}^{J_3}(h)\, ds\;.$$

- The final **observation equation** for the geometry-free GPS observation $\phi_4(t)$ reads

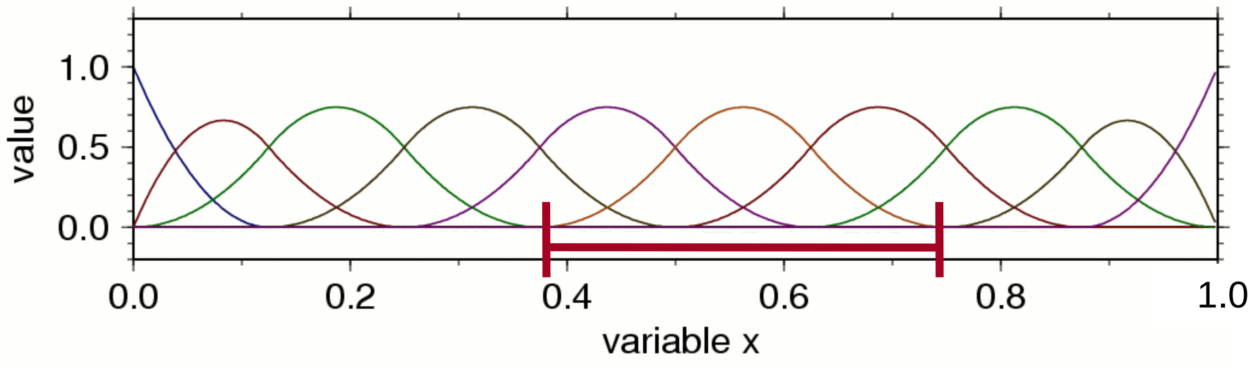
$$\boxed{\begin{aligned}\phi_4(t) - \beta_{S,R} - \alpha \cdot STEC_{ref}(t) \\ = \alpha \cdot \sum_{k_1=0}^{K_1-1} \sum_{k_2=0}^{K_2-1} \sum_{k_3=0}^{K_3-1} \boxed{d_{k_1,k_2,k_3}(t)}\; k_{k_1,k_2,k_3}(R,S) + \boxed{\beta_R} + \boxed{\beta_S} + e(t)\;.\end{aligned}}$$

unknown scaling coefficients

unknown delays

B-spline modeling

- As 1D scaling functions $\phi_k^J(x)$ of level J we choose **normalized endpoint-interpolating quadratic B-spline functions** shown in the figure below for level J = 3. (# = 2^J + 2)



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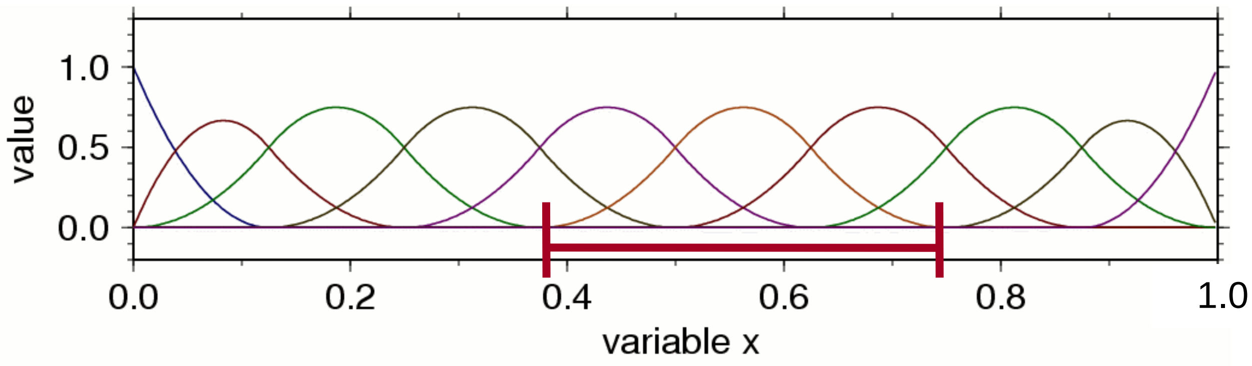
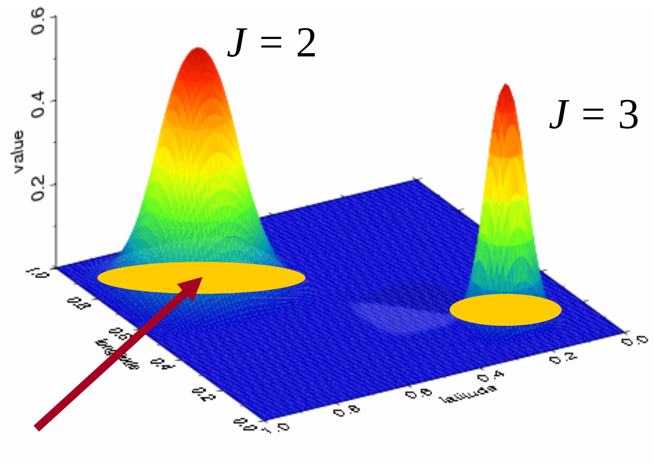
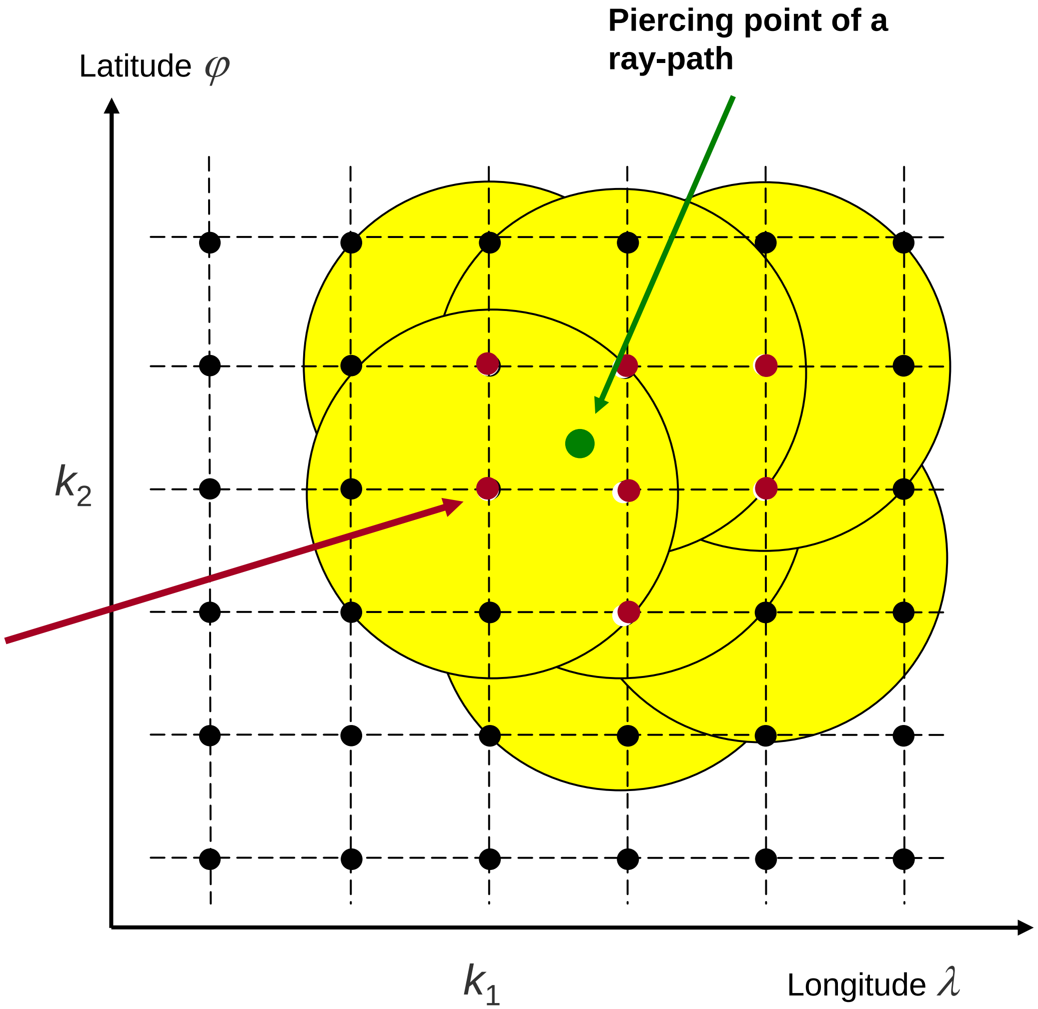




- The figure on the left-hand side demonstrates the **localizing feature** of the 2D B-splines $\phi_{k_1}^{J_1}(x)\,\phi_{k_2}^{J_2}(y)$, since the higher the level value J is chosen the sharper is the peak.
- Mathematically spoken B-splines are **compactly supported**. The **„non-zero“ zone** (influence zone, **foot print**) in the (x,y)-plane is similar to a **circle** or an **ellipse**.
- In the 3D case these non-zero zones are similar to **spheres** or **ellipsoids** (biaxial or triaxial).

