Inductively Coupled In-Circuit Measurement of Multiport Admittance Parameters

Simone Negri, Giordano Spadacini, Flavia Grassi, and Sergio A. Pignari

Abstract – This letter proposes an inductively coupled in-circuit measurement method for characterizing the admittance matrix of multiport systems at radio frequencies. The measurement setup involves inductive probes connected to a vector network analyzer, each defining a longitudinal port along the clamped wire. The rationale of the method is first presented in general terms for an arbitrary number N of probes, allowing measurement of an $N \times N$ admittance matrix. Then, the involved equations are fully developed in closed form for the case $N = 2$, which assumes special importance because multiple two-port measurements can manage any number of ports using the admittance matrix definition. The proposed method is experimentally verified using a set of passive networks.

1. Introduction

The need to characterize the impedance or admittance of industrial equipment at radio frequencies (well above functional low frequencies, e.g., dc or 50-Hz alternating current) is recently emerging in several application fields, including power-converter stability analysis, motor fault diagnosis, state-of-charge estimation of electrochemical batteries, and electromagnetic interference modeling. Unfortunately, conventional radio-frequency instrumentation is scarcely suited to such needs. On the one hand, impedance meters are limited to single-port (self-impedance) measurements. On the other hand, multiport vector network analyzers (VNA) involve coaxial connectors, which are scarcely compatible with industrial wiring harnesses. Moreover, their connection to a system in operation (in the following referred to as in-circuit measurement) is prevented by the potentially dangerous low-frequency voltage/current levels unless ad hoc coupling networks are designed.

In this respect, in-circuit measurement methods based on inductive probes are promising [1, 2] because they are minimally intrusive (probes are clamped on cables) and do not establish galvanic connections to the system under test. Of note, both monitor and bulk current injection probes normally exploited in electromagnetic compatibility laboratories for quite different purposes can be conveniently reused in this context. However, well-established methods—that is, the two-probe setup [2, 3] and the single-probe setup (SPS) [4, 5]—are both limited to a single measurement port. Recently, a different two-probe setup has been proposed in [6] to characterize both the self and mutual admittances in a two-port system.

In this letter, the rationale of inductively coupled measurements is fully generalized for N ports and N probes (i.e., just one probe per port), thus defining a multiple-probe setup (MPS), which reduces to [6] for $N = 2$ and to [4, 5] for $N = 1$. Furthermore, the special two-port case is shown here to be of particular importance because it enables the characterization of N-port networks at the expense of multiple measurements by the properties of the admittance matrix.

Manuscript received 7 February 2024. This study was carried out within the MOST – Sustainable Mobility Center and received funding from the European Union Next-Generation EU (PIANO NAZIONALE DI RIPRESA E RESILIENZA (PNRR) - MISSIONE 4 COMPONENTE 2, INVESTIMENTO 1.4 - D.D. 1033 17/06/2022, CN00000023). This manuscript reflects only the authors' views and opinions, neither the European Union nor the European Commission can be considered responsible for them.

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2. In-Circuit Measurement of the Admittance Matrix by Inductive Probes

2.1 Multiple Probe Setup

Let us consider the setup depicted in Figure 1, consisting of a VNA, N inductive probes, and an N-port network. The target of the proposed method is to determine the $N \times N$ unknown admittance matrix $\mathbf{Y}_x$ of the network from the knowledge of the S-parameter matrix $\mathbf{S}_m$ of the whole setup measured by the VNA, which relates the incident power waves $a_1, a_2, \ldots a_N$ and reflected power waves $b_1, b_2, \ldots b_N$, according to:

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} = \underbrace{\begin{bmatrix} S_{m11} & S_{m12} & \cdots & S_{m1N} \\ S_{m21} & S_{m22} & & S_{m2N} \\ \vdots & & \ddots & \vdots \\ S_{mN1} & S_{mN2} & \cdots & S_{mNN} \end{bmatrix}}_{\mathbf{S}_m} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} \tag{1}$$

