Plane-Wave Diffraction by a Slit Formed by Two Semi-Infinite Parallel-Plate Waveguides—Part I: The Case of *H* Polarization

Takashi Nagasaka, Wataru Takahara, and Kazuya Kobayashi

Abstract – The diffraction by a slit formed by two semi-infinite parallel-plate waveguides is rigorously analyzed using the Wiener–Hopf technique for the *H*-polarized plane-wave incidence. Introducing the Fourier transform for the scattered field and applying boundary conditions in the transform domain, the problem is formulated in terms of the simultaneous Wiener–Hopf equations, which are solved via the factorization and decomposition procedure leading to exact and high-frequency asymptotic solutions. Taking the inverse Fourier transform and applying the saddle point method, the scattered field in real space is derived. Numerical examples of the far-field intensity are presented and scattering characteristics of the slit are discussed.

1. Introduction

The analysis of electromagnetic scattering by slits formed by parallel-plate waveguides (i.e., tandem slits) is important in electromagnetic theory, as well as microwave and optical instrumentation. In the past, Alldredge [1] analyzed the plane-wave diffraction by a perfectly conducting tandem slit using the variational procedure introduced by [2]. The same diffraction problem was subsequently analyzed by [3], with the aid of the Wiener–Hopf technique, leading to a high-frequency asymptotic solution. The solution procedure in [3] was further extended to the analysis of a tandem slit with impedance waveguide walls by [4] and a tandem slit with material loading by [5]. In [4, 5], the authors used the Wiener–Hopf technique to solve the problems together with the iteration procedure by assuming that the slit width is large compared with the wavelength.

In this two-part article, we reconsider the tandem slit problem analyzed by [3] from new aspects and rigorously analyze the plane-wave diffraction for both *H* and *E* polarizations using the Wiener–Hopf technique based on the Jones' method [6]. The case of *H* polarization is considered in this first part, whereas the second part [7] concerns *E* polarization. The method of solution based on the Wiener–Hopf technique in this article is very different from that in previous works [3–5] in the sense that the multiple diffraction between the edges of the waveguides are rigorously taken into account.

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Takashi Nagasaka, Wataru Takahara, and Kazuya Kobayashi are with the Department of Electrical, Electronic, and Communication Engineering, Chuo University, 1-13-27 Kasuga, Bunkyo-ku, Tokyo 112-8551, Japan; e-mail: nagasaka@elect.chuo-u.ac.jp, a17.dam3@g.chuo-u.ac.jp, kazuya@tamacc.chuo-u.ac.jp.

Introducing the Fourier transform for the unknown scattered field and applying boundary conditions in the transform domain, the problem is formulated in terms of the simultaneous Wiener–Hopf equations satisfied by unknown spectral functions. Using the factorization and decomposition procedure [6], the Wiener–Hopf equations are solved leading to an exact solution. However, the solution is formal because infinite branch-cut integrals with unknown integrands are involved. Applying a novel asymptotic approach developed in our previous articles [8, 9], we derive high-frequency solutions in the form of complete asymptotic series. The scattered field in real space is derived by taking the Fourier inverse of the solution in the transform domain and evaluating resulting integrals. This procedure gives transverse magnetic modes for the field inside the waveguides, whereas for the region outside the waveguide, the scattered far field is derived using the saddle point method. Numerical examples of the far-field intensity are presented for various physical parameters and scattering characteristics of the slit are discussed in detail. The time factor is assumed to be $\exp(-i\omega t)$ and suppressed throughout the article.

2. Formulation of the Problem

We consider the diffraction of an *H*-polarized plane wave by a slit formed by two semi-infinite parallel-plate waveguides, as shown in Figure 1, where the waveguide plates are infinitely thin, perfectly conducting, and uniform in the y direction.

Let the total magnetic field $\phi^t(x,z)$ $[\equiv H_y^t(x,z)]$ be defined by

$$\begin{aligned} \phi^t(x,z) &= \phi^i(x,z) + \phi(x,z) + R\phi^r(x,z)\,, \quad x > b \\ &= \phi(x,z)\,, \quad |x| < b \\ &= \phi(x,z)\,, \quad x < -b \end{aligned} \tag{1}$$

where

$$R = e^{-2ikb\sin\theta_0} \tag{2}$$

$$\phi^i(x,z) = e^{-ik(x\sin\theta_0 + z\cos\theta_0)} \tag{3}$$

$$\phi^r(x,z) = e^{ik(x\sin\theta_0 - z\cos\theta_0)} \tag{4}$$

for $0 < \theta_0 < \pi/2$ with $k = \omega(\mu_0\varepsilon_0)^{1/2}$ being the free-space wavenumber. Here, $\phi^i(x,z)$ and $R\phi^r(x,z)$ denote