B-spline modeling — 2

- The centers (k_1,k_2) of **the foot prints** of the 2D B-spline functions $\phi_{k_1}^{J_1}(\lambda)\,\phi_{k_2}^{J_2}(\varphi)$ define a **regular grid** within the unit square.
- Due to their compact support (yellow **foot prints**) only B-splines related to the red dots, have **non-zero entries** in the observation equation shown before.
- The **series (scaling) coefficient** $d_{k1,k2}$ can be determined by the GNSS observation related to the piercing point (green dot).



B-spline modeling — 2

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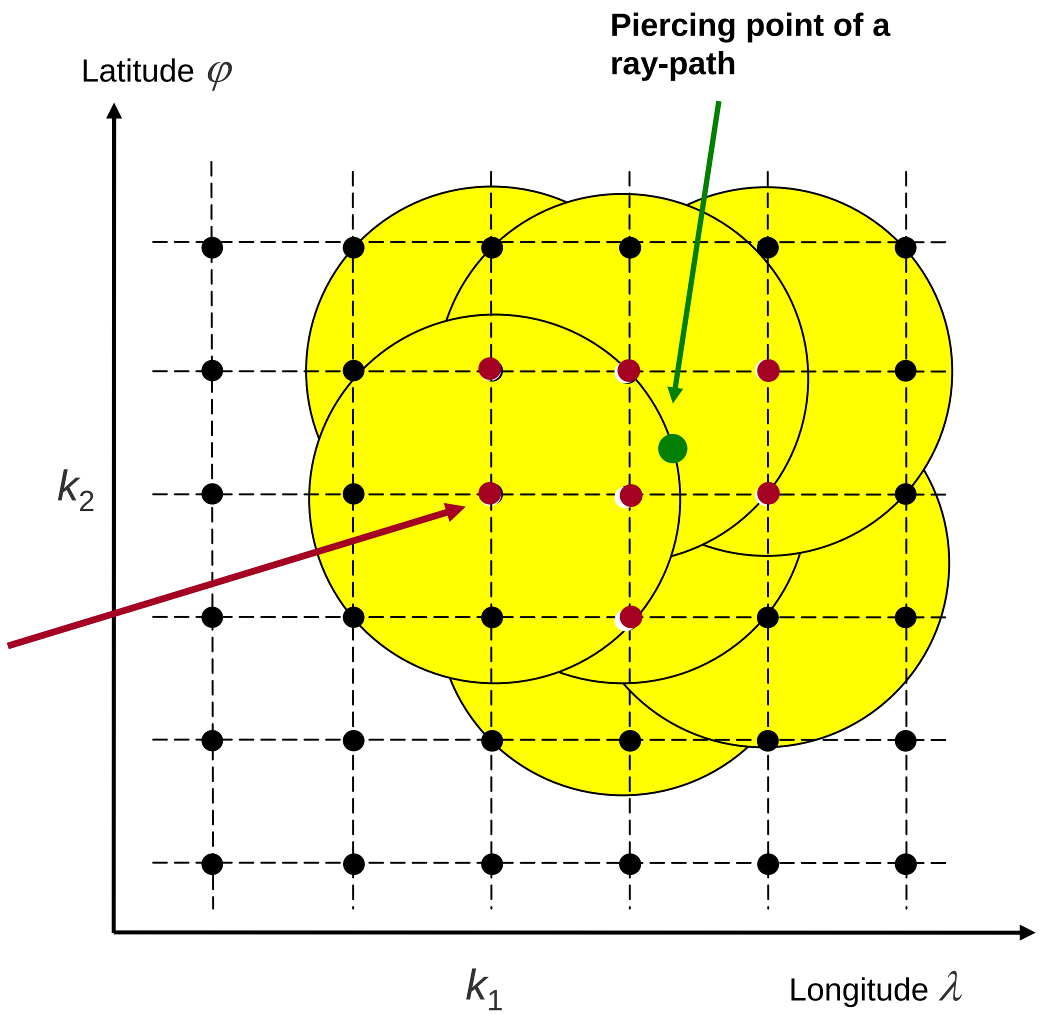
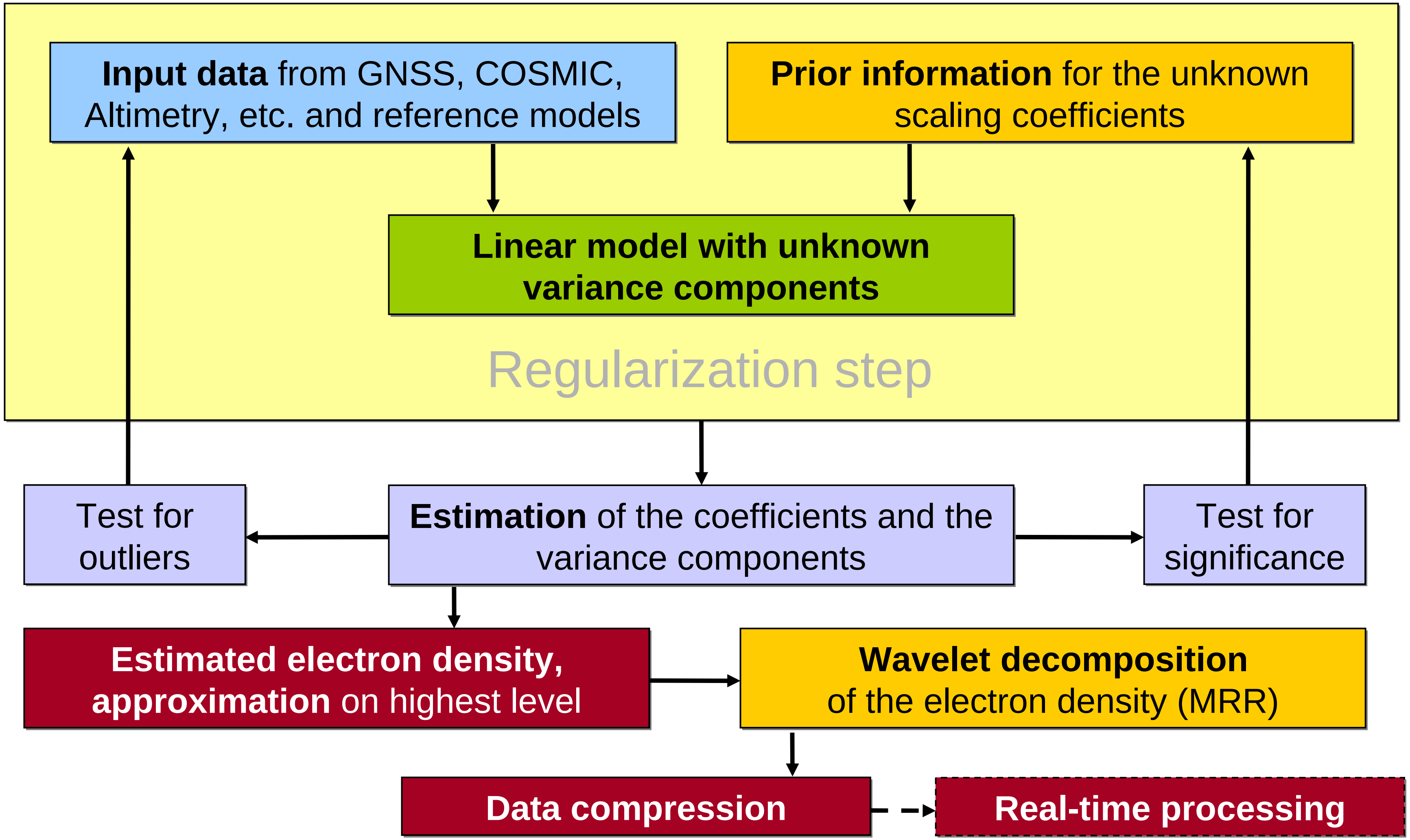
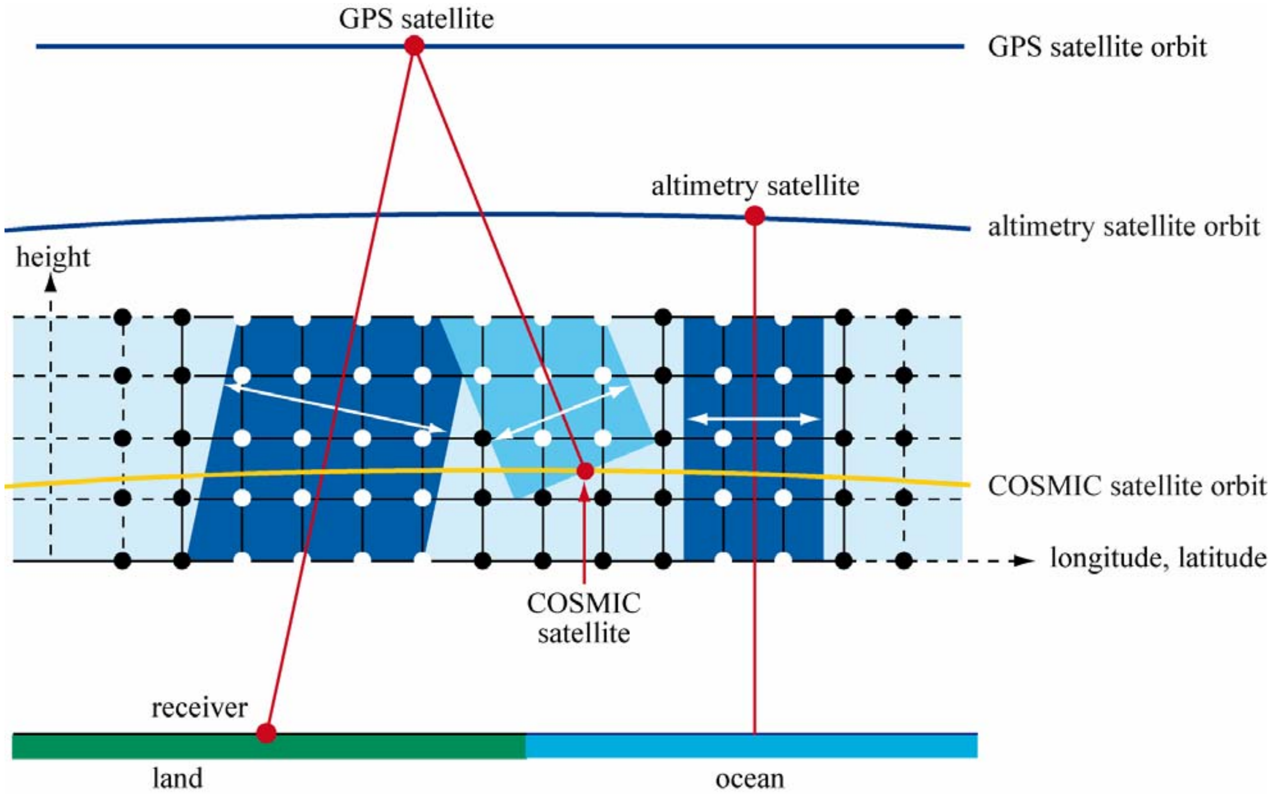
- The **series (scaling) coefficient** $d_{k1,k2}$ can be determined by the GNSS observation related to the piercing point (green dot).
- If the location of the observation lies close to the border of the foot print the coefficient is weakly determined $\rightarrow$ **prior information** for $d_{k1,k2}$.



