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Optimal aperture synthesis radar imaging

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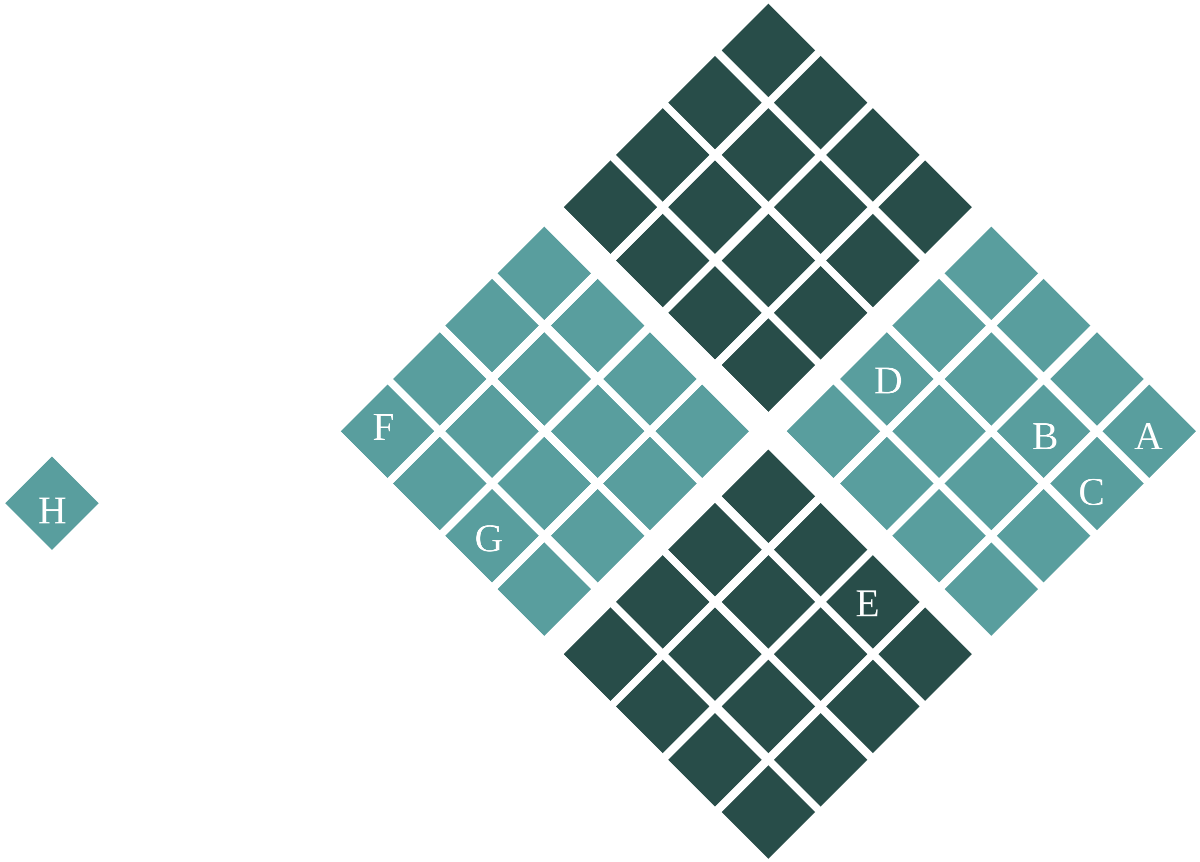
Basic problem



$$\underbrace{V(k\mathbf{d})}_{g_j} = \int_{2\pi} \underbrace{A_N(\sigma)B(\sigma)}_{f_i} \underbrace{e^{jk\mathbf{d}\cdot\sigma}}_{h_{ij}} d\Omega$$

