Aperture Efficiency Calculation of Multibeam Axisymmetric Dual-Reflector Antenna of Cassegrain Type

Makoto Nagai and Hiroaki Imada

Abstract – The antenna efficiency of aperture type is one of the important figure of merits of radio telescopes in the field of millimeter-wave to terahertz astronomy. Based on an aperture efficiency theory for general dual-reflector antennas, we obtained an expression of the aperture efficiency applicable to a multibeam axisymmetric dual-reflector antenna of Cassegrain type. We also derived a minimum blockage condition for a multibeam Cassegrain-type telescope. We propose a procedure to optimize the configuration under constraints on a main reflector diameter, field-of-view radius, main reflector focal distance, and the distance between the subreflector and the focal plane. This approach serves as a valuable tool in the initial design stages of a multibeam Cassegrain-type telescope.

1. Introduction

Next-generation radio telescopes with large reflectors are planned in the field of millimeter-wave to terahertz astronomy. To obtain imaging survey capability, a new radio telescope is desired to have a wide field of view (FoV) for radio cameras with a large format of detector arrays. Design of a multibeam radio telescope is a new challenge because the design of traditional radio telescopes is based on methods to treat a single pencil beam. The antenna efficiency of aperture type, known as *aperture efficiency* in field of radio astronomy, is one of the important figure of merits of such radio telescopes. When the incident plane wave perfectly couples with a receiver beam of the fundamental Gaussian mode at a circular aperture, the aperture efficiency becomes the product of the taper efficiency and the spillover efficiency. This result is generalized for the axisymmetric system with blockage by the subreflector [1]. Though this is a well-known result of a single-beam system, the corresponding formalism of a multibeam system was not clear, even for the center beam of an axisymmetric system. Recently, we developed a theory that factorizes the aperture efficiency of a general dual-reflector antenna into three factors: the beam coupling efficiency; the exit pupil spillover efficiency; and the entrance pupil spillover efficiency [2]. The entrance pupil spillover efficiency is a new factor that is essential for a multibeam system.

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Makoto Nagai and Hiroaki Imada are with the Advanced Technology Center, National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan; e-mail: makoto.nagai@nao.ac.jp.

In this article, we apply this theory to multibeam axisymmetric dual-reflector antennas. We also derive a multibeam minimum blockage condition (MBC) for a Cassegrain-type telescope. We obtain an analytical expression of the aperture efficiency of the center beam, while the aperture efficiencies of off-center beams can be calculated by using numeric integrations. The calculation assumes an ideal coupling in polarization and phase that may potentially be affected by aberrations [3] but takes Gaussian illumination by the feed and the blocking by the subreflector into account, thus giving a best efficiency for the given setup. We propose a procedure to optimize the telescope geometry and the illumination under constraints on the main reflector diameter, FoV radius, main reflector focal distance, and distance between the subreflector and the focal plane.

2. Telescope Optics

Figure 1 shows the antenna geometry of Cassegrain type and design parameters considered in our procedure. The components are main reflector M_1, subreflector M_2, and the focal plane. The main reflector focus F_1, the composite focus F_{1+2}, and the symmetrical axis l are indicated. We adopted commonly used parameters [4]: main reflector diameter D_m, subreflector diameter D_s, focal distance of the main reflector F, and distance from the focal plane to the subreflector L_s. Focal plane diameter is D_f, and the distance from the main reflector to the subreflector is L_2. Mirror shapes are not explicitly specified in this procedure, because the perfect beam coupling is just assumed.

We consider the image of subreflector made by the main reflector that becomes the entrance pupil, as shown in Figure 2. The position and the diameter of the entrance pupil is calculated from the lens equation and the magnification equation based on geometrical optics

$$L_\mathrm{en} = FL_2/(F - L_2) \tag{1}$$

$$D_\mathrm{en} = FD_\mathrm{s}/(F - L_2) \tag{2}$$

where L_en is the distance from the main reflector. The entrance pupil and the telescope aperture defined by the main reflector determine the FoV of the telescope. The FoV radius without vignetting ϕ_FoV is given by