Procedure (adjustment, MRR, data compression)



Input data



- The relative **GPS-LEO geometry** is changing rapidly (including **occultation** measurements).
- LEO measurements **stabilize** the estimation process.
- Dual-frequency **altimetry** satellites measure always the **vertical electron content** (more or less only over the oceans).

Combination of input data

- For each of the **different observation types** (including altimetry and terrestrial measurements) it is possible to derive an **observation equation** for estimating the unknowns of the electron density B-spline model within a specific spatio-temporal (4D) region $\mathbb{B}^4$.

- We assume now that for each of the altogether p techniques an **observation vector** $\boldsymbol{y}_i$, $i = 1, \ldots, p$ is available, e.g. $\boldsymbol{y}_1$ may contain all reduced GNSS geometry-free observations measured from ground stations within $\mathbb{B}^4$.

- Hence, for each observation vector $\boldsymbol{y}_i$ a **Gauss-Markov model**

$$\boxed{\boldsymbol{y}_i + \boldsymbol{e}_i = \boldsymbol{A}_i\,\boldsymbol{\beta} \qquad \text{with} \qquad D(\boldsymbol{y}_i) = \sigma^2_{y,i}\,\mathbf{P}^{-1}_{y,i}}$$

  can be established.

- The quantities $\sigma^2_{y,i}$ and $\mathbf{P}_{y,i}$ are the unknown variance factor and the given positive definite weight matrix.

- The vector $\boldsymbol{\beta}$ consists of the unknown **B-spline scaling coefficients** $d_{k_1,k_2,k_3}(t)$, the unknown delays β_R and β_S and other auxiliary parameters. Note, the scaling coefficients have to be discretized w.r.t. the time.

Combination of input data — 2

- With the **prior information** for the expectation vector $E(\boldsymbol{\beta}) = \boldsymbol{\mu}_\beta$ and the covariance matrix $D(\boldsymbol{\beta}) = \mathbf{P}_\beta^{-1}$ for the unknown B-spline scaling coefficients the additional linear model

$$\boldsymbol{\mu}_\beta + \mathbf{e}_\beta = \boldsymbol{\beta} \qquad \text{with} \qquad D(\boldsymbol{\mu}_\beta) = \sigma_\beta^2\,\mathbf{P}_\beta^{-1}$$

  can be formulated; $\mathbf{e}_\beta$ = error vector of the prior information, σ_β^2 = unknown variance factor.