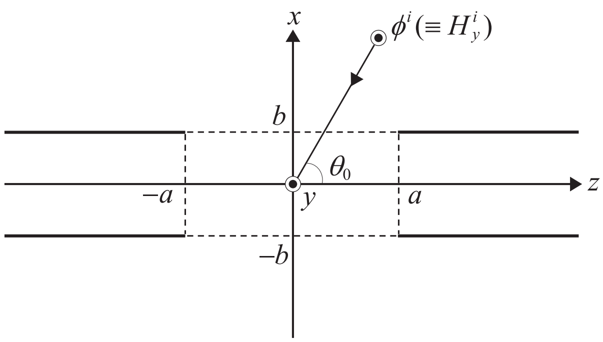


Figure 1. Geometry of the problem.

the incident field and the field reflected from the waveguide plates at $z = b$, respectively. The term $\phi(x, z)$ is the scattered field and satisfies the two-dimensional wave equation:

$$\left(\partial^2/\partial x^2 + \partial^2/\partial z^2 + k^2\right)\phi(x, z) = 0 \tag{5}$$

Once the scattered field is found, nonzero components of electromagnetic fields are derived from

$$\left(H_y^t, E_x^t, E_z^t\right) = \left(\phi^t, \frac{1}{i\omega\varepsilon_0}\frac{\partial \phi^t}{\partial z}, \frac{i}{\omega\varepsilon_0}\frac{\partial \phi^t}{\partial x}\right) \tag{6}$$

where the superscript t implies the total field.

For convenience of analysis, we introduce a small loss into the medium as in $k = k_1 + ik_2$ with $0 < k_2 \ll k_1$. Let us now define the Fourier transform of $\phi(x, z)$, with respect to z as

$$\Phi(x, \alpha) = (2\pi)^{-1/2}\int_{-\infty}^{\infty} \phi(x, z)e^{i\alpha z}dz \tag{7}$$

where $\alpha(\equiv \mathrm{Re}\alpha + i\mathrm{Im}\alpha) = \sigma + i\tau$. We also define the Fourier integrals as

$$\Phi_\pm(x, \alpha) = \pm(2\pi)^{-1/2}\int_{\pm a}^{\pm\infty} \phi(x, z)e^{i\alpha(z\mp a)}dz \tag{8}$$

$$\Phi_1(x, \alpha) = (2\pi)^{-1/2}\int_{-a}^{a} \phi(x, z)e^{i\alpha z}dz \tag{9}$$

In view of the radiation condition at infinity, we see that $\Phi(x, \alpha)$ and $\Phi_\pm(x, \alpha)$ are regular in $|\tau| < k_2 \cos\theta_0$ and $\tau \gtrless \mp k_2 \cos\theta_0$, respectively. We also see that $\Phi_1(x, \alpha)$ is an entire function. Taking the Fourier transform of (5) and using the radiation condition, we derive the transformed wave equation

$$\left(d^2/dx^2 - \gamma^2\right)\Phi(x, \alpha) = 0, \quad \gamma = \left(\alpha^2 - k^2\right)^{1/2} \tag{10}$$

in the strip $|\tau| < k_2 \cos\theta_0$. Because γ is a double-valued function of α, we choose a proper branch such that $\gamma = -ik$ for $\alpha = 0$.

In view of the boundary conditions for tangential electromagnetic fields, we see that

$$H_y^t(\pm b + 0, z) = H_y^t(\pm b - 0, z), \quad |z| < a \tag{11}$$

$$E_z^t(\pm b + 0, z) = E_z^t(\pm b - 0, z), \quad |z| < a \tag{12}$$

$$E_z^t(\pm b, z) = 0, \quad |z| > a \tag{13}$$

The tangential component of the total electric field is continuous across $x = \pm b$ for $-\infty < z < \infty$. In contrast, the tangential component of the total magnetic field is continuous across $x = \pm b$ only for $|z| < a$.

Considering the boundary condition of $E_z^t(\pm b, z)$, the solution of (10), bounded for $|x| \to \infty$, can now be expressed as

$$\begin{aligned}\Phi(x, \alpha) &= A(\alpha)e^{-\gamma(x-b)}, \quad x > b \\ &= -A(\alpha)\cosh[\gamma(x + b)]/\sinh(2\gamma b) \\ &\quad - B(\alpha)\cosh[\gamma(x - b)]/\sinh(2\gamma b), |x| < b \\ &= B(\alpha)e^{\gamma(x+b)}, \quad x < -b\end{aligned} \tag{14}$$

where $A(\alpha)$ and $B(\alpha)$ are all unknown spectral functions that are to be determined via the Wiener−Hopf technique.

