Interpretation Method for Response Waveform of Transient Scattered Magnetic Field by a Coated Cylinder Covered with a Thick Material

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Abstract—We briefly propose an interpretation method for a response waveform of a transient scattered magnetic field by a two-dimensional coated cylinder by using a novel time-domain (TD) asymptotic-numerical solution (TD-ANS). We assume that the thickness of a coating medium is thick as compared with the wavelength of a central angular frequency of a high-frequency (HF) pulse wave radiated from a magnetic line source. Multiple reflected waves passing through a coating medium are added to the novel TD-ANS as compared with a conventional TD-ANS. The TD-ANS can extract each pulse wave element from a response waveform. The validity of an interpretation method for a response waveform is confirmed by comparing with a reference solution.

Keywords—coated cylinder, transient scattered magnetic field, response waveform, interpretation method

I. INTRODUCTION

The study on the high-frequency (HF) scattering by a coated conducting cylinder (Hereinafter referred to as a coated cylinder) has been an important research subject in the application area such as low-observability techniques for the radar cross section of an aircraft fuselage [1] – [3].

We have recently derived a frequency-domain (FD) uniform asymptotic solution (FD-UAS) [4] and a time-domain (TD) asymptotic-numerical solution (TD-ANS) [5], [6] for a (transient) scattered field by a two-dimensional (2-D) coated cylinder covered with a thin lossy dielectric material. The FD-UAS [4] is effective in understanding scattering phenomena and is applicable in a conducting and an impedance problem. The TD-ANS [5], [6] can extract each pulse wave element, which reaches an observation point, from the response waveform. The FD-UAS and the TD-ANS are convenient for engineering applications due to their simple representations.

However, the conventional FD-UAS in [4] (the conventional TD-ANS in [5], [6]) has the problem whose accuracy is gradually decreased with increasing of the thickness of a coating medium as compared with the wavelength of an incident wave (the wavelength of a central angular frequency of an incident HF pulse wave).

To overcome the above-mentioned problem, we have newly derived a FD-UAS and a TD-ANS for a (transient) scattered magnetic field by a 2-D coated cylinder covered with a thick material [7]. The novel FD-UAS and the novel TD-ANS in [7] differ from the conventional FD-UAS and the conventional TD-ANS in that the multiple reflected waves passing through a coating medium are added.

In this study, we briefly propose an interpretation method for a response waveform of a transient scattered magnetic field by a 2-D coated cylinder by using the TD-ANS in [7]. We assume that the thickness of a coating medium is thick as compared with the wavelength of a central angular frequency of a HF pulse wave radiated from a magnetic line source. The TD-ANS can extract each pulse wave, which has a different propagation path, from a response waveform. The validity of an interpretation method for a response waveform is confirmed by comparing with a reference solution. The time convention \( \exp(-i\omega t) \) is assumed and suppressed in this paper.

II. TIME-DOMAIN ASYMPOTIC-NUMERICAL SOLUTION

A. Formulation and Scattering Phenomena

Fig. 1 shows a 2-D coated cylinder with radius of curvature \( \rho = a \) covered with a medium 2 (\( \epsilon_2^*, \mu_2^* \)) of thickness \( t (= a - b) \), coordinate systems \( (x, y, z) \) and \( (\rho, \phi) \), and a magnetic line source \( Q(\rho_0, \phi_0) \). The \( \epsilon_2^* \) denotes a permittivity of a medium 2 and is given by \( \epsilon_2^* = \epsilon_2 + i\sigma/\omega \) where \( \sigma \) is a conductivity and \( \mu_2^* \) a permeability of free space. We assume that the thickness \( t \) of the medium 2 \( (b \leq \rho \leq a) \) is thick \( (t > \lambda_0) \) as compared with the wavelength \( \lambda_0 \) of a central angular frequency of a HF pulse source function (see (6)).

Fig. 2 depicts geometrical boundaries (GBs) \( GB_0^\omega (= GB_0^p) \), \( p = 0, 1, 2 \), transition regions (TRs) \( TR^\omega_p (= TR^p) \), \( p = 0, 1, 2 \), and a schematic figure for scattering phenomena, which are formed from a counterclockwise direction \( (n = 0) \).
of a multiple reflected wave and coordinate systems (x, y, z) and (ρ, φ). P(ρ, φ): observation point.

without encircling the coated cylinder, after radiated from the point Q. Notation p represents the number of reflection on a conducting surface (ρ = b) of a multiple reflected wave passing through the medium 2 (b ≤ p ≤ a).

In Fig. 2, the surrounding space in a medium 1 (ε1, μ0) is divided into 4 regions A0−p (= A0−p) − D0−p (= D0−p) by the GBs; namely, GB0, p = 0, 1, 2. The region A0 is the lit side of the GB0. The region B0 (C0) represents the space between the GB0 (GB1) and the GB1 (GB2). The region D0 is the shadow side of the GB2. We omit a schematic figure for scattering phenomena formed from a clockwise direction (n = −1) by a coated cylinder in this paper because scattering phenomena from a clockwise direction (n = −1) are similar to those shown in Fig. 2.