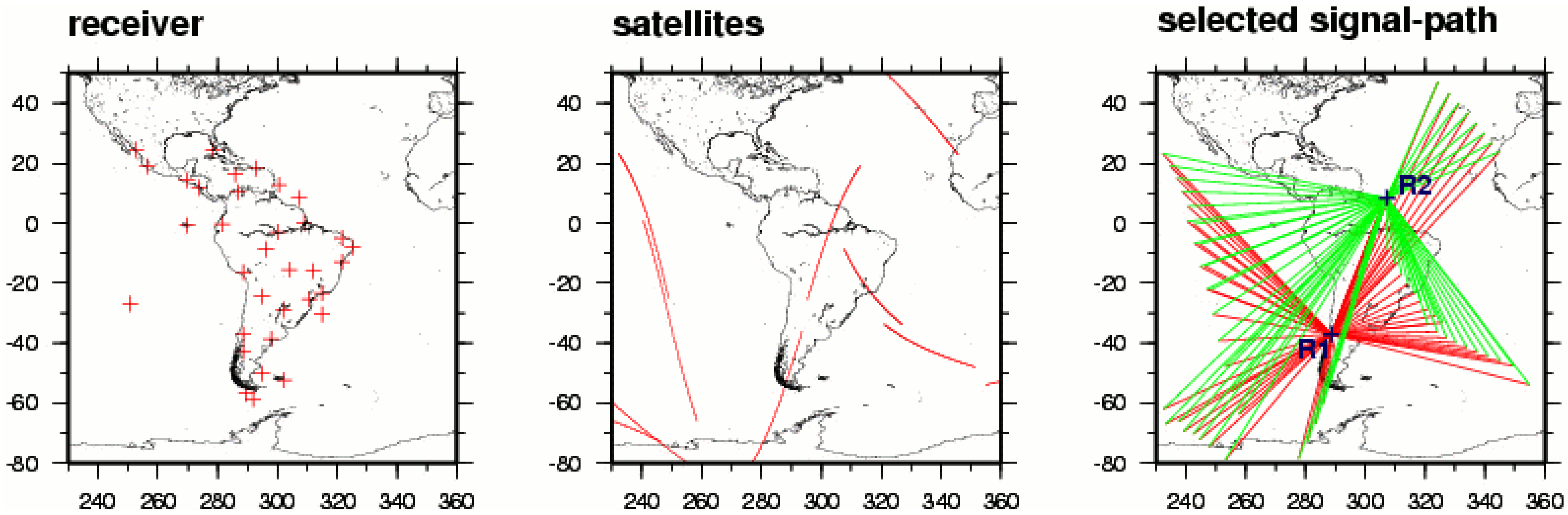
- The combination of the models yields the **normal equation system**

$$\boxed{\left(\sum_{i=1}^{p} \frac{1}{\sigma_{y,i}^2}\,\mathbf{A}_i^T\,\mathbf{P}_{y,i}\,\mathbf{A}_i + \frac{1}{\sigma_\beta^2}\mathbf{P}_\beta\right)\widehat{\boldsymbol{\beta}} = \sum_{i=1}^{p} \frac{1}{\sigma_{y,i}^2}\,\mathbf{A}_i^T\,\mathbf{P}_{y,i}\,\boldsymbol{y}_i + \frac{1}{\sigma_\beta^2}\,\mathbf{P}_\beta\,\boldsymbol{\mu}_\beta}$$

  for the **unknown parameter vector** $\boldsymbol{\beta}$ with **unknown variance components** $\sigma_{y,i}^2$, $i = 1, \ldots, p$ and σ_β^2.

- For solving this problem we apply a **fast Monte-Carlo implementation** of the iterative maximum-likelihood **V**ariance **C**omponent **E**stimation (MCVCE, Koch and Kusche, 2001).

GNSS modeling



- We chose 34 stations in Central and South America as well as 69 positions along the GPS satellite orbits. Hence, we evaluate altogether 34 · 69 = 2346 simulated observations.
- The ray-paths with a zenith angle larger then 80° were neglected.
- For each measurements we have to evaluate the integrals over the 3-dimensional B-spline functions along the ray-paths. We perform this integration by **Gauss quadrature**.

For more details see:

Zeilhofer, C. et al. (2008): Regional 4-D modeling of the ionospheric electron density from satellite data and IRI. Advances in Space Research (JASR), accepted, in press

Zeilhofer, C. (2008): Multi-dimensional B-spline modeling of spatio-temporal ionospheric signals. DGK, Series A, 123

GNSS modeling — 2

- To test our B-spline approach we simulate GNSS observations. For that purpose we consider the **residual electron density**

$$\Delta N(\mathbf{x},t) = N_{sim}(\mathbf{x},t) - N_{ref}(\mathbf{x},t)$$

wherein $N_{sim}(\mathbf{x},t)$ are simulated electron density values and $N_{ref}(\mathbf{x},t)$ are the values of the reference model, namely a low-pass filtered IRI version.

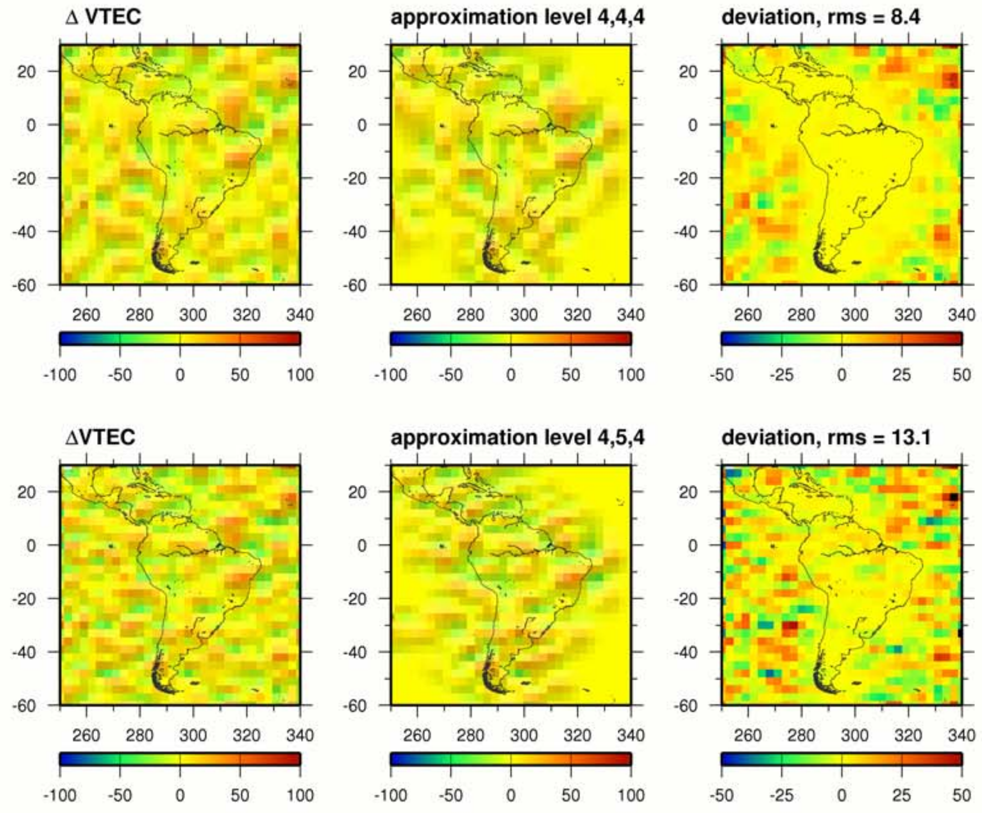
- To simulate GPS observations we set time $t = t_0$ and integrate along the ray-paths over the residual electron density $\Delta N(\mathbf{x},t)$ and get **residual STEC** values

$$\Delta STEC(R,S) = \int_R^S \Delta N(\mathbf{x},t)\, ds$$

- Replacing the integrand by the 3D series expansion for the residual electron density yields

$$\Delta STEC(R,S) + e(R,S) = \sum_{k_1=0}^{K_1-1} \sum_{k_2=0}^{K_2-1} \sum_{k_3=0}^{K_3-1} d_{k_1,k_2,k_3}\; k_{k_1,k_2,k_3}(R,S).$$