MaxEnt algorithm

Shannon's entropy suggests model form

$$S = -f_i \ln f_i / F$$

$$E(f(\lambda_j, \gamma)) = S + \lambda_j (g_j - f_i h_{ij}) + \gamma \left(I_i f_i - F \right)$$

$$f_i = F \frac{e^{-\lambda_j h_{ij}}}{Z}$$

$$Z = I_i e^{-\lambda_j h_{ij}}$$

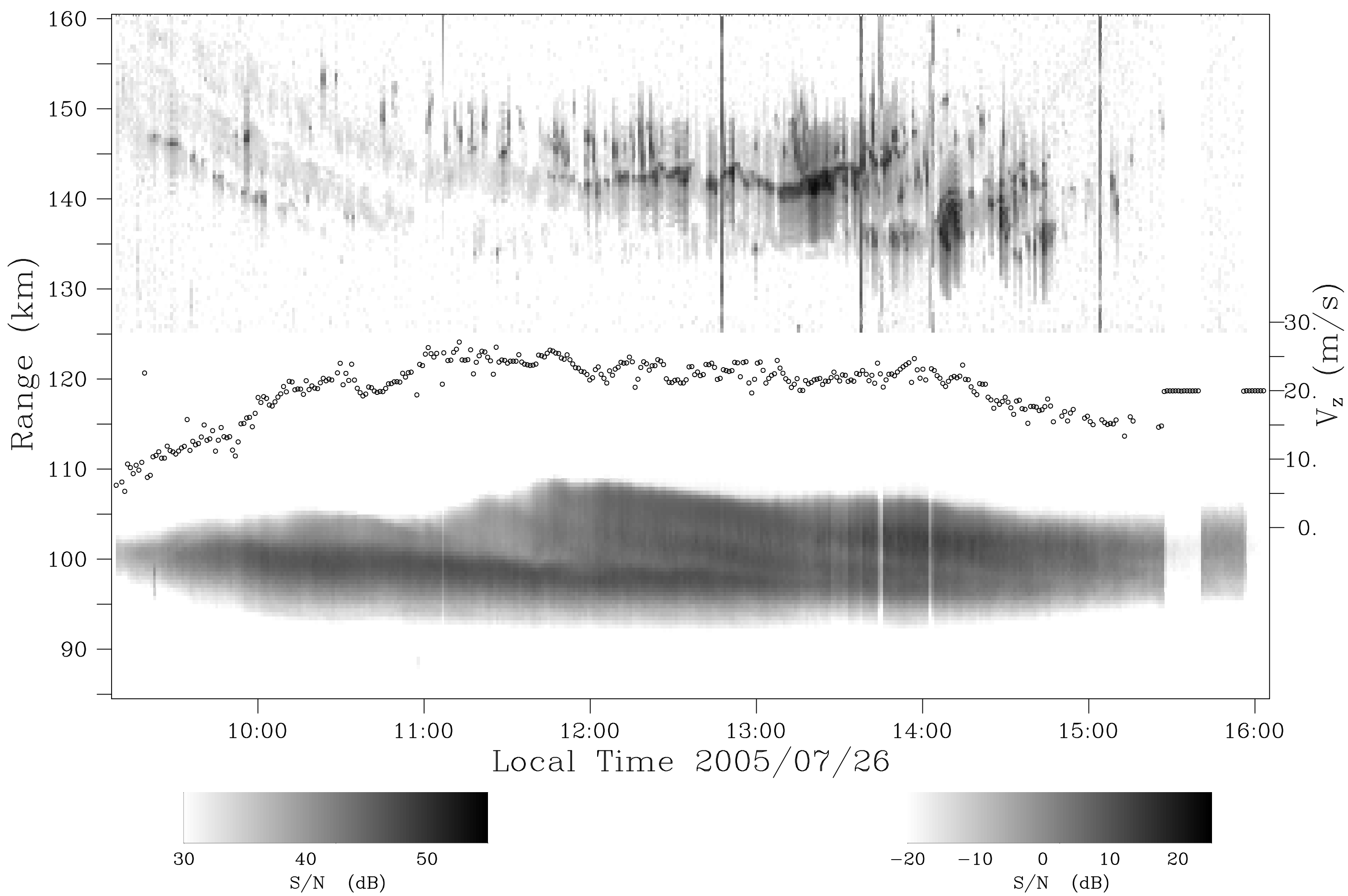
MaxEnt algorithm

$$
\begin{aligned}
E(f(e_j,\lambda_j,\Lambda)) \\
&= S + \lambda_j(g_j + e_j - f_i h_{ij}) + \Lambda\left(e_j^2\sigma_j^{-2} - \Sigma\right) \\
&= \lambda_j(g_j + e_j) + F\ln Z + \Lambda\left(e_j^2\sigma_j^{-2} - \Sigma\right)
\end{aligned}
$$

... resulting is a system of coupled nonlinear equations to solve for Σ = 2N$+1$ Lagrange multipliers λ_j. We use a globally convergent Hybrid method and specify the Jacobian matrix explicitly.

