Four Paths From Perfect Electric Conductor (PEC) to Perfect Magnetic Conductor (PMC)

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Abstract – This article focuses on various paths that lead from perfect electric conductor (PEC) to perfect magnetic conductor (PMC). These paths are located in a multidimensional space of electromagnetic materials and surfaces. By varying certain material parameters, one creates continuous transitions through this space, with end points at PEC and PMC. Four particular routes are given attention, those for which the following parameters vary: the intrinsic impedance of a simple isotropic medium; the degree of the impedance boundary condition; the admittance of a perfect electromagnetic conductor; and the chirality of a simple skewon medium.

1. Introduction

The perfect electric conductor (PEC) and perfect magnetic conductor (PMC) are fundamental concepts in electromagnetics and radio science. An electric conductor is a material on whose surface the electric field vector cannot have a tangential component; it is perpendicular. Likewise, on the surface of the magnetic conductor, the magnetic field has to be normal, again with no tangential component. Although many natural materials in nature can be considered as PEC (or at least very good conductors, such as copper, silver, and superconducting materials), PMC media do not naturally appear. However, at least in the microwave region, artificial structures have been devised with which it is possible to simulate the PMC response to electromagnetic fields [1].

In the following, the PEC and PMC boundary conditions are considered as two points in a space with several dimensions. Natural paths leading from one to the other are presented in this space, where the materials to be encountered are treated as either penetrable media or as absolute boundaries.

2. PEC and PMC as Limits of Dielectric–Magnetic Materials

Consider isotropic lossless materials with dielectric and magnetic response. They are characterized by the relative permittivity ε and relative permeability μ with the following constitutive relations between the electric and magnetic fields **E, H** and the electric and magnetic flux densities **D, B:**

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$$\mathbf{D} = \varepsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H} \tag{1}$$

Here, the fields are normalized with free-space constants ε_0, μ_0 as follows: $\mathbf{E} = \sqrt{\varepsilon_0}\,\mathbf{E}_{\text{abs}}$, $\mathbf{H} = \sqrt{\mu_0}\,\mathbf{H}_{\text{abs}}$, $\mathbf{D} = \mathbf{D}_{\text{abs}}/\sqrt{\varepsilon_0}$, and $\mathbf{B} = \mathbf{B}_{\text{abs}}/\sqrt{\mu_0}$, so they all have the same unit of the square root of energy density ($\sqrt{\text{VAs/m}^3}$), instead of the units of the absolute quantities ($[\mathbf{E}_{\text{abs}}] = \text{V/m}$, $[\mathbf{H}_{\text{abs}}] = \text{A/m}$, $[\mathbf{D}_{\text{abs}}] = \text{As/m}^2$, $[\mathbf{B}_{\text{abs}}] = \text{Vs/m}^2$, Vs/m^2). This leads to the convenience that ε and μ are dimensionless.

To simulate the electromagnetic response of a PEC medium, a usual approach is to let the permittivity grow toward infinity. However, one needs at the same time to let μ decrease to zero [2]. One argument for this observation comes from Faraday's law $\nabla \times \mathbf{E}_{\text{abs}} = -\,\text{j}\omega \mathbf{B}_{\text{abs}}$ (here using the time-harmonic notation $\exp(\text{j}\omega t)$). Hence, if the electric field vanishes (as it should due to the definition of a perfectly conducting medium), also the magnetic flux density **B** has to vanish. However, because there is no restriction for the magnetic field **H**, the permeability must vanish to satisfy (1). Analogously, the PMC medium corresponds to the case $\varepsilon \to 0, \mu \to \infty$.

Two derived quantities can be defined, the refractive index n and the intrinsic impedance η:

$$n = \sqrt{\varepsilon\mu}, \quad \eta = \sqrt{\frac{\mu}{\varepsilon}} \tag{2}$$

Using these four quantities $(\varepsilon, \mu, n, \eta)$ and assuming them to be real and positive, we define and categorize extreme parameter materials, based on the magnitudes. Each of the parameters can be very large (VL) or very small, near zero (NZ), leading to the eight classes shown in Figure 1.