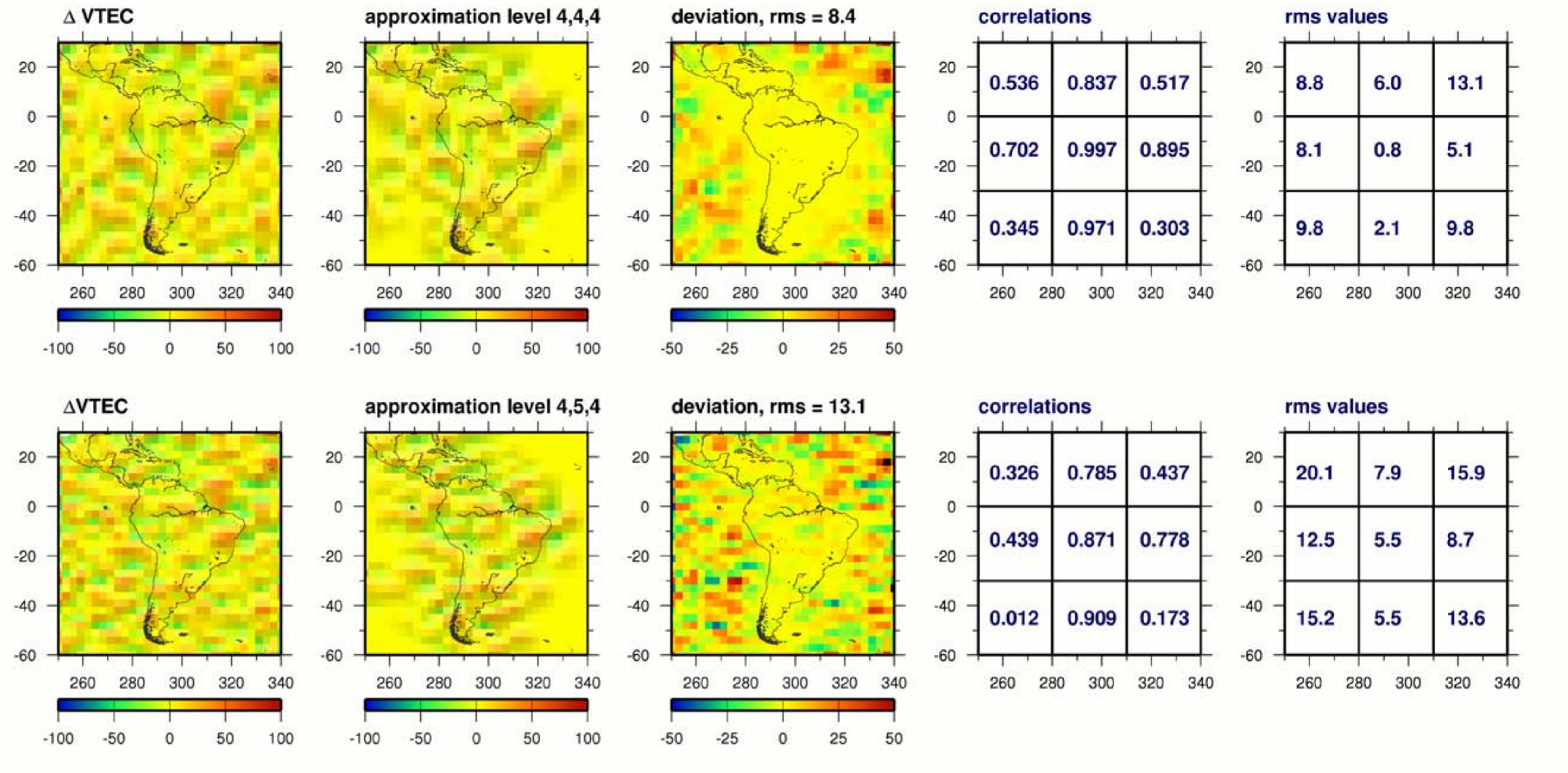
GNSS modeling — 3



- The two rows show respectively:
  - **left panels**: 2 simulated residual VTEC data sets (bottom: higher frequency content in latitude)
  - **mid panels**: estimated B-spline models of the levels (4,4,4) and (4,5,4) respectively
  - **right panels**: deviations of the input data and the estimations

- The distribution of the data (receiver- and satellite-positions) influences the deviations.
- We divide the area into 9 subareas and calculate the correlations and rms values of the deviations.
- Over the continent we have high correlations and low rms values, this is due to the advantageous distribution of the ray-paths.

GNSS modeling — 3

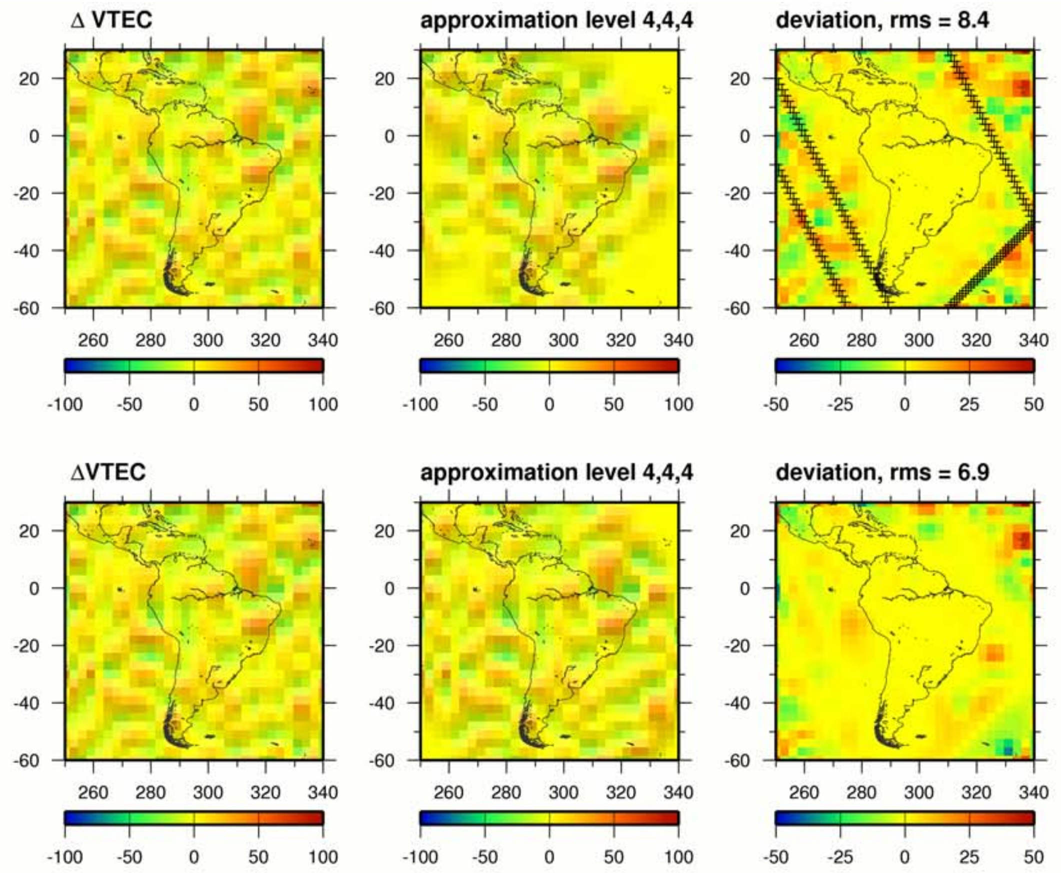


- The distribution of the data (receiver- and satellite-positions) influences the deviations.
- We divide the area into 9 subareas and calculate the correlations and rms values of the deviations.
- The combination of **altimetry** and GNSS observations combines observations that lie mainly above the ocean and above the continent.

GNSS modeling — 4

- We add an additional linear model for altimetry observations. The observation equation reads

$$\Delta VTEC(R,S)+e(R,S)=\sum_{k_1=0}^{K_1-1}\sum_{k_2=0}^{K_2-1}\sum_{k_3=0}^{K_3-1} d_{k_1,k_2,k_3}\ k_{k_1,k_2,k_3}(R,S).$$



- The black crosses show the observation sites of the simulated altimetry measurements.
- The combination improves clearly the accuracy of the results over the oceans.
- The bottom panels show the combination results.
- The rms value decreases from 8.4 TECU to 6.9 TECU.