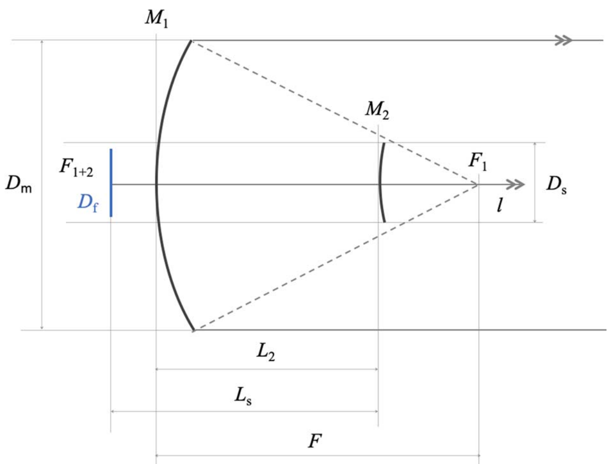

Figure 1. Antenna geometry and design parameters.

$$\tan\phi_{\mathrm{FoV}} = (D_{\mathrm{m}} - D_{\mathrm{en}})/(2L_{\mathrm{en}}) \tag{3}$$

Eliminating D_{en} and L_{en} in the previous equations, we obtain a relation

$$L_2 = (D_{\mathrm{m}} - D_{\mathrm{s}})F/(D_{\mathrm{m}} + 2F\tan\phi_{\mathrm{FoV}}) \tag{4}$$

A schematic of the whole optics with an equivalent lens is shown in Figure 3. Paths of the center beam are shown in this figure. The incident plane wave partially blocked by the subreflector enters the telescope aperture and reaches the entrance pupil. The feed beam from the focal plane array illuminates the exit pupil. The incident beam and the feed beam are coupled on the pupils.

3. The MBC for a Multibeam Telescope

To reduce the blockage effect due to the subreflector, the size of the subreflector is desired to be smaller. The lower limit of the subreflector size is given by MBC [5]. Here, we derive the MBC for a multibeam telescope.

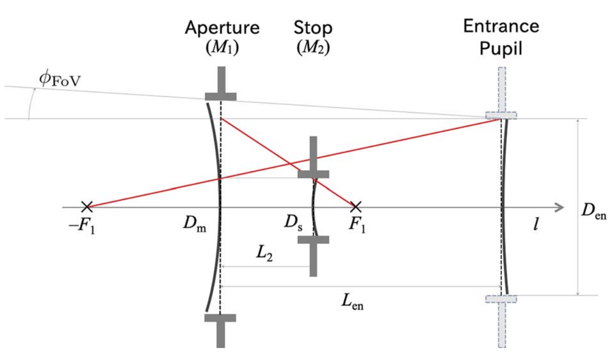

Figure 2. Position and size of the entrance pupil. Incident plane wave propagates from left to right.

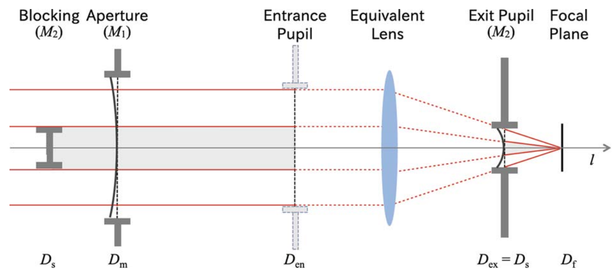

Figure 3. Schematic of beam path. Path of the center beam, entering the entrance pupil, focused on the focal plane, is shown. Shadow made by the blockage of the subreflector is shaded in gray.

The fundamental relation for the telescope is

$$\lambda^2 = \eta_{\mathrm{ap}}(\pi D_{\mathrm{m}}^2/4)\Omega \tag{5}$$

where λ is the target wavelength, η_{ap} is the aperture illumination efficiency, and Ω is the beam solid angle. Values of η_{ap} and Ω depend on beams, and we assume that the beams have comparable values. The fundamental relation for each feed is

$$\lambda^2 = \eta_{\mathrm{fd,ap}} A_{\mathrm{fd}} \Omega_{\mathrm{fd}} \tag{6}$$

where $\eta_{\mathrm{fd,ap}}$ is the feed aperture illumination efficiency, A_{fd} is the feed aperture area, and Ω_{fd} is the feed beam solid angle. We assume that every feed has the same values of $\eta_{\mathrm{fd,ap}}$, A_{fd}, and Ω_{fd}.