In recent metamaterials research, much interest has been on the class of epsilon-near-zero (ENZ) materials due to the potential for achieving tunneling of waves through narrow subwavelength channels [3]. However, to effectively define PEC or PMC, it is rather the intrinsic impedance whose magnitude matters. Choosing $\mu = 1/\varepsilon$, a one-parameter road from PEC to PMC (η changing from zero to infinity) goes from impedance near zero (ZNZ) to impedance very large (ZVL). This choice (in other words $n = 1$) is natural in the sense that for $\eta = 1$, it corresponds to vacuum (the origin of the map in Figure 1). Such a nonrefractive simple isotropic medium (SIM) is *isorefractive*, according to the terminology in [4]. With the requirement of isorefractivity, the constitutive relations (1) simplify into

$$\mathbf{D} = \frac{\mathbf{E}}{\eta}, \quad \mathbf{B} = \eta \mathbf{H} \tag{3}$$

Hence, the PEC medium can be also termed ZNZ, while the PMC corresponds to ZVL. Indeed,

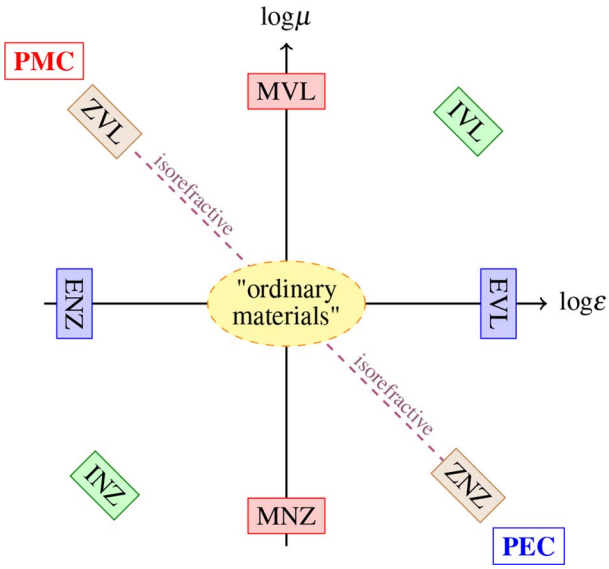

Figure 1. Extreme materials in the plane where the axes are the logarithms of the relative permittivity ε and relative permeability μ. Notation: the permittivity (E), permeability (M), the refractive index (I), the intrinsic impedance (Z), very small (near zero [NZ]), or very large (VL). The origin represents vacuum $\varepsilon = \mu = 1$. Modified from [2].

artificial magnetic conductor boundaries are sometimes called *high impedance surfaces* [5].

To understand how electromagnetic waves interact with materials characterized by (3), consider scattering of plane waves by a homogeneous sphere made of such a medium. A parameter describing the scattering behavior of particles is the scattering efficiency Q_{sca} of a sphere (the efficiency is equal to the scattering cross section of the sphere divided by its geometrical cross section) [6]. This quantity depends on the optical size (size parameter $x = 2\pi a/\lambda$ with the radius a and wavelength λ) of the sphere and its material. As is well-known, for a PEC sphere (as well as for a PMC sphere), the cross section begins with an x^4 dependence for small scatterers, then slightly oscillates, and finally settles at the value of twice the geometric cross section for large x (a phenomenon known as the *extinction paradox*) [7, 8].

Once the material of the sphere varies on its path from PEC to PMC (the diagonal from ZNZ to ZVL in Figure 1), the scattering behavior also changes. This is shown in Figure 2. The end values on the impedance axis reproduce the known PEC and PMC scattering curves. Because the scattering curves for PEC and PMC are the same, the efficiencies of spheres with η and η^{-1} are equal, too. It is also evident that the scattering efficiency vanishes for $\eta = 1$, independently of x.

3. PEC and PMC as Impedance Surfaces

Another way to approach PEC and PMC is to treat the medium as a boundary. Indeed, PEC and PMC are special cases of the so-called *impedance boundary*,

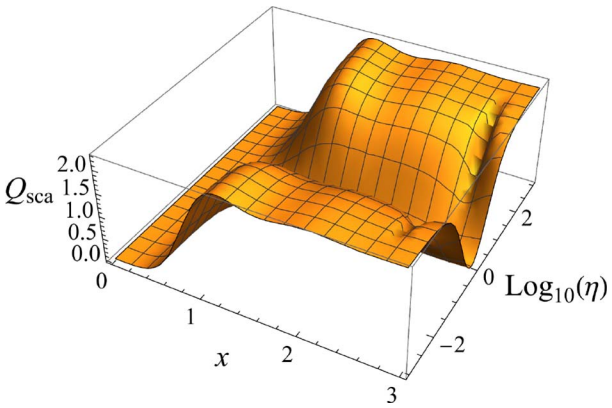

Figure 2. The scattering efficiency of a sphere with $\mu = 1/\varepsilon$, as a function of the intrinsic impedance η (logarithmically varying from 0.001 to 1000) and the size parameter x.