To this end, $\mathbf{S}_\text{m}$ is first converted into the measured admittance matrix $\mathbf{Y}_\text{m}$ by

$$\mathbf{Y}_m = Z_0{}^{-1}(\mathbf{I}_N - \mathbf{S}_m)(\mathbf{I}_N + \mathbf{S}_m)^{-1} \tag{2}$$

where $\mathbf{I}_\text{N}$ is the $N \times N$ identity matrix, and $Z_0 = 50\ \Omega$ is the standard S-parameter reference impedance. For the port voltages and currents defined in Figure 1, the following relationships hold:

$$\underbrace{\begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}}_{\mathbf{i}} = \underbrace{\begin{bmatrix} Y_{m11} & Y_{m12} & \ldots & Y_{m1N} \\ Y_{m21} & Y_{m22} & & Y_{m2N} \\ \vdots & & \ddots & \vdots \\ Y_{mN1} & Y_{mN2} & \ldots & Y_{mNN} \end{bmatrix}}_{\mathbf{Y}_m} \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}}_{\mathbf{v}} \tag{3}$$

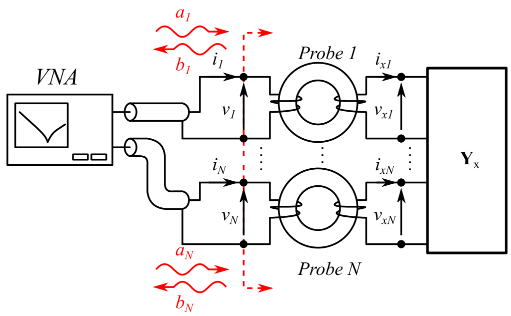

Figure 1. Schematic of the proposed inductively coupled MPS method.

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}}_{\mathbf{T}_1} \begin{bmatrix} v_{x1} \\ i_{x1} \end{bmatrix}, \quad \dots \quad \begin{bmatrix} v_N \\ i_N \end{bmatrix} = \underbrace{\begin{bmatrix} A_N & B_N \\ C_N & D_N \end{bmatrix}}_{\mathbf{T}_N} \begin{bmatrix} v_{xN} \\ i_{xN} \end{bmatrix} \tag{4}$$

where $\mathbf{T}_1, \dots \mathbf{T}_N$ are the transmission (also known as ABCD) matrices representing the behavioral models of probes 1 to N, respectively, whose parameters satisfy, by reciprocity,

$$\det[\mathbf{T}_i] = 1, \;\; \forall i \tag{5}$$

By substituting $v_1, \dots v_\mathrm{N}$, and $i_1, \dots i_\mathrm{N}$ from (4) in (3) and collecting the currents $i_{x1}, \dots i_{x\mathrm{N}}$ and the voltages $v_{x1}, \dots v_{x\mathrm{N}}$, the following relation is obtained:

$$(\mathbf{D} - \mathbf{Y}_m \circ_H \mathbf{B})\mathbf{i}_x = (\mathbf{Y}_m \circ_H \mathbf{A} - \mathbf{C})\mathbf{v}_x \tag{6}$$

where

$$\mathbf{D} = diag(D_1, D_2, \dots D_N), \;\; \mathbf{C} = diag(C_1, C_2, \dots C_N) \tag{7}$$

$$\mathbf{B} = [B_1 \cdot \mathbf{1}_{(N\times1)}, \;\; B_2 \cdot \mathbf{1}_{(N\times1)}, \;\; \dots \;\; B_N \cdot \mathbf{1}_{(N\times1)}] \tag{8}$$

$$\mathbf{A} = [A_1 \cdot \mathbf{1}_{(N\times1)}, \;\; A_2 \cdot \mathbf{1}_{(N\times1)}, \;\; \dots \;\; A_N \cdot \mathbf{1}_{(N\times1)}] \tag{9}$$

$$\mathbf{i}_x = [i_{x1}, \;\; i_{x2}, \;\; \dots \;\; i_{xN}]^T, \;\; \mathbf{v}_x = [v_{x1}, \;\; v_{x2}, \;\; \dots \;\; v_{xN}]^\mathrm{T} \tag{10}$$