Let n_{bm} be the number of beams. We define filling factors as shown in Figure 4. The ratio of the total beam solid angle over the FoV k_{bm}, the ratio of the beam cross section at the subreflector over the subreflector area k_{s}, and the ratio of the total feed aperture area over the focal plane are k_{f}. These filling factors can be written as follows:

$$k_{\mathrm{bm}} = n_{\mathrm{bm}}\Omega/(\pi\phi_{\mathrm{FoV}}^2) \tag{7}$$

$$k_{\mathrm{s}} = L_{\mathrm{s}}^2\,\Omega_{\mathrm{fd}}/(\pi D_{\mathrm{s}}^2/4) \tag{8}$$

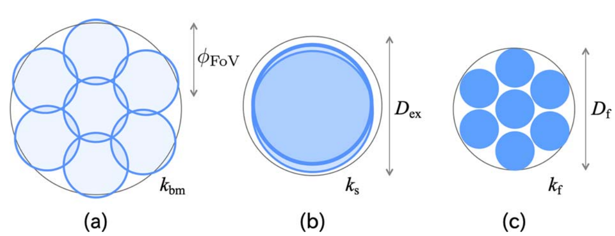

Figure 4. Beam cross sections of a multibeam telescope. (a) At the sky, the telescope FoV (the gray circle) is filled with the telescope beams (blue circles). (b) At the exit pupil, the subreflector area (the gray circle) is illuminated by the feed beams (the blue circle). (c) At the focal plane, the focal plane (the gray circle) is filled with the feed apertures (blue-filled circles).

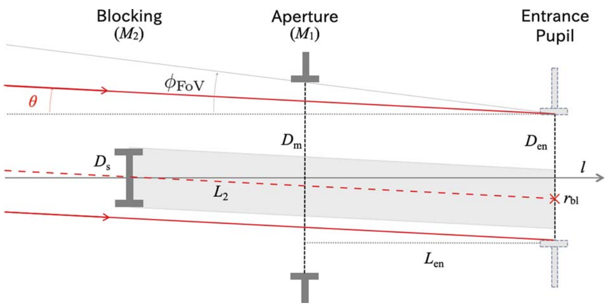

Figure 5. Path of an off-center beam of angle θ, entering the entrance pupil. Rays are drawn based on geometrical optics. Shadow made by the blockage of the subreflector is shaded in gray.

$$k_{\rm f} = n_{\rm bm} A_{\rm fd} / (\pi D_{\rm f}^2/4) \tag{9}$$

These factors can be about 0.8 to 0.9 in an efficient design. For example, if the beam cross sections fills the FoV and the focal plane as the densest packing of equal circles, $k_{\rm bm}$ and $k_{\rm f}$ become 0.907.

Eliminating Ω, $\Omega_{\rm fd}$, and $A_{\rm fd}$ in the previous equations, we obtain the MBC for a multibeam system,

$$(D_{\rm s} D_{\rm f})^2 = k_{\rm bm}\eta_{\rm ap}/(k_{\rm s}k_{\rm f}\eta_{\rm fd,ap}) \cdot (2\phi_{\rm FoV} L_{\rm s} D_{\rm m})^2 \tag{10}$$

We consider only the case in which the focal plane is behind the main reflector, because the focal plane array should be installed in a receiver cabin behind the main reflector structure. In this case, the minimum $D_{\rm s}$ is achieved when $D_{\rm s} = D_{\rm f}$. We thus obtain

$$D_{\rm s}^{\rm min} = \sqrt[4]{k_{\rm bm}\eta_{\rm ap}/(k_{\rm s}k_{\rm f}\eta_{\rm fd,ap})}\sqrt{2\phi_{\rm FoV} L_{\rm s} D_{\rm m}} \tag{11}$$

Because the inside of the fourth root is a product of factors about 1 for an efficient telescope and the fourth root is much closer to unity, we can omit this factor.

$$D_{\rm s}^{\rm min} \approx \sqrt{2\phi_{\rm FoV} L_{\rm s} D_{\rm m}} \tag{12}$$

We take this value for $D_{\rm s}$ and $D_{\rm f}$ in our procedure.

4. Calculation of Aperture Efficiency

The antenna efficiency of aperture type $\eta_{\rm ant}$ for a multibeam telescope is factorized into three factors [2]: the entrance pupil spillover efficiency $\eta_{\rm en}$; the beam coupling efficiency $\eta_{\rm bcp}$; and the exit pupil spillover efficiency $\eta_{\rm ex}$. Blockage can be considered in the beam coupling efficiency as $\eta_{\rm bcp} = \eta_{\rm bcp0}\eta_{\rm bl}(\theta)$, where $\eta_{\rm bcp0}$ is the beam coupling efficiency without the blockage and $\eta_{\rm bl}(\theta)$ is the blockage efficiency, which depends on the incident beam inclination angle θ.