which is defined by the requirement it enforces on electric **E** and magnetic fields **H** at its surface [9]:

$$\mathbf{E}_{\text{tan}} = Z_s\, \mathbf{n} \times \mathbf{H}_{\text{tan}} \tag{4}$$

Here, the subscript "tan" is the component of the vector tangential to the surface. The unit normal of the boundary is **n**, and Z_s is the (dimensionless) scalar surface impedance. Obviously, for PEC, $Z_s = 0$, and by duality, it follows that $Z_s^{-1} = 0$ for PMC.

For an isotropic boundary to be lossless, the surface impedance has to be purely imaginary [10]. Hence, two natural paths from PMC to PEC open up: the surface impedance Z_s changes from a large value to zero, being either purely imaginary (a lossless boundary), or purely real, depending on the sign of the value Z_s; in this case, the boundary is dissipative ($Z_s > 0$) or active ($Z_s < 0$).

Figure 3 shows how this scattering behavior varies with the (real and positive) surface impedance Z_s. Because this type of object is lossy, it also displays absorption, measured with the absorption efficiency Q_{abs}. The added effect of scattering and absorption gives the extinction efficiency $Q_{\text{ext}} = Q_{\text{sca}} + Q_{\text{abs}}$ [6]. Because PEC and PMC are lossless, the absorption efficiency vanishes there, and scattering equals extinction, as can be seen in the behavior for Z_s being close to zero or infinity.

4. Perfect Electromagnetic Conductor

The perfect electromagnetic conductor (PEMC) [11] is a natural generalization of PEC and PMC. The constitutive relations of PEMC read as

$$\mathbf{D} = M\,\mathbf{B}, \quad \mathbf{H} = -M\,\mathbf{E} \tag{5}$$

between the normalized flux densities and fields, where the material parameter M stands for the dimensionless PEMC admittance. The very particular character of these relations is that they do not connect a flux density to the field, but instead one flux density to another, and one field to another.

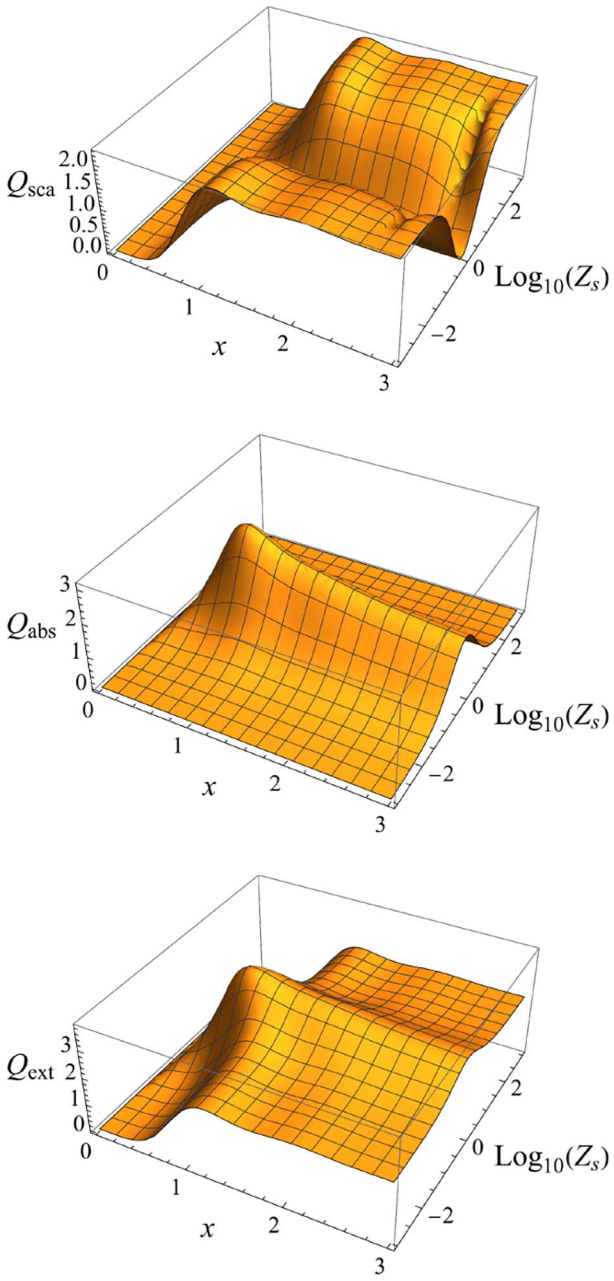

Figure 3. Scattering, absorption, and extinction efficiencies of a sphere with size parameter x and the surface impedance Z_s (logarithmically varying from 0.001 to 1000).