and $\circ_H$ denotes the Hadamard matrix product (entrywise product) [7]. The currents $i_{x1}, \dots i_{xN}$ can now be collected in (6) and expressed as functions of voltages $v_{x1}, \dots v_{xN}$, resulting in

$$\mathbf{i}_x = \underbrace{(\mathbf{D} - \mathbf{Y}_m \circ_H \mathbf{B})^{-1} (\mathbf{Y}_m \circ_H \mathbf{A} - \mathbf{C})}_{\mathbf{Y}_x} \mathbf{v}_x \tag{11}$$

which provides the expression of the desired $N \times N$ admittance matrix $\mathbf{Y}_x$ as a function of the measured S-parameter matrix and the ABCD parameters (4). The latter represent known and invariable frequency responses associated with the specific probes; however, their use is unpractical as they are not standard information in the data-sheet. Consequently, it is necessary to establish an overall calibration procedure for the MPS. To this end, by tedious yet simple algebraic calculus, the ABCD parameters in $\mathbf{Y}_x$ can be recollected into a minimum number of coefficients k_n, $n = 1, 2, \dots, n_{par}$, where

$$n_{par} = 3N + N(N-1)/2 = N(N+5)/2 \tag{12}$$

that is, three terms for each port plus one term for each possible couple of ports. Hence, coefficients k_n can be identified by designing suitable calibration networks with known admittance parameters, by measuring their S-parameters and establishing a linear system of n_{par} equations, as shown in Section 2.3.

2.2 Special Cases

For exemplification, it is straightforward to develop (11) in closed form for $N = 2$, getting the following expressions of the admittance-matrix entries:

$$\begin{aligned} Y_{x11} &= (k_1 + k_2 S_{m11} + k_2 k_7 \det[\mathbf{S}_m] + k_1 k_7 S_{m22})/\Delta \\ Y_{x12} &= -k_3 S_{m12}/\Delta, \;\; Y_{x21} = -k_3 S_{m21}/\Delta, \\ Y_{x22} &= (k_4 + k_4 k_6 S_{m11} + k_5 k_6 \det[\mathbf{S}_m] + k_5 S_{m22})/\Delta \\ \Delta &= 1 + k_6 S_{m11} + k_6 k_7 \det[\mathbf{S}_m] + k_7 S_{m22}, \end{aligned} \tag{13}$$

where $n_{par} = 7$ coefficients $k_1, k_2, \dots, k_7$ are defined as:

$$k_1 = \frac{-A_1 + C_1 Z_0}{B_1 - D_1 Z_0}, \; k_2 = \frac{A_1 + C_1 Z_0}{B_1 - D_1 Z_0}, \; k_3 = -\frac{2Z_0}{(B_1 - D_1 Z_0)(B_2 - D_2 Z_0)}, \\ k_4 = \frac{-A_2 + C_2 Z_0}{B_2 - D_2 Z_0}, \; k_5 = \frac{A_2 + C_2 Z_0}{B_2 - D_2 Z_0}, \; k_6 = \frac{B_1 + D_1 Z_0}{-B_1 + D_1 Z_0}, \; k_7 = \frac{B_2 + D_2 Z_0}{-B_2 + D_2 Z_0} \tag{14}$$

(13) and (14) coincide with the formulation derived in [6] for two probes. Of note, by further reducing the number of ports to $N = 1$, one simply gets the self-admittance

$$Y_x = (k_1 + k_2 S_{m11})/(1 + k_6 S_{m11}) \tag{15}$$

which is the same formulation of the SPS [4, 5] in a slightly different but equivalent form. Hence, the proposed MPS includes past methods as special cases.