Table 1. Efficiencies in case of $\phi_{\rm FoV}$ is 0.5° (%)

	Center beam $\theta = 0°$	Beam on edge $\theta = 0.5°$
$\eta_{\rm en}$	81.7	
$\eta_{\rm bcp0}$	89.9	
$\eta_{\rm bl}$	89.6	89.8
$\eta_{\rm ex}$	90.4	
$\eta_{\rm ant}$	59.5	59.6

$$\eta_{\rm ant}(\theta) = \eta_{\rm en}\eta_{\rm bcp0}\eta_{\rm bl}(\theta)\eta_{\rm ex}\cos\theta \tag{13}$$

The center beam has an inclination of $\theta = 0$, while off-center beams need an inclination factor $\cos\theta$ (Figure 5).

Incident beam $u_{\leftarrow}$ is uniform plane wave, and feed beam $u_{\rightarrow}$ is a Gaussian beam with an illumination parameter α $(= 1/k_{\rm s})$ at the subreflector [6]. We assume the perfect beam coupling, where polarization efficiency and phase efficiency are both unity. Factors of the antenna efficiency are given as follows:

$$\begin{aligned} \eta_{\rm en} &= D_{\rm en}^2/D_1^2 \\ \eta_{\rm bcp0} &= (2/\alpha)(1-\exp(-\alpha))^2/(1-\exp(-2\alpha)) \\ \eta_{\rm ex} &= 1-\exp(-2\alpha) \end{aligned} \tag{14}$$

The blockage efficiency is

$$\eta_{\rm bl} = |1 - c_{\rm bl}|^2 \tag{15}$$

where

$$c_{\rm bl} = \int_B u_{\rightarrow}(p)u_{\leftarrow}(p)\mathrm{d}^2p \Big/ \int_{A_{\rm en}} u_{\rightarrow}(p)u_{\leftarrow}(p)\mathrm{d}^2p \tag{16}$$

is the blockage coefficient. The divisor of the blockage coefficient is

$$\int_{A_{\rm en}} u_{\rightarrow}(p)u_{\leftarrow}(p)\mathrm{d}^2p = (\alpha/\pi)(1-\exp(-\alpha)) \tag{17}$$

The numerator of the blockage coefficient for the center beam becomes $(\alpha/\pi)(1-\exp(-\beta\alpha))$, and the blockage efficiency for the center beam is written as

$$\eta_{\rm bl}(0) = (\exp(-\alpha\beta) - \exp(-\alpha))^2/(1-\exp(-\alpha))^2 \tag{18}$$

where β is the blockage area fraction $\beta = (D_{\rm s}/D_{\rm en})^2$. The numerator of the blockage coefficient can be obtained with numeric integrations. For given β, the value of α that maximizes factor $\eta_{\rm bcp0}\eta_{\rm bl}(0)\eta_{\rm ex} = (2/\alpha)(\exp(-\alpha\beta) - \exp(-\alpha))^2$ is determined. We take this best α value for design illumination.

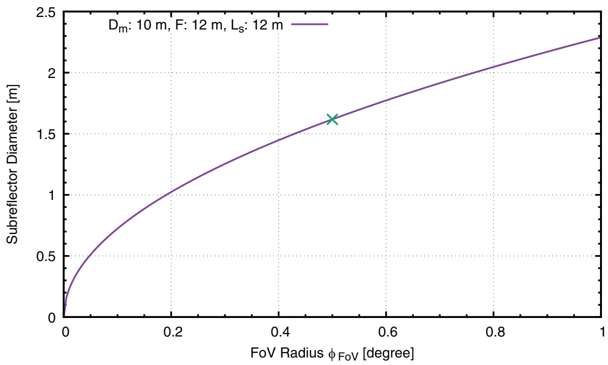

Figure 6. Example of a plot of subreflector diameter versus FoV radius. Point at $\phi_{\mathrm{FoV}} = 0.5^\circ$ is indicated by a cross.

5. Design Procedure

The antenna geometry has four degrees of freedom. We propose a design procedure that starts with specifying four parameters: the main reflector diameter D_m; the main reflector focal distance F; the distance between the focal plane and the subreflector L_s; and the FoV radius ϕ_{FoV}.

Given four parameters D_m, F, L_s, and ϕ_{FoV}, we can calculate subreflector diameter D_s with the multibeam MBC (12) and distance from the main reflector to the subreflector L_2 with the FoV condition (4). We can determine D_{en} using magnification (2) and the blockage area fraction β. For this β, calculate the optimum illumination parameter α. Now, we can obtain the four factors of the antenna efficiency, as described in the previous section.

At the heart of this procedure is the multibeam MBC for Cassegrain-type antenna with an efficient and uniform focal plane array, based on the fundamental relation for the telescope and each feed. Thus, (2) and (4) are based on geometrical optics. The illumination of the subreflector is by the same Gaussian beams from each feed. The resulting antenna efficiency is an ideal value under the perfect beam coupling with aberrations and cross polarization ignored.