To write (5) in a form in which the fluxes appear as functions of fields, the relation appears clumsier; for details, see [12]:

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = \lim_{q\to\infty} q \begin{pmatrix} M(1+q^{-2}) & 1 \\ 1 & 1/M \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \tag{6}$$

This matrix form means that PEMC is a special case of a bi-isotropic medium [13], where magnetoelectric cross coupling is present. In particular, because the matrix in (6) is symmetric, the medium it represents is a form of a nonreciprocal Tellegen material [14]. Furthermore, due to the infinite limit it contains, it is a form of extreme Tellegen medium, also called *axion medium* [15].

By varying the parameter M in PEMC, another path is formed from PEC ($M=\infty$) to PMC ($M=0$).

Unlike the case for scalar impedance boundary, where lossless surfaces required purely imaginary values for Z_s, a lossless PEMC boundary displays a real value of M. For lossless boundaries, the impedance dyadic has to be anti-symmetric [10].

To illustrate the continuous change through this PEMC route, consider the reflection of a linearly polarized wave, normally incident on a planar PEMC surface characterized by admittance M. It turns out that the polarization of the reflected wave will be rotated and forms the angle α for the incident wave vector. The rotation angle is 2arctan$M$, and this angle goes from 0 to π, as M grows from zero to infinity (the sign of M determines the sign of α). For PEC, the reflection is 180° because the conductor short-circuits the tangential electric field, while PMC corresponds to an open circuit, with the reflection in the same phase as the incidence. Note the nonreciprocity in the interaction, provided that the admittance is finite ($M\neq 0$, $M^{-1}\neq 0$).

5. PEC and PMC as Extreme Bi-Isotropic Media

As seen from (6), the PEMC medium, considered as material, has isotropic constitutive relations with magnetoelectric coupling. This is a special case of bi-isotropic media [13]. However, because a 2×2 matrix response has four degrees of freedom, there can be, in addition to permittivity ε and permeability μ, two magnetoelectric parameters, of which PEMC represents the character of nonreciprocity (axion-type response) [15]. Another type of cross coupling is present in handed matter (Pasteur media) [13], and allowing for these chiral responses opens up another dimension in the description of matter. In the following, let us combine both effects into the constitutive relations.

A generalization of the PEMC medium (6) is the so-called *Minkowskian isotropic medium* (MIM) [16] or *Gibbsian isotropic medium* [17]. It has the form

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = \frac{1}{\cos\vartheta} \begin{pmatrix} 1/\eta & \sin\vartheta \\ \sin\vartheta & \eta \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \tag{7}$$

where $\sin\vartheta = \eta M$. The choice $\vartheta=\pi/2$ leads to PEMC.

To include the class of Pasteur media into the description, an analogous expansion can be made for reciprocal magnetoelectric cross coupling. The so-called *simple skewon* (SS) medium [18] is another extreme type of bi-isotropic media for which the chirality plays the same role as nonreciprocity in the PEMC medium. With the imaginary unit j, the SS relations are $\mathbf{D} = -\mathrm{j}N\mathbf{B}$, $\mathbf{H} = -\mathrm{j}N\mathbf{E}$. Following [12], we can

generalize this into two-parameter chiral bi-isotropic constitutive relations

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = \frac{1}{\cos\vartheta} \begin{pmatrix} 1/\eta & \mathrm{e}^{-\mathrm{j}\psi}\sin\vartheta \\ \mathrm{e}^{\mathrm{j}\psi}\sin\vartheta & \eta \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \tag{8}$$

where the angle ψ includes the Pasteur parameter N:

$$M - \mathrm{j}N = \frac{\sin\vartheta}{\eta}\exp(-\mathrm{j}\psi) \tag{9}$$

In other words, the material parameters (relative permittivity, permeability, Tellegen [nonreciprocity], and Pasteur [chirality] parameters) [13] are

$$\begin{aligned} \varepsilon &= \frac{1}{\eta\cos\vartheta} & \chi &= \cos\psi\ \tan\vartheta \\ \mu &= \frac{\eta}{\cos\vartheta} & \kappa &= \sin\psi\ \tan\vartheta \end{aligned} \tag{10}$$