GNSS modeling — 4

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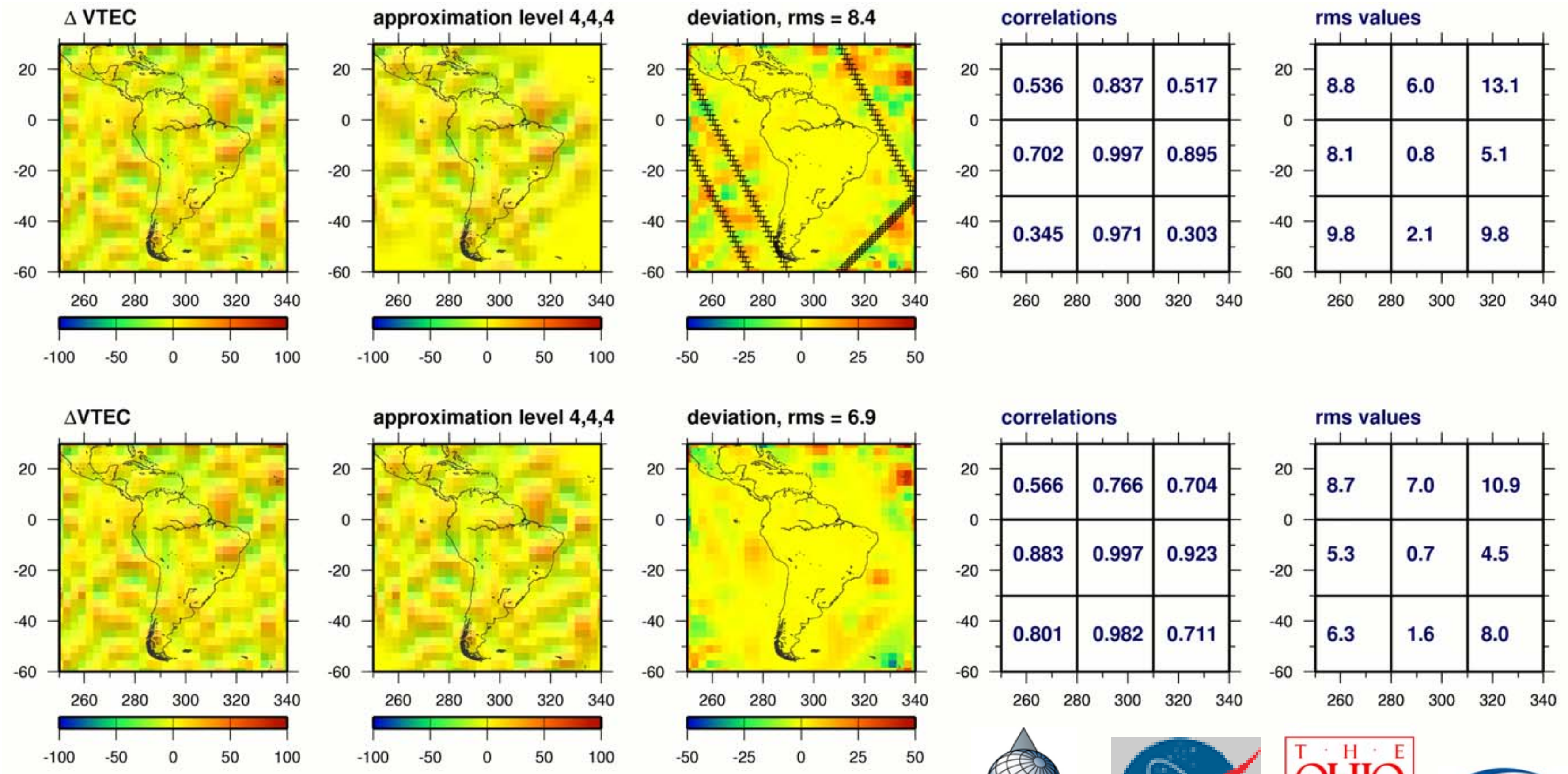
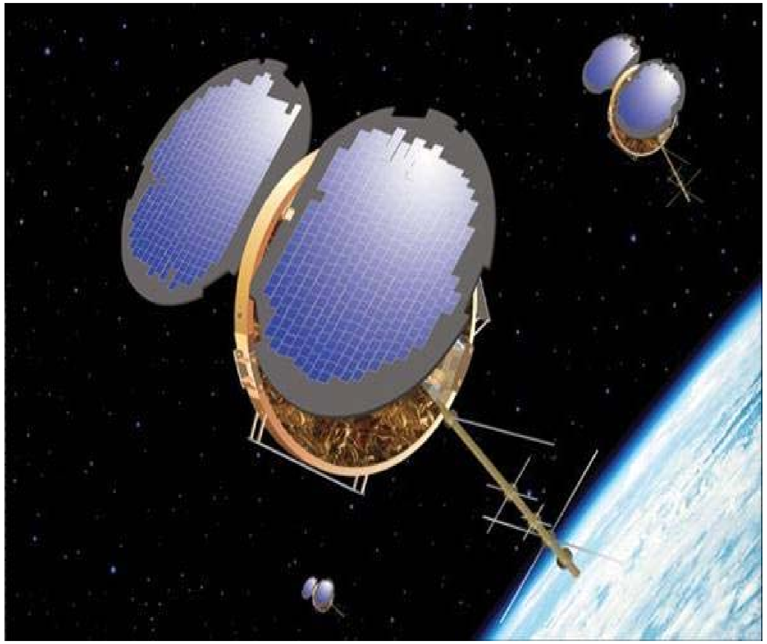
$$\Delta VTEC(R,S) + e(R,S) = \sum_{k_1=0}^{K_1-1} \sum_{k_2=0}^{K_2-1} \sum_{k_3=0}^{K_3-1} d_{k_1,k_2,k_3}\; k_{k_1,k_2,k_3}(R,S).$$



COSMIC input data

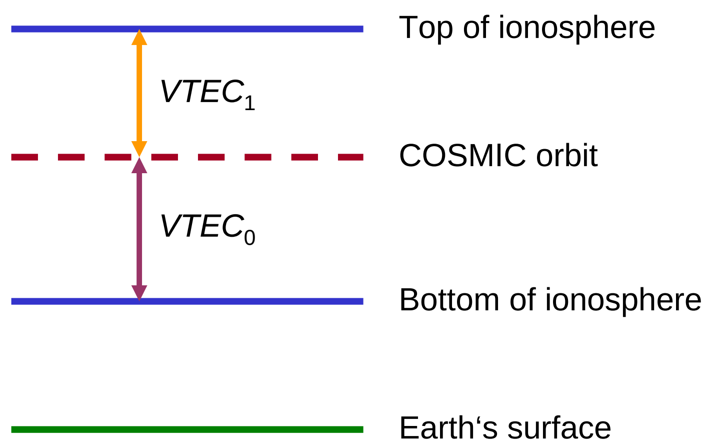
- **COSMIC/FORMOSAT-3** is the **C**onstel-lation **O**bserving **S**ystem for **M**eteorology, **I**onosphere and **C**limate and Taiwan's **Formo**sa **Sat**ellite Mission #**3**, a joint Taiwan-U.S. project.
- The six COSMIC satellites were success-fully launched on April 14th, 2006. Over the first year, the satellites had been gradually boosted from their initial orbit of 400 km to the final orbit of 800 km.
- Each spacecraft is equipped with three instruments, namely a **GPS (occultation) receiver** (GOX), a **tiny ionospheric pho-tometer** (TIP) and a **tri-band beacon** (TBB).



from: L.-C. Tsai,C.H. Liu, and T.T. Hsiao

COSMIC input data — 2

- In our investigation we calculate the electron density in a pre-processing step from so-called **compensated STEC** values by an "improved" Abel transform.
- To be more specific, we do not assume locally spherical symmetry within the ionosphere, but consider the effect of **large-scale horizontal gradients** and/or **inhomogeneous electron density distribution**.
- For more details see the paper: Tsai, Lung-Chih and Wei-Hsiung Tsai (2004): Improvement of GPS/MET ionospheric profiling and validation using the Chung-Li ionosonde measurements and the IRI model. (TAO,15(4), 589-607)
- A "**measured**" **VTEC** value (observation) mean the sum of two parts:
  - $VTEC_0$ is the integration of the calculated electron density along the vertical from bottom to orbital height.
  - $VTEC_1$ is an extrapolated value provided by Lung-Chih Tsai using a Chapman layer model.