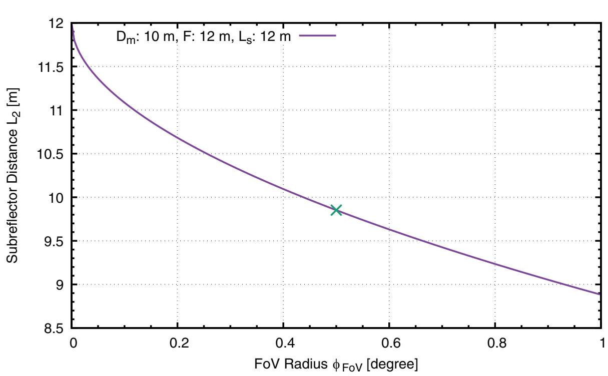

Figure 7. Example of a plot of subreflector distance versus FoV radius. Point at $\phi_{\mathrm{FoV}} = 0.5^\circ$ is indicated by a cross.

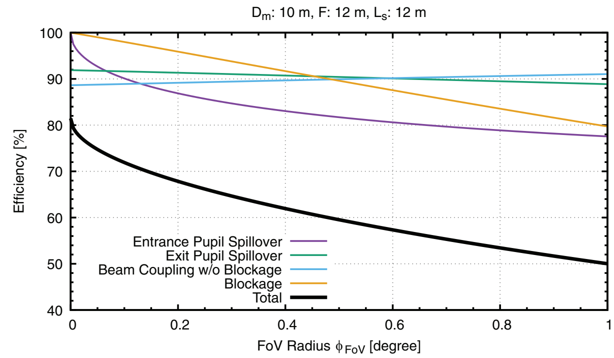

Figure 8. Example of a plot of efficiency factors versus FoV radius.

6. Example

Let us consider a telescope with a main reflector whose diameter is 10 m and the focal distance F is 12 m. Let the focal plane position L_s be 12 m, so the focal plane is just behind the main reflector structure. Let us set the FoV radius ϕ_{FoV} to be 0.5°; and D_s and D_f become 1.62 m, and L_2 becomes 9.58 m. The diameter and the position of the entrance pupil D_{en} and L_{en} are 9.04 m and 55.04 m, respectively. The blockage area fraction β is 0.032, and the best illumination parameter α becomes 1.17. The efficiencies are summarized in Table 1.

We can plot design parameters as a function of FoV radius ϕ_{FoV}. Figures 6 and 7 show the subreflector diameter D_s and distance L_2, respectively. The plots imply that the subreflector becomes larger and is closer to the main reflector to achieve a larger FoV. Figure 8 shows resulting efficiencies. The entrance pupil spillover efficiency quickly decreases as the FoV radius increases.

7. Conclusion

We developed a design procedure for a multibeam radio telescope with an axisymmetric dual-reflector antenna. We described equations for the Cassegrain-type system, although the calculation is similar for the Cassegrain type and the Gregorian type. A design obtained with this procedure can be a starting point for further investigation using ray tracing or simulations based on physical optics.

8. References

1. P. F. Goldsmith, "Radiation Patterns of Circular Apertures With Gaussian Illumination," *International Journal of Infrared and Millimeter Waves*, **8**, 7, July 1987, pp. 771–781.
2. M. Nagai, H. Imada, and T. Okumura, "Factorization of Antenna Efficiency of Aperture-Type Antenna: Beam

Coupling and Two Spillovers," *IEEE Transactions on Antennas and Propagation*, **69**, 7, July 2021, pp. 3750–3757.

3. H. Imada and M. Nagai, "Analytical Expression of Aperture Efficiency Affected by Seidel Aberrations," *Optics Express*, **28**, 16, August 2020, pp. 23075–23090.

4. C. Granet, "Designing Axially Symmetric Cassegrain or Gregorian Dual-Reflector Antennas From Combinations of Prescribed Geometric Parameters," *IEEE Antennas and Propagation Magazine*, **40**, 2, April 1998, pp. 76–82.

5. P. Hannan, "Microwave Antennas Derived From the Cassegrain Telescope," *IRE Transactions on Antennas and Propagation*, **9**, 2, March 1961, pp. 140–153.

6. J. W. Lamb, "Quasioptical Coupling of Gaussian Beam Systems to Large Cassegrain Antennas," *International Journal of Infrared and Millimeter Waves*, **7**, October 1986, pp. 1511–1536.