In the framework of a general N-port formulation, the two-port case (13) assumes special relevance by the properties of the admittance matrix and fully justifies the preference in this work for admittance parameters instead of the impedance used in [2–5]. Namely, an $N \times N$ admittance matrix $\mathbf{Y}_x$ can be identified by a number $N(N-1)/2$ of repeated two-port evaluations. In each, two probes are clamped on some ports ij, $i \neq j$ (e.g., ports 1-2, 1-3, and 2-3 for $N = 3$), and the relevant 2×2 submatrix $\mathbf{Y}_{ij}$ is measured. In the absence of a probe clamped on a wire, the longitudinal port defined on that wire is inherently in a short-circuit condition (i.e., zero voltage), which is consistent with the definition of admittance-matrix entries:

$$Y_{hk} = \left. \frac{i_h}{v_k} \right|_{v_j = 0, \forall j \neq k} \tag{16}$$

Hence, just two probes and a simple two-port VNA (a common VNA configuration) can be used to characterize any multiport network.

2.3 Calibration Procedure

For clarity, the determination of the n_{par} coefficients k_n by calibration measurements is discussed for the two-port case (14), as the extension to any port number follows the same approach. For a two-port network, the fully coupled equations (13) degenerate into separate equations if ports are uncoupled ($S_{m12} = S_{m21} = 0$ and $Y_{x12} = Y_{x21} = 0$ in (13)). After basic algebra, self-admittances can be expressed as:

$$Y_{x11}\big|_{S_{m12}=S_{m21}=0} = \frac{k_1 + k_2 S_{m11}}{1 + k_6 S_{m11}} \tag{17}$$

$$Y_{x22}\big|_{S_{m21}=S_{m12}=0} = \frac{k_4 + k_5 S_{m22}}{1 + k_7 S_{m22}} \tag{18}$$

which represent an alternative formulation of the equations of the SPS [4, 5] at each port. Hence, one can adapt the same SPS calibration procedure [5]. The one-port calibration procedure relies on three known one-port reference loads Y_A, Y_B, Y_C. By connecting in turn these loads to probe 1, the reflection S-parameters S_{1A}, S_{1B}, S_{1C}, respectively, are measured by the VNA at port 1. By enforcing (17), a linear system in the variables k_1, k_2, k_6 is obtained as:

$$\begin{cases} Y_A(1 + k_6 S_{1A}) = k_1 + k_2 S_{1A} \\ Y_B(1 + k_6 S_{1B}) = k_1 + k_2 S_{1B} \\ Y_C(1 + k_6 S_{1C}) = k_1 + k_2 S_{1C} \end{cases} \tag{19}$$

which yields one unique solution. By connecting in turn reference loads Y_A, Y_B, Y_C to probe 2, the corresponding reflection S-parameters S_{2A}, S_{2B}, S_{2C} are measured at port 2, and coefficients k_4, k_5, k_7 can be similarly obtained. The last coefficient, k_3, can be determined by connecting a reference two-port network to the probes, whose known admittance matrix and measured S-parameters are

$$\mathbf{Y}_D = \begin{bmatrix} Y_{D11} & Y_{D12} \\ Y_{D21} & Y_{D22} \end{bmatrix}, \ \mathbf{S}_D = \begin{bmatrix} S_{D11} & S_{D12} \\ S_{D21} & S_{D22} \end{bmatrix} \tag{20}$$

By enforcing data (20) in the mutual admittances (13), coefficient k_3 is readily obtained.

The same principle applies to the calibration of N probes and ports. There will be 1) three reference loads to carry out three single-port measurements separately for each probe, thus identifying $3N$ coefficients; and 2) a reference two-port network to carry out two-port measurements for each possible couple of ports, thus identifying the remaining $N(N-1)/2$ coefficients. In the end, the number of calibration measurements and the number of identified coefficients k_n coincide with (12).

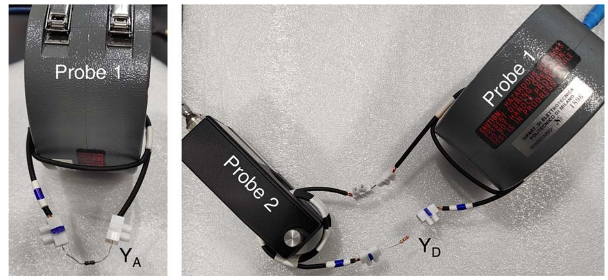

Figure 2. (a) One-port calibration setup with bulk current injection probe, (b) two-port calibration setup.

