Optimum Antenna Height for Single-hop Oblique Incidence (NVIS) Propagation

B.A.Austin

Hon. Senior Fellow, Dept. Of Electrical Engineering & Electronics

University of Liverpool, UK

Introduction

While doing some analysis of the effects of antenna height on its radiation patterns an interesting and potentially useful result emerged.

It was shown theoretically (and confirmed by subsequent NEC analysis) that for any prevailing critical frequency (f_{oF2}), there is an optimum height yielding maximum antenna gain for a horizontal, single element antenna when used in NVIS single-hop, oblique incidence applications to a distance of about 500km. This height (in metres) when the antenna is above perfect ground is given by $h_{opt} = \frac{88}{fo_{F2}}$, where f_{oF2} is the critical frequency of the F2 region of the ionosphere in MHz. Thus, for a critical frequency at the point of reflection of 5.87 MHz, for example, the optimum antenna height is 15m.

At this height the angle at which the maximum radiation occurs from the antenna will be optimum for all distances from virtually zero out as far as the usually assumed NVIS oblique incidence limit of 500 km. Naturally, the optimum transmission frequency changes as the range increases since it is influenced by the prevailing MUF, which is both a function of the critical frequency and the transmission range. This frequency is easily determined if the critical frequency of the F2 layer is known from ionospheric soundings. The effect of real ground below the antenna is taken into account when determining the optimum angle of radiation.

Analysis

Near vertical incidence skywave (NVIS) propagation [1] is typically analysed by assuming a flat earth model. This is valid up to a distance of about 500 km between transmitter and receiver [2]. It will be appreciated that for a typical F2 layer height of 250 km, this maximum range implies that the angle of incidence is 45 degrees, which suggests that the term NVIS is being used rather loosely at that limiting distance. However, with due allowance for such terminological inexactitude, “NVIS” will be taken to mean continuous, uninterrupted propagation to a maximum distance of about 500 km radius around the transmitting antenna, as illustrated in Figure 1 below.

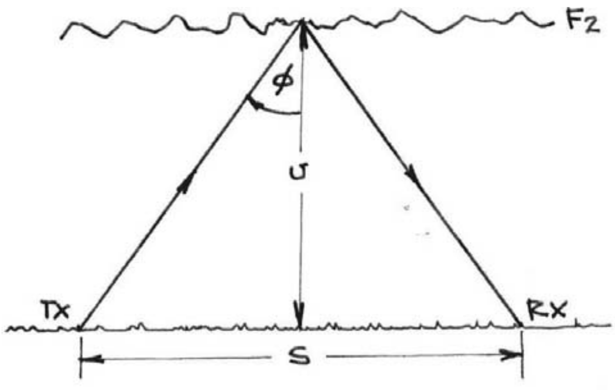

Figure1: NVIS propagation geometry.

The optimum traffic frequency (FOT) for any such path is given by FOT $= 0.85\,MUF$, where MUF is the maximum usable frequency for the path in question. It is based on the lower decile of the daily values of the operational MUF at a given time. This means that use of the FOT will ensure propagation over a given path during 90% of a specified period, usually a month. In the analysis that follows, the FOT will therefore be the chosen operating frequency and will henceforth be designated as f. Since the MUF and critical frequencies are related by the so-called "secant law", we can write the following: $f = 0.85\, f_{oF2}$ sec Ø where Ø is the angle of incidence between the upward propagating ray and the F2 region of the ionosphere. From the diagram above it is obvious that Ø $= tan^{-1}(\,{}^{S}/_{2v})$, where s is the skip distance for that angle and ionospheric layer height of v.

In practice, every antenna erected above the earth operates in conjunction with its image that is situated an equal distance below the air-earth interface as the antenna is above it. This is illustrated in Figure 2 below, where the horizontal antenna (viewed end-on) is at height h above the ground and its image is therefore at a distance h below the surface. The distance between them is $d = 2h$.