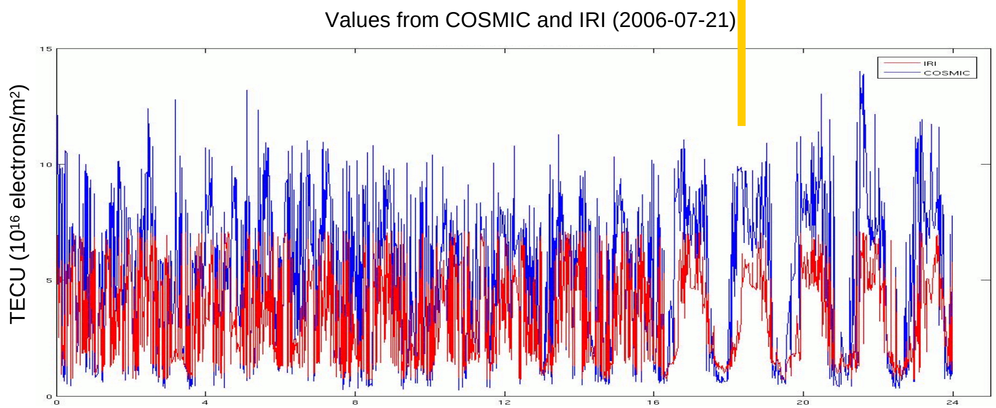


COSMIC input data — 3

- The table shows the statistical values of the COSMIC input data and the corresponding VTEC values from IRI.

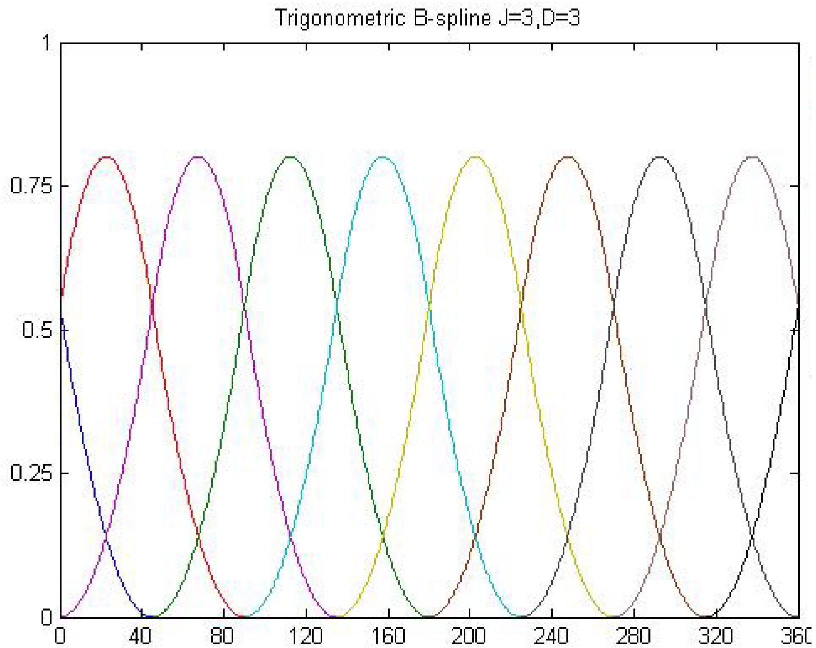
Edited (2547 points)		
	COSMIC	IRI
Mean	5.18	3.45
Standard Deviation	3.22	2.03
RMS	6.10	4.00
Correlation coefficient	0.76	

- The lower figure shows the "measured" COSMIC VTEC input data $VTEC_0$ between orbital height and the bottom of the ionosphere and the corresponding $VTEC_0$ values from IRI. (outlier detection was performed)

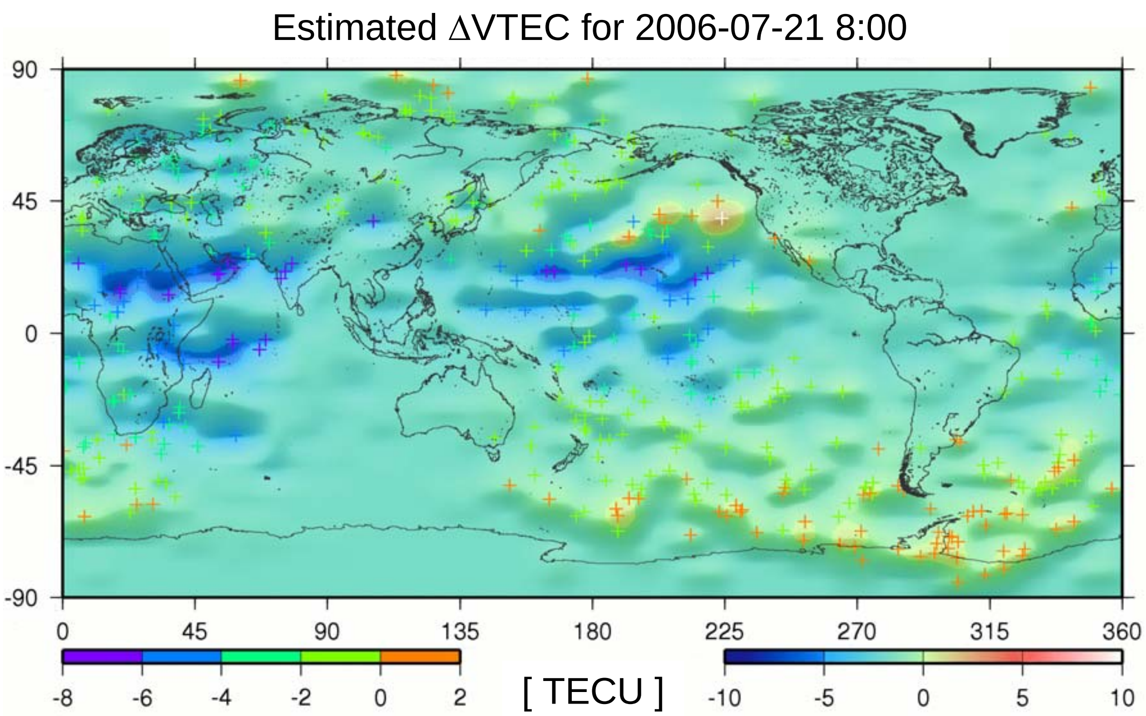


COSMIC modeling

- In case of **regional modeling** the ray-paths of **occultation** measurements might leave the region of investigation → **global model approach.**
- The figure shows the **trigonometric B-spline** functions defined on the interval between 0° to 360°.
- These functions are also **compactly supported**, i.e. the non-zero (influence) zone is finite.
- In opposite to the endpoint-interpolating B-splines the trigonometric B-splines are "**wrapping around**".