3. Experimental Validation

An experimental setup corresponding to Figure 1 was realized using a Keysight E5061B VNA connected to two inductive probes by coaxial cables. Probe 1 was a bulk current injection probe FCC F-120-2, and probe 2 was a monitor probe Solar 9123-1N, chosen for availability. Probes 1 and 2 were clamped on two turns and six turns of wire, respectively, to match the secondary turns with the unchangeable primary turns inside the probe to optimize measurement sensitivity [5]. The VNA was set to measure 1601 points from 150 kHz to 30 MHz, with 100-Hz resolution bandwidth and 8-dBm forward power. The frequency range was chosen to comply with probe bandwidth and consider the weak inductive coupling, which impairs sensitivity at low frequency, and setup parasitics, which affect measurement reproducibility at high frequency.

Each probe was first calibrated by three one-port measurements using Y_A, Y_B, Y_C, as shown in Figure 2a. The final two-port calibration measurement was realized by connecting the probe's secondary windings, as shown in Figure 2b, with the longitudinal insertion of a reference load Y_D, realizing the two-port reference network (20), where $Y_{D11} = Y_{D22} = Y_D$, $Y_{D12} = Y_{D21} = -Y_D$. In principle, any load values could be used for calibration, according to Section 2.3. The empirical experience suggests choosing resistive loads spanning the admittance range of interest. In total, the proposed calibration procedure requires four reference loads, which were chosen as

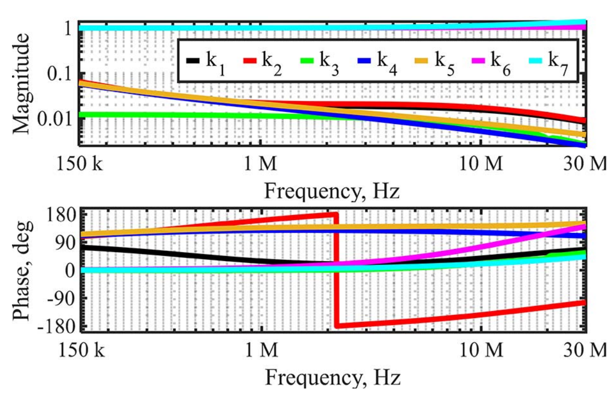

Figure 3. Magnitude and phase of coefficients k_1, k_2, k_3, k_4, k_5 (S), k_6, k_7 (unitless).

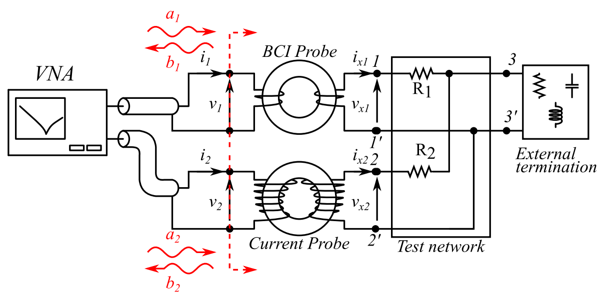

Figure 4. Principle representation of the experimental setup for calibration and verification.

resistors of nominal values 1.1 Ω, 50 Ω, 1 kΩ, and 220 Ω, whose admittances Y_A, Y_B, Y_C, Y_D, respectively, are determined by independent reflectometric measurements to include parasitic effects (series inductance, parallel capacitance). The magnitude and phase of coefficients k_n obtained from the calibration procedure are plotted in Figure 3.

A passive test network was constructed to verify the proposed measurement method, whose circuit can be seen in Figure 4, along with a principle representation of the whole setup. The test network is a three-port network with $R_1 = 0$ Ω, $R_2 = 125$ Ω, which was used as a two-port network by terminating the third port with an inductor (10 μH) or a capacitor (47 nF). The test networks were enclosed in a metallic box with external SMA connectors, enabling direct VNA connection to measure S-parameters, which are converted into admittances (2) and used as reference. As shown in Figure 5, the connection of the test networks to the probes needed two short pieces of twisted-wire pair, which were separately characterized in terms of lumped capacitance and inductance to allow for embedding their contribution to the reference admittance values.