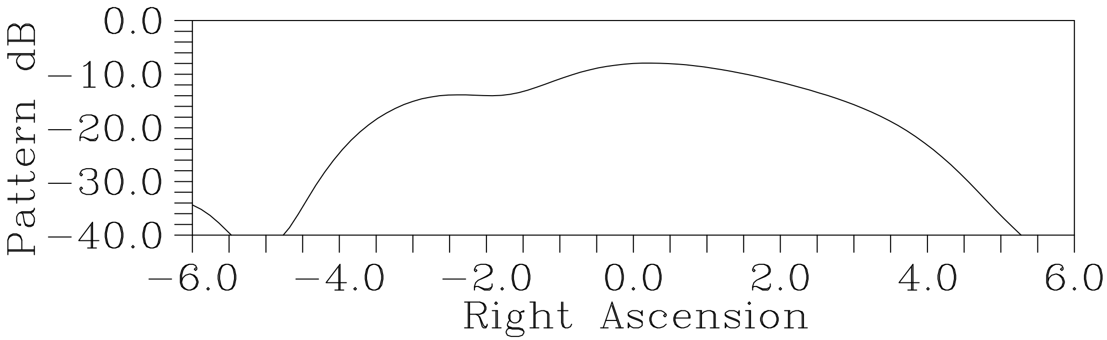
Solutions are found for each Doppler bin in each range gate in each time step of an image on the basis of data (g_j) and their theoretical variances σ_j.

Large scale electrojet waves



July 26, 2005

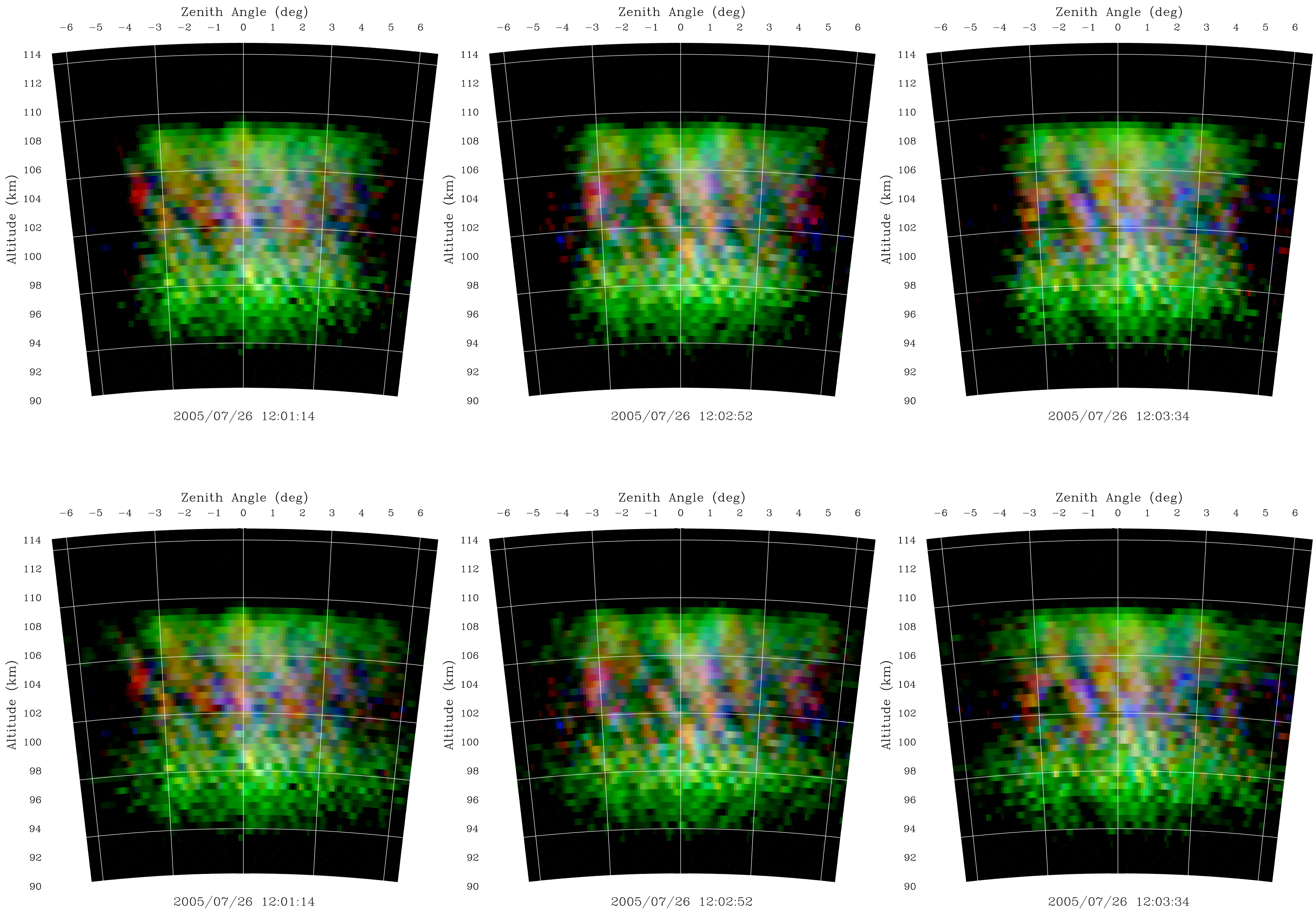
Upgrade 1. Specify two-way antenna radiation pattern