In the following sections, only the final results for the TD-ANS [7] needed in an interpretation method of a response waveform in Section III are summarized.

B. TD-ANS for GO Series Solution

The TD-ANS for a geometric optical ray (GO) series solution for the z-component of a transient scattered magnetic field can be expressed as the sum of the z-component of a transient scattered magnetic field from a counterclockwise direction (n = 0) and that from a clockwise direction (n = −1).

\[ y(t) \sim y_{\text{TD-ANS}}(t) = \left( y_{\text{GO}_{\text{series}}}(t) \right) = \sum_{n=1}^{\infty} \left[ U_{\text{DG0}}(P)y_{\text{DG0}}^n(t) + \sum_{p=0}^{M} U_{\text{RGO}_p}(P)y_{\text{RGO}_p}^n(t) \right] \]  

\[ y_{\text{DG0}}^p(t) = \frac{1}{2\pi} \int_{0}^{\infty} U_{\text{DG0}}(P)\text{DG0}^p(\omega)\text{S}(\omega)e^{-(i\omega t)d\omega} \]  

\[ y_{\text{RGO}_p}^n(t) = \frac{1}{2\pi} \int_{0}^{\infty} U_{\text{RGO}_p}(P)\text{RGO}_p^n(\omega)\text{S}(\omega)e^{-(i\omega t)d\omega} \]  

where y_{\text{DG0}}^p(t) and y_{\text{RGO}_p}^n(t), which are pulse wave elements of the y_{\text{GO}_{\text{series}}}(t), denote a direct GO (DG0) and a p times reflected GO (RGO), respectively. Notations DG0^p(ω) and RGO_p^n(ω) represent the FD scattered field elements of the FD-UAS. The unit step function U_{\text{DG0}}(P), j = DG0, RGO_p, is defined as U_{\text{DG0}}(P) = 1 (or U_{\text{RGO}_p}(P) = 0) when y_{\text{j}}(t), j = DG0, RGO_p, can (or cannot) reach an observation point P(ρ, φ). The integer M^n is the truncation number of the y_{\text{RGO}_p}^n(t) series. In (2) and (3), notation S(ω) represents a frequency spectrum of a HF pulse source function s(t) (see (6) and (7)). The y_{\text{j}}(t), j = DG0, RGO_p, represented by the integral form in (2) and (3) is calculable independently by applying the fast Fourier transform (FFT) numerical code [8].

As shown in Fig. 2, the propagation paths of the DG0^−p (≡ DG0), RGO_p^−p (≡ RGO0), RGO_p^−1 (≡ RGO1), and RGO_p^−2 (≡ RGO2), are Q → P0, Q → P1, Q → P2, Q → Q, R1 → Q, R1 → P1, and Q → Q, R1 → Q, R1 → R2 → Q, respectively. The TD-ANS for the GO series solution in (1) is effective in the deep lit regions away from the GBs.

C. TD-ANS for extended UTD Series Solution

The TD-ANS for an extended uniform geometrical theory of diffraction (extended UTD) series solution for the z-component of a transient scattered magnetic field can be expressed as the sum of the z-component of a transient scattered magnetic field from a counterclockwise direction (n = 0) and that from a clockwise direction (n = −1).

\[ y(t) \sim y_{\text{TD-ANS}}(t) = \left( y_{\text{extended UTD series}}(t) \right) = \sum_{n=1}^{\infty} \left[ U_{\text{DG0}}(P)y_{\text{DG0}}^n(t) + \sum_{p=0}^{M} U_{\text{RSD}_p}(P)y_{\text{RSD}_p}^n(t) \right] \]  

Fig. 2 Schematic figure for scattering phenomena, GBs: GB0, p = 0, 1, 2, and TRs: TR0−p (≡ TR0−p), p = 0, 1, 2, formed from a counterclockwise direction (n = 0) after radiated from a magnetic line source Q.
\[ y_{\text{RSD}}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} U_{\text{RSD}}^p(P)\text{RSD}_p^\omega(\omega)\exp(-i\omega t)d\omega \quad (5) \]

where \( y_{\text{DGO}}(t) \) and \( y_{\text{RSD}}(t) \), which are pulse wave elements of the \( y_{\text{extended UTD series}}(t) \), denote a DGO and a \( p \) times reflected-surface diffracted ray (RSD), respectively. Notation \( \text{RSD}_p^\omega(\omega) \) represents the FD scattered field element of the FD-UAS. The unit step function \( U_{\text{RSD}}^p(P) = 1 \) (or \( U_{\text{RSD}}^p(P) = 0 \)) when \( y_{\text{RSD}}(t) \) can (or cannot) reach an observation point \( P \). The integer \( N^\omega \) is the truncation number of the \( y_{\text{RSD}}(t) \) series. The \( y_{\text{RSD}}(t) \) represented by the integral form in (5) is calculable independently by applying the FFT numerical code in the same manner as the \( y_{\text{DGO}}(t) \) in (2).