Figures 6 and 7 show the comparison between reference values and admittance parameters obtained by the proposed inductively coupled measurement method, in magnitude and phase, for capacitive and inductive terminations. The overall accuracy of the proposed measurement method was very good. The variable frequency response of phases was well captured. With additional experiments, a dynamic range extended over six orders of magnitude was ascertained from tens of micro-

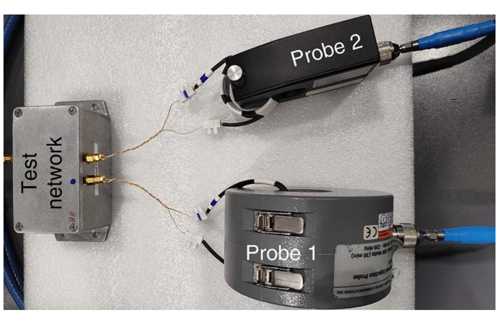

Figure 5. Experimental setup for verification of the proposed admittance measurement method on passive test loads.

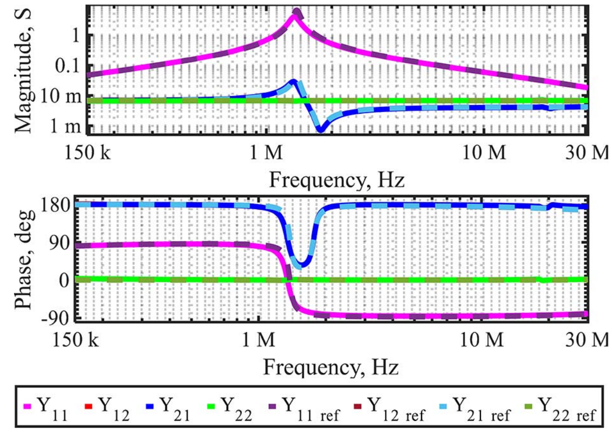

Figure 6. Comparison between reference (dashed lines) and measured (solid lines) values of the coefficients of the admittance matrix of the test load with capacitive termination.

Siemens to ten Siemens (in relationship with the specific probes used).

4. Conclusion

An original inductively coupled in-circuit MPS method for multiport networks is proposed, which includes the SPS [4, 5] as a special case. In general, a number N of inductive probes connected to an N port VNA allows measurement of an $N \times N$ admittance matrix. If only a two-port VNA is available, the same purpose can be obtained by two inductive probes and $N(N-1)/2$ measurements of the 2×2 admittance submatrices. The method requires a preliminary calibration of the MPS setup by using three known reference one-port loads and a known reference two-port network to carry out $N(N+5)/2$ calibration tests. Passive networks experimentally validated the proposed method. Prospective applications include identifying electromagnetic-interference behavioral models at radio frequency in power electronics systems.

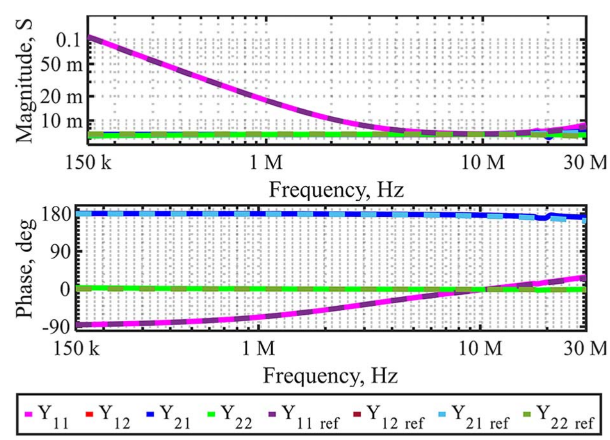

Figure 7. Comparison between reference (dashed lines) and measured (solid lines) values of the coefficients of the admittance matrix of the test load with inductive termination.

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