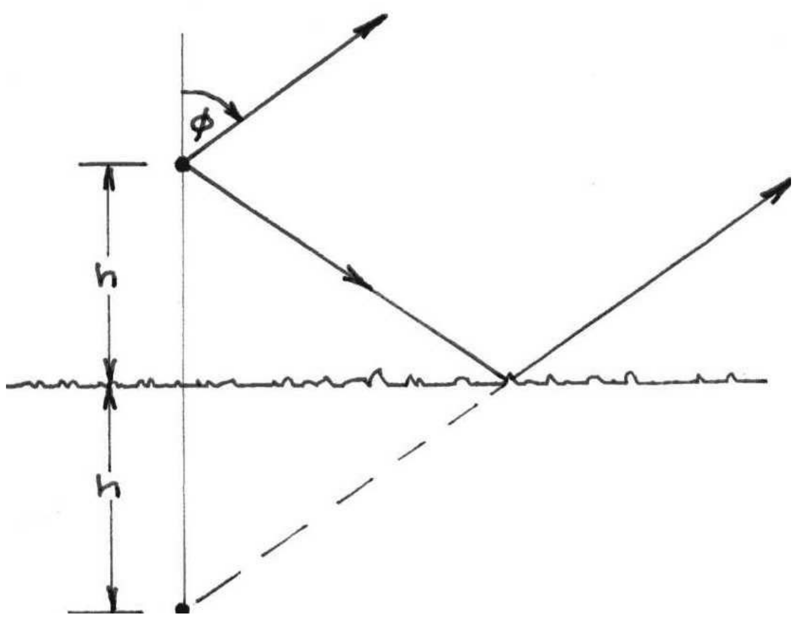

Figure 2: Horizontal antenna plus its image viewed end on.

The angle at which the electromagnetic radiation leaves the antenna (relative to the vertical) for propagation over any given distance is given by ∅. It will be seen to be the same angle as the angle of incidence upon the ionosphere in Figure 1. This fact leads to an important practical result that does not appear to have been noted before.

The antenna plus its image, when viewed in this elevation plane, constitutes what is essentially a two-element array of point sources spaced a distance $d = 2h$ apart. In practice, of course, the typical NVIS antenna would be a horizontally polarised dipole or end-fed element. Its current distribution must favour radiation towards the zenith when erected close to the ground. In the elevation plane the radiation pattern is independent of that distribution of current. On the assumption that the ground plane is a perfect conductor, the boundary condition at that surface requires that the resulting tangential electric field go to zero ($E_{tan} = 0$), hence the fields produced by the antenna and its image must be out of phase (as would be the currents that produced them in the equivalent two-element array in free space). From this we have that the total electric field E_T is given by the phasor sum of the fields produced by the antenna E_a and its image E_i. Thus, on the assumption that those currents are of equal amplitude we have, for the total phase shift φ between the antenna and its image that:

$E_T = E_a + E_i = E\left(1 + e^{j\varphi}\right);\quad \text{where} \quad \varphi = \frac{4\pi hf}{c}\cos\emptyset - \pi$ and c is the speed of light.

Naturally this expression for E_T is a maximum when φ is zero. It is known as the *principal maximum* of the array [3]. The angle at which maximum radiation occurs, $\emptyset_{\max sig}$, is related to the antenna's height h and the operating frequency f. It follows directly from this relationship above with the frequency expressed in MHz. Thus it is easy to show that

$$\emptyset_{\max sig} = cos^{-1}\frac{75}{h\,f_{MHz}}$$

Since that angle ∅ is common to both the equations for the operating frequency in terms of the MUF, and that for the antenna situated above the ground, they can be combined as follows:

$f = 0.85 f_{oF2}\, sec\emptyset = {0.85 f_{oF2}}/{cos\emptyset}$, while $cos\emptyset = {75}/{hf}$, hence we have that

$h = \frac{75}{0.85}\frac{1}{f_{oF2}} = 88/f_{oF2}$.

This extremely simple relationship $h = \frac{88}{f_{oF2}}$ between the height of the antenna and the critical frequency of the ionosphere has interesting and important implications. It is in fact the optimum height that a horizontal antenna should be erected above the ground so that it always radiates its maximum signal at the optimum angle required for oblique incidence propagation over a single-hop path up to 500 km from the antenna at a frequency equal to the FOT for the path. It should be noted that this result is independent of the length of the antenna. This is because a horizontal antenna above its image behaves, in the elevation plane, as an array of two point sources.

The transmitting frequency (the FOT for the required path) required to satisfy this condition follows simply from the critical frequency and path geometry as

$f = 0.85\, f_{oF2}\ \sec\emptyset = 0.85\, f_{oF2}\sec[tan^{-1}(\frac{s}{2v})]$.

A numerical example

As an example both to illustrate the usefulness of these results and to confirm their accuracy by means of an EZNEC simulation, consider the case where the critical frequency is assumed to be 5 MHz and the virtual height of the F2 region is 250km (a typical, average value). The optimum height of the antenna is then $h_{opt} = \frac{88}{5} = 17.6m$.