As shown in Fig. 2, the propagation paths of the \( \text{RSD}_p^\omega \) (\( \text{RSD}_p^\omega(\omega) \)), \( \text{RSD}_p^{\omega=0}(\omega) \), and \( \text{RSD}_p^{\omega=\infty}(\omega) \) are \( Q \rightarrow Q_1 \cap Q_2 \rightarrow P_1 \), \( Q \rightarrow Q_2 \rightarrow R_2 \rightarrow Q_1 \cap Q_2 \rightarrow P_2 \), and \( Q \rightarrow Q_2 \rightarrow R_2 \rightarrow Q_3 \rightarrow R_4 \rightarrow Q_1 \cap Q_2 \rightarrow P_2 \), respectively. Notation \( Q_1 \cap Q_2 \equiv \cap \), denotes the propagation path of a creeping wave along the coating surface \( \rho = a \) from the point \( Q_1 \) to the point \( Q_2 \). The TD-ANS for the extended UTD series solution in (4) is effective in the TRs adjacent to the GBs and the shadow side of the GBs.

III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we perform numerical calculations required to assess the validity of an interpretation method for a response waveform of a transient scattered magnetic field by a coated cylinder covered with a thick material.

In numerical calculations, we use the following truncated Gaussian-type modulated pulse source function \( s(t) \) [9]:

\[ s(t) = \begin{cases} 
\exp\left[-i\omega_0(t-t_0)^2/4d^2\right] & \text{for } 0 \leq t \leq 2t_0 \\
0 & \text{for } t < 0, t > 2t_0 
\end{cases} \quad (6) \]

where \( \omega_0 = 2\pi c/\lambda_0 \), where \( c \): speed of light, \( \lambda_0 \): wavelength) denotes a central angular frequency, and \( t_0 \) and \( d \) are constant parameters. The frequency spectrum \( S(\omega) \) of the \( s(t) \) in (6) is given by

\[ S(\omega) = 2d\sqrt{\pi} \text{Re}[\text{erf}(\beta(\omega))]\exp[i\omega_0 t_0 - d^2(\omega - \omega_0)^2] \quad (7) \]

\[ \beta(\omega) = \frac{t_0}{2d} - i d(\omega - \omega_0) \quad (8) \]

where \( \text{erf} z \) is the error function [10] defined by

\[ \text{erf} z = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2)dt. \quad (9) \]

Figs. 3(a) and 3(b) illustrate the real part of the \( s(t) \) in (6) and the absolute value of the \( S(\omega) \) in (7), respectively. Numerical parameters used in the calculations are given in the caption of Fig. 3.
Fig. 4 Response waveforms observed in the region $A^0$ (Fig. 4(a)) and the region $B^0$ (Fig. 4(b)). Numerical parameters: $a = 5.0\,\text{m}$, $t = 8.0\lambda_0 = 0.314\lambda_0 = 0.196\,\text{m}$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_0 = 1.96$, $\sigma = 3.849 \times 10^{-3}\,\text{S/m}$, source point: $P(\rho, \phi) = (1.4a, 0.0\,\text{m})$, and observation point: $P(\rho, \phi) = (2.4a, \phi) = 30.0^\circ$ (Fig. 4(a)) and $\phi = 160.0^\circ$ (Fig. 4(b)).

Fig. 4(b), we observe four pulse waves in the response waveform $\text{Re}[y_{\text{TD-ANS}}(t)]$. By comparing Fig. 4(b) with the $\text{Re}[y_{\text{TD-ANS}}(t)]$ in (11), we can distinguish that the four pulse waves are the $\text{Re}[y_{\text{RSD}}^{0}(t)]$, the $\text{Re}[y_{\text{RSD}}^{1}(t)]$, the bundle of $\text{Re}[y_{\text{RSD}}^{0}(t)]$ and $\text{Re}[y_{\text{RSD}}^{1}(t)]$, and the $\text{Re}[y_{\text{RSD}}^{2}(t)]$ are waiting in order to the observation point. We can confirm that the TD-ANSs in (10) and (11) can extract each pulse wave, which has a different propagation path, from a response waveform even when pulse wave elements overlap mutually at the observation point.

From the above-mentioned discussions, we can conclude that the TD-ANSs in (1) and (4) for the transient scattered magnetic field by a coated cylinder covered with a thick material are useful in understanding the scattering phenomena.

IV. CONCLUSION

We have briefly proposed the interpretation method for a response waveform of a transient scattered magnetic field by a two-dimensional (2-D) coated cylinder covered with a thick material. We showed that the time-domain (TD) asymptotic-numerical solution (TD-ANS) including a multiple reflection effect can extract each pulse wave from a response waveform. We confirmed the validity of the interpretation method for a response waveform using the TD-ANS by comparing with the reference solution.

REFERENCES