Choosing $\vartheta = 0$ returns us the isorefractive SIM, discussed in Section 2, compare (3). On the other hand, the limits of the impedance parameter η lead to perfect electric ($\eta \to 0$) and perfect magnetic ($1/\eta \to 0$) conductors. By replacing η with the angular parameter $\tan\xi$, the three-dimensional space spanned by these parameters are visualized as a sphere, as shown in Figure 4. The class of materials within this sphere is called *unimodular* media, thanks to the unity determinant of the material matrix in (8). This forms a three-dimensional subclass of all possible bi-isotropic media with four material degrees of freedom. For example, the previously mentioned MIM (Gibbsian) (6) forms a circular disk, a vertical cut of the sphere that includes the SIM line and the meridians with longitudes $\psi = 0$ and $\psi = \pi$. The angles have intuitive interpretations:

- The angle of intrinsic impedance $\xi = \arctan\eta$ runs from PEC ($\xi = 0$) to PMC ($\xi = \pi$).
- The angle of magnetoelectricity vanishes for simple isotropic materials (the diagonal axis from the north pole to the south pole of the sphere), and is $\pi/2$ at the surface of extreme bi-isotropic materials, such as PEMC and SS.
- The angle of reciprocity ψ separates the Tellegen and Pasteur character of the magnetoelectric interaction: $\psi = 0, \pi$ return purely nonchiral media (such as PEMC), and $\psi = \pi/2, 3\pi/2$ correspond to reciprocal chiral media (such as simple-skewon right-handed and simple-skewon left-handed chiralities) on the surface of the sphere.

6. Conclusion

PEC and PMC are fundamental concepts in electromagnetics and also the natural or artificial realizations are extremely useful in engineering applications. In this article, several ways were presented on how PEC and PMC can be seen as end points on a path through the multidimensional space of electromagnetic materials and surfaces: 1) varying the intrinsic impedance of isorefractive media; 2) using an impedance boundary condition on the surface of the material; 3) following the PEMC domain with varying admittance parameter; and 4) through the space of chiral media along the SS path.

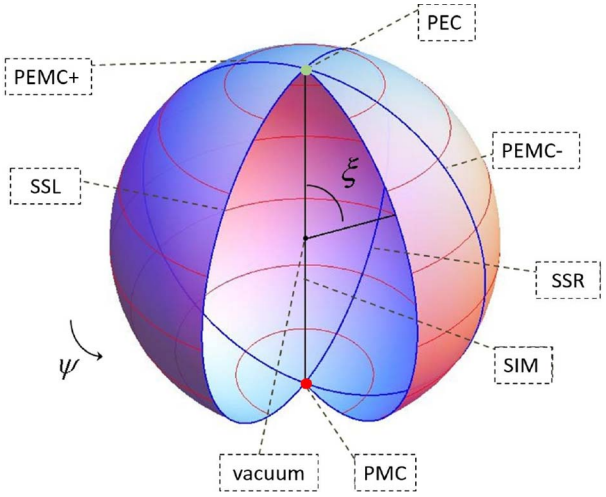

Figure 4. Visualization of the space of bi-isotropic media in three dimensions that contain SIM, PEMC media, and simple-skewon media (with left-handed [SSL] and right-handed [SSR] chiralities), as well as perfect electric and magnetic conductors (PEC and PMC) as north and south poles. The intrinsic impedance parameter is connected to the angle ξ as $\eta = \tan\xi$.

These are the pathways from PEC to PMC, where only one parameter is changing. Of course, side routes can be found that combine various mechanisms. For example, it is possible in Figure 4 to travel from the north pole to the south pole also along other meridians than pure PEMC or SS and also go below and through the surface of the sphere. Likewise, the space spanned by the boundary condition in Section 3 is not one dimensional; the impedance can be lossy or lossless. Furthermore, note that all the media and surfaces treated in the article were isotropic (or bi-isotropic). Allowing anisotropy, a plethora of additional routes from PEC to PMC become available.

7. References

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