Table 1 below then shows the required operating frequencies and the calculated angles ∅ for propagation over various distances between 100 and 500km. It also includes the value of ∅ given by EZNEC at which the gain of the antenna is a maximum. The exact agreement between them should be noted.

S (km)	f (MHz)	h (m)	h(λ)	Φ(degrees)	Φ (deg) EZNEC Perfect ground
	$(4.25 \sec tan^{-1} [^{S}/_{500}]$	$(^{88}/_{f_{oF2}})$	$[^{h\, f_{MHz}}/_{300}]$	$[\emptyset = tan^{-1}(s/500)]$	
100	4.33	17.6	0.25	11	11
200	4.58	17.6	0.27	22	22
300	4.96	17.6	0.29	31	31
400	5.44	17.6	0.32	39	39
500	6.01	17.6	0.35	45	45

Table 1: Operating frequencies (FOT) and angles for various distances.

Note that the antenna is seen to be at a height of a quarter wavelength when the signal is semi-vertically incident on the ionosphere (when s = 100km, $\emptyset = 11^0$). This is true for the case of a perfectly conducting ground beneath the antenna. In reality, however, when the antenna is above typical rural ground ($\sigma = 5mS/m;\ \epsilon_r = 13$), its height will be slightly different because of the penetration of the electromagnetic fields into the ground due to the skin effect.

<u>Correction for real ground</u>

Placing the antenna at 17.6m above real ground with those characteristics and simulating the effect using EZNEC yielded the following changes in the angle at which the gain was a maximum. These results (shown in Table 2) enabled an approprriate correction factor g for the effects of real ground to be calculated as follows.

f (MHz)	$\emptyset(\boldsymbol{EZNEC\ real\ ground})^0$	$(\emptyset_{\boldsymbol{real}} - \emptyset_{\boldsymbol{perfect}})^{\mathbf{0}}$
4.33	29	29 - 11 = 18
4.58	34	34 – 22 = 12
4.96	40	40 – 31 = 9
5.44	45	45 – 39 = 6
6.01	50	50 – 45 = 5

Table 2: Change in the angle ∅ due to real ground beneath the antenna.

Real ground, for the same physical antenna height, causes the angle $\emptyset$ to increase (or the so-called radiation angle $\alpha = 90 - \emptyset$ to decrease). The antenna thus appears to be at an electrically greater height $h_{eff} = gh$ above the interface, where g is the appropriate correction factor. It can be calculated from the expression for the Principal Maximum of this simple two-element array of point sources where $\frac{ghf}{75} cos\emptyset = 1$, from which $g = {}^{4.3}/_{f\ cos\emptyset}$ when the antenna is 17.6m above the real ground interface. Thus $g \sim 1.12$ for the various angles $\emptyset$ across the 4.33 to 6.01 MHz frequency range and h_{eff} turns out to be essentially constant and equal to 19.6m.

This result allows us to modify the expression used to predict the angle at which the antenna gain above real earth is a maximum and by so doing this produces very good agreement with the EZNEC computation. Maximum radiation above typical real ground therefore occurs at $\emptyset_{\max rad} = cos^{-1}[{}^{67}/_{f_{MHz}\ h_{opt}}]$ .This particular result is not overly sensitive to ground conductivity changes by an order of magnitude. It is somewhat more sensitive to a change in relative permittivity and also more so at lower frequencies.

Summary

For optimum propagation over both NVIS and more oblique paths via the ionospheric F2 region, the optimum frequency in MHz (for a given f_{oF2}) is given by:

$f = 0.85\ f_{oF2} \sec[\tan^{-1}(s/2v)]$, where s is the ground (or skip) distance and v is the virtual height of the F2 region.

The optimum height (in metres) of the antenna above perfect ground is: $h_{opt} = 88/f_{oF2}$.

The angle of maximum antenna gain is: $\emptyset_{\max rad} = cos^{-1}[67/f_{MHz} h_{opt}]$ over typical real ground.

References

1. J.M Goodman, *HF Communications – Science and Technology,* 1992, Van Nostrand Reinhold, N.Y

2. K.Davies, *Ionospheric Radio,* 1990, Peter Peregrinus Ltd, London.

3. E.C.Jordan, *Electromagnetic Waves and Radiating Systems,* 1964, Prentice-Hall, N.J.

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