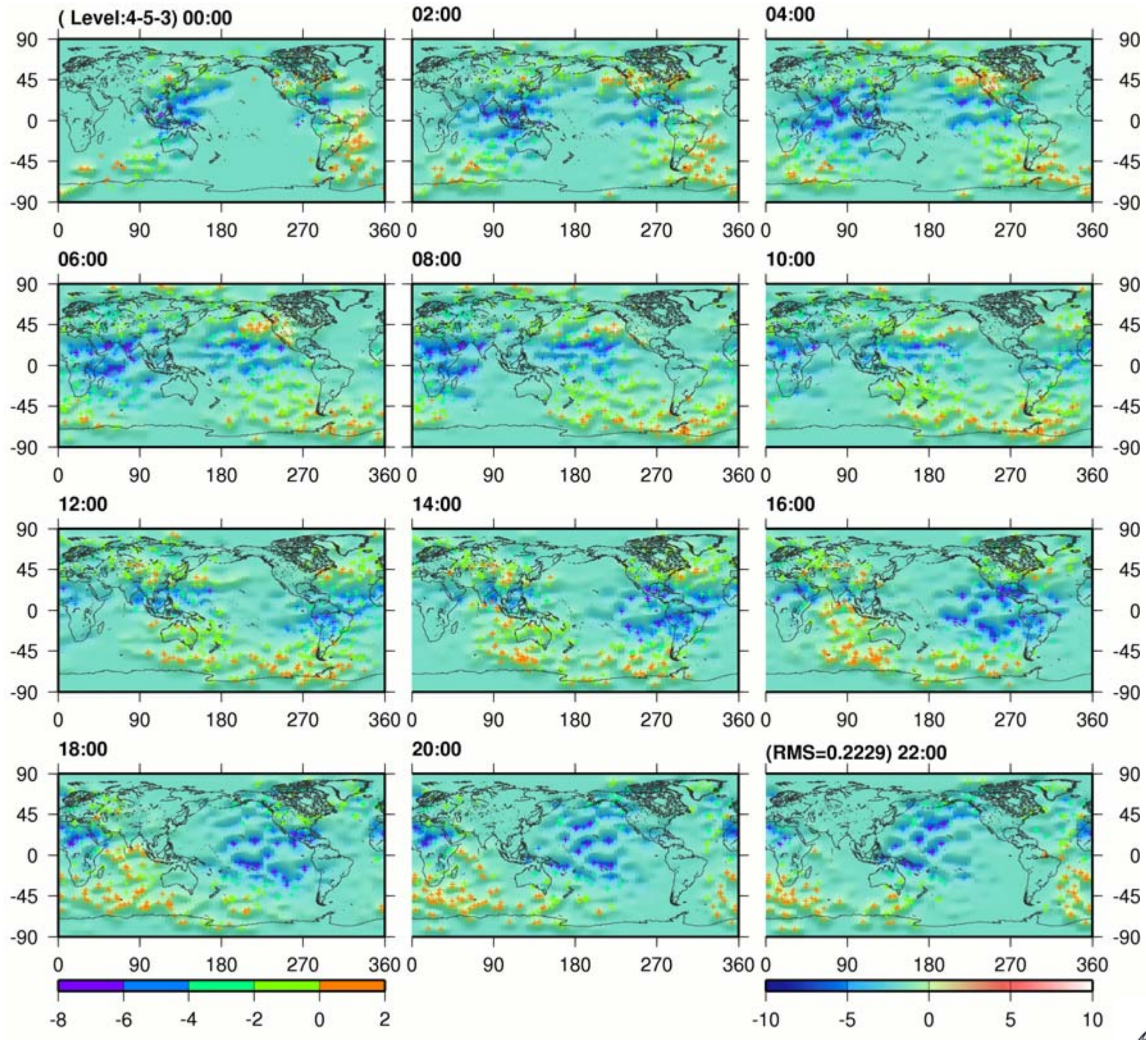
COSMIC modeling — 2



- We apply this global approach for modeling ΔVTEC, i.e. we introduce **trigonometric B-splines** in longitude (level 4).
- For modeling the latitude- and time-dependency we again use **endpoint interpolating B-splines** (level 5 and level 3, resp.).
- In the figures the crosses indicate the observation sites (blue and green colours mean negative values, orange positive values).
- Structures are modelable only in regions where observations are available.

COSMIC modeling — 2

Estimated ΔVTEC 2006-07-21



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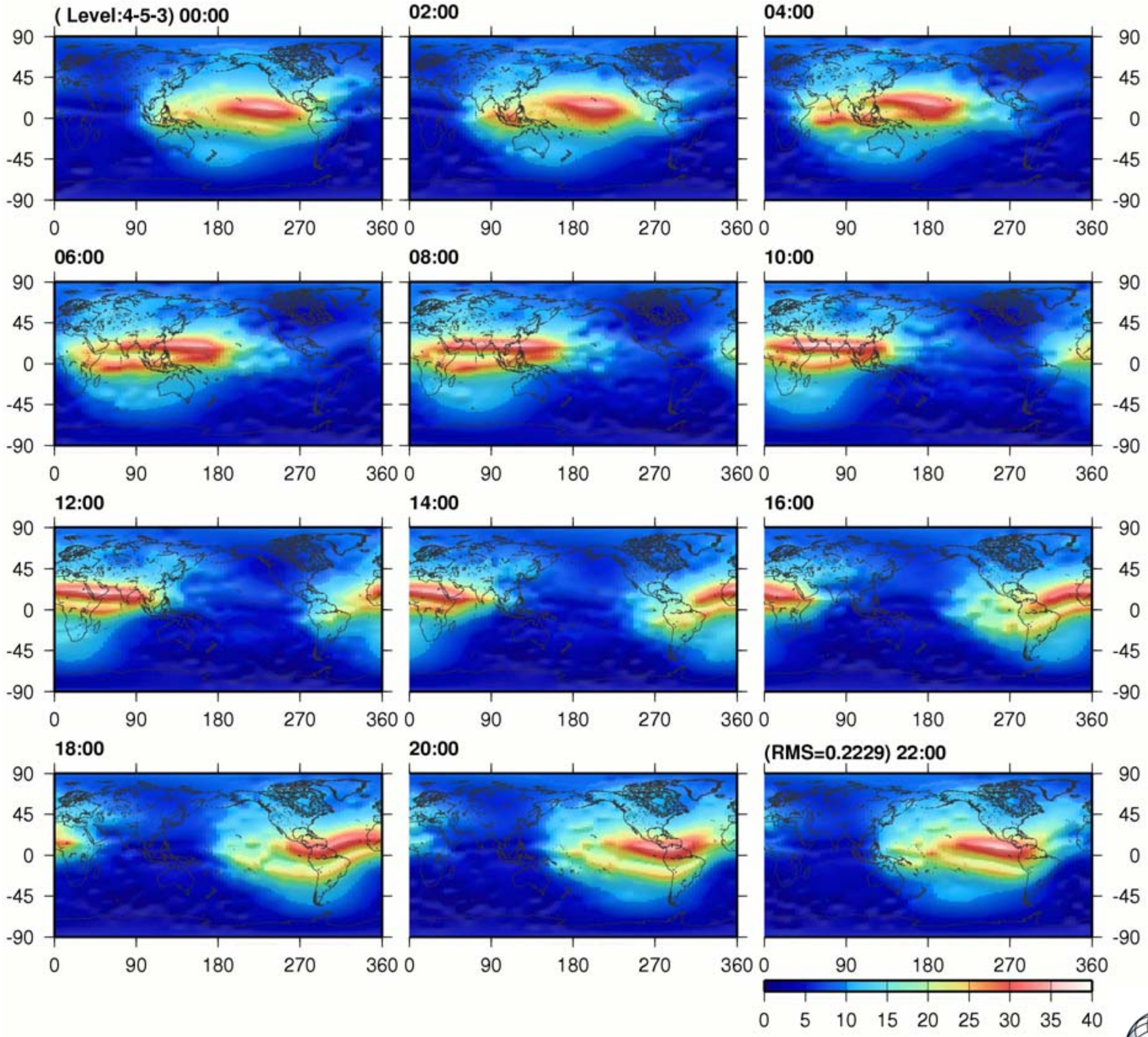
COSMIC modeling — 3

Estimated VTEC 2006-07-21



- The panels show the **estimated VTEC** from COSMIC by adding the estimation of ΔVTEC to the reference values from IRI.
- The shown results are "**snapshots**" with a temporal resolution of 2 hours.
- Estimated VTEC values can be calculated **for each time** t of July 21st, 2006.
- **Validation**, e.g. by ionosondes necessary.

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Final remarks

- We demonstrated how a **multi-dimensional** model of **ionospheric signals** can be performed.
- We presented first a **3D B-spline approach** with **time-dependent coefficients** for GNSS geometry-free observations.
- The accuracy was improved by incorporating **altimetry observations**.
- We further presented a **3D trigonometric/B-spline approach** for modeling VTEC data from COSMIC.
- We showed just some **components** of the procedure. The **next step** will focus on the **joint evaluation** of the real data from GNSS, altimetry and COSMIC.
- Since the presented procedure can be applied to **regional** or **local** areas, features like the **Equatorial Anomaly** can be handled appropriately.
- Finally, an **update of climatological parameters** of IRI can be performed.