$$S' \quad = \quad -f_i \ln(f_i/p_i F)$$

$$\begin{aligned} f_i &= F p_i \frac{e^{-\lambda_j h_{ij}}}{Z} (\mathrm{no\,sum}) \\ Z &= p_i e^{-\lambda_j h_{ij}} \end{aligned}$$

Comparison


Zenith Angle (deg)
Altitude (km)
2005/07/26 12:01:14
Zenith Angle (deg)
Altitude (km)
2005/07/26 12:02:52
Zenith Angle (deg)
Altitude (km)
2005/07/26 12:03:34
Zenith Angle (deg)
Altitude (km)
2005/07/26 12:01:14
Zenith Angle (deg)
Altitude (km)
2005/07/26 12:02:52
Zenith Angle (deg)
Altitude (km)
2005/07/26 12:03:34

Upgrade 2. Utilize full error covariance matrix

$$
\begin{aligned}
\delta^2 &= \langle (\hat{\rho}_{12} - \langle \hat{\rho}_{12} \rangle) (\hat{\rho}_{34} - \langle \hat{\rho}_{34} \rangle)^* \rangle \\
&= \frac{1}{m} \Bigg[\rho_{13}\rho_{24}^* - \frac{1}{2}\rho_{34}^* (\rho_{13}\rho_{23}^* + \rho_{14}\rho_{24}^*) \\
&\quad - \frac{1}{2}\rho_{12} (\rho_{13}\rho_{14}^* + \rho_{23}\rho_{24}^*) \\
&\quad + \frac{1}{4}\rho_{12}\rho_{34}^* \left(|\rho_{13}|^2 + |\rho_{14}|^2 + |\rho_{23}|^2 + |\rho_{24}|^2 \right) \Bigg]
\end{aligned}
$$

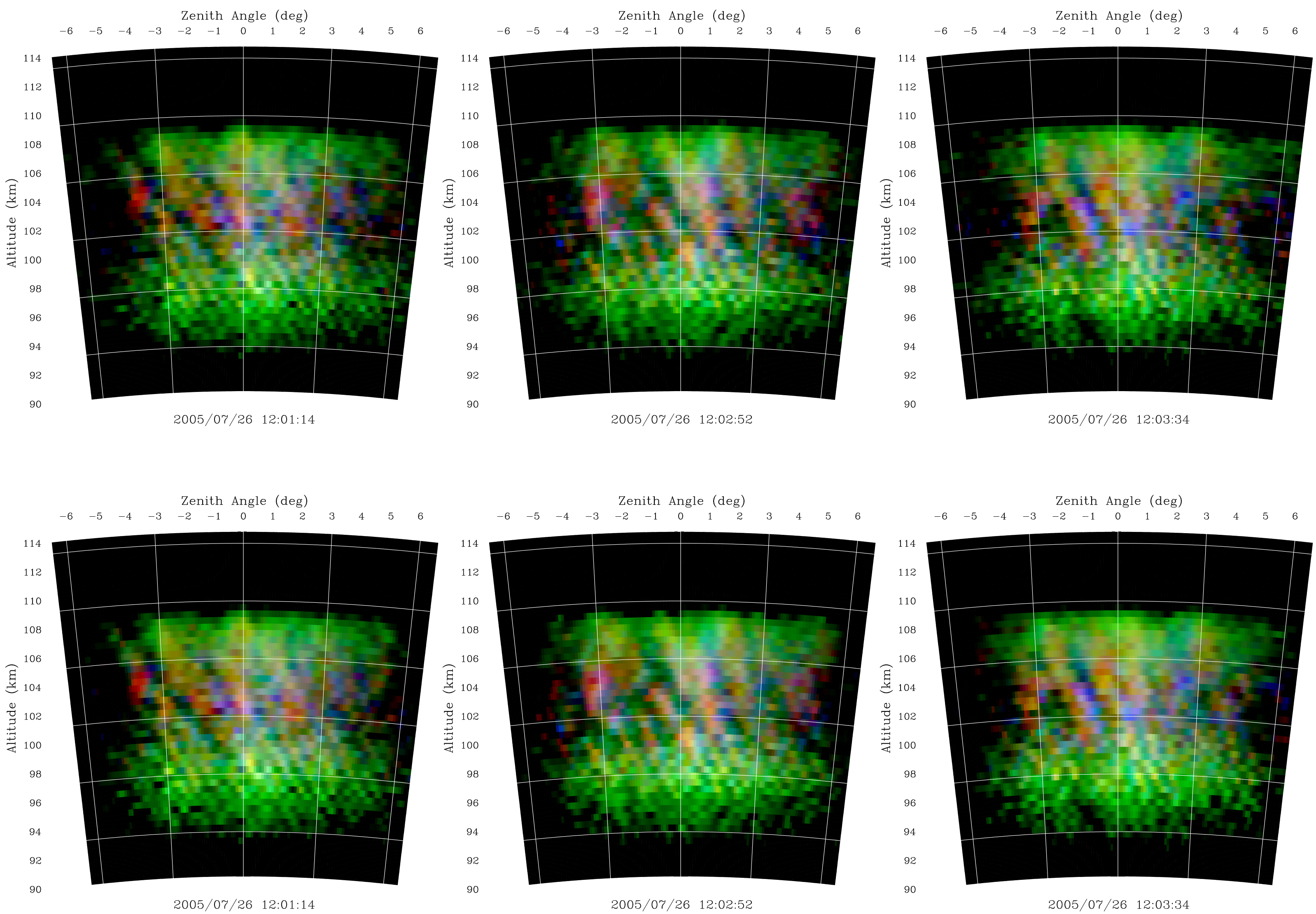
$$
\delta'^2 = \langle (\hat{\rho}_{12} - \langle \hat{\rho}_{12} \rangle) (\hat{\rho}_{34} - \langle \hat{\rho}_{34} \rangle) \rangle
$$

Real and imaginary parts separately

$$
\begin{aligned}
\langle e_{r12} e_{r34} \rangle &= \Re(\delta^2 + \delta'^2)/2 \\
\langle e_{i12} e_{i34} \rangle &= \Re(\delta^2 - \delta'^2)/2 \\
\langle e_{r34} e_{i12} \rangle &= \Im(\delta^2 + \delta'^2)/2 \\
\langle e_{r12} e_{i34} \rangle &= \Im(\delta'^2 - \delta^2)/2
\end{aligned}
$$

Perform similarity transformation, deriving new variances from eigenvalues of old error covariance matrix and new visibilities, point spread function from transformation composed of the eigenvectors.

Comparison



Zenith Angle (deg)
−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6
Altitude (km)
114 112 110 108 106 104 102 100 98 96 94 92 90
2005/07/26 12:01:14
2005/07/26 12:02:52
2005/07/26 12:03:34
2005/07/26 12:01:14
2005/07/26 12:02:52
2005/07/26 12:03:34

Summary

- The MaxEnt strategy is to limit the space of candidate images to those that are positive definite, consistent with data to the degree required by experimental confidence limits, smooth (in some sense), and most representative of the class of possible solutions.
- It is designed to make maximum use of available information while remaining minimally committal to things unknown.
- The prior probability of the image should be modified to reflect the actual two-way radiation pattern of the imaging experiment.
- The full error covariance matrix is not diagonally dominant in the high SNR, high coherence limit and should be incorporated in the inversion by similarity transformation.
- Future improvements include incorporation about correlated image features and optimal selection of Σ.