Given the boundary conditions in the Fourier transform domain, we obtain that

$$A(\alpha) + B(\alpha) = -S_1(\alpha)/\gamma \tag{15}$$

$$A(\alpha) - B(\alpha) = -D_1(\alpha)/\gamma \tag{16}$$

$$\begin{aligned}&e^{-i\alpha a}J_-(b, \alpha) + e^{i\alpha a}J_+(b, \alpha) \\ &\quad - \left(A_1 e^{i\alpha a} - A_2 e^{-i\alpha a}\right)/(\alpha - k\cos\theta_0) \\ &= A(\alpha)[\cos(2\gamma b)/\sin(2\gamma b) + 1] \\ &\quad - B(\alpha)/\sin(2\gamma b)\end{aligned} \tag{17}$$

$$\begin{aligned}&e^{-i\alpha a}J_-(-b, \alpha) + e^{i\alpha a}J_+(-b, \alpha) \\ &= A(\alpha)/\sin(2\gamma b) \\ &\quad - B(\alpha)[\cos(2\gamma b)/\sin(2\gamma b) + 1]\end{aligned} \tag{18}$$

where

$$S_1(\alpha) = \Phi_1'(b, \alpha) + \Phi_1'(-b, \alpha) \tag{19}$$

$$D_1(\alpha) = \Phi_1'(b, \alpha) - \Phi_1'(-b, \alpha) \tag{20}$$

$$J_\pm(\pm b, \alpha) = \Phi_\pm(\pm b + 0, \alpha) - \Phi_\pm(\pm b - 0, \alpha) \tag{21}$$

$$A_{1,2} = 2e^{-ikb\sin\theta_0}e^{\mp ika\cos\theta_0}\Big/\left(i\sqrt{2\pi}\right) \tag{22}$$

The prime in (19) and (20) denotes differentiation with respect to x.

Carrying out some manipulations with the aid of boundary conditions, we are led to

$$-S_1(\alpha)/L(\alpha) = e^{i\alpha a}U_{(+)}(\alpha) + e^{-i\alpha a}U_-(\alpha) \tag{23}$$

$$-D_1(\alpha)/N(\alpha) = e^{i\alpha a}V_{(+)}(\alpha) + e^{-i\alpha a}V_-(\alpha) \tag{24}$$

where

$$\left.\begin{matrix} U_{(+)}(\alpha) \\ U_-(\alpha) \end{matrix}\right\} = J_+^s(\alpha) \mp A_{1,2}/(\alpha - k\cos\theta_0) \tag{25}$$

$$\left.\begin{matrix} V_{(+)}(\alpha) \\ V_-(\alpha) \end{matrix}\right\} = J_+^d(\alpha) \mp A_{1,2}/(\alpha - k\cos\theta_0) \tag{26}$$

$$J_\pm^{s,d}(\alpha) = J_\pm(b,\alpha) \pm J_\pm(-b,\alpha) \tag{27}$$

$$L(\alpha) = \gamma e^{-\gamma b}\cosh(\gamma b) \tag{28}$$

$$N(\alpha) = \gamma e^{-\gamma b}\sinh(\gamma b) \tag{29}$$

In the previously mentioned notations, the subscripts $+$ and $-$ imply that the functions are regular in the upper $(\tau > -k_2\cos\theta_0)$ and the lower $(\tau < k_2\cos\theta_0)$ half plane, respectively, whereas the subscript $(+)$ implies that the functions are regular in $\tau > -k_2\cos\theta_0$, except for a simple pole at $\alpha = k\cos\theta_0$. The Wiener–Hopf equations, (23) and (24), are satisfied by the unknown spectral functions and hold in the strip $|\tau| < k_2\cos\theta_0$.

3. Solution of the Wiener–Hopf Equations

The kernel functions $L(\alpha)$ and $N(\alpha)$ defined by (28) and (29) can be factorized as [6]

$$L(\alpha) = L_+(\alpha)L_-(\alpha) \tag{30}$$

$$N(\alpha) = N_+(\alpha)N_-(\alpha) \tag{31}$$

where $L_\pm(\alpha)$ and $N_\pm(\alpha)$ are split functions given by

$$\begin{aligned} L_\pm(\alpha) &= (\cos kb)^{1/2}e^{i3\pi/4}\exp\left[\frac{i\gamma b}{\pi}\ln\left(\frac{\pm\alpha-\gamma}{k}\right)\right] \\ &\cdot\exp\left\{\pm\frac{i\alpha b}{\pi}\left[1-C+\ln\left(\frac{\pi}{2kb}\right)+i\frac{\pi}{2}\right]\right\} \\ &\cdot(k\pm\alpha)^{1/2}\prod_{\substack{n=1\\ \text{odd}}}^{\infty}\left(1\pm\frac{\alpha}{i\gamma_n}\right)\exp\left(\pm\frac{2i\alpha b}{n\pi}\right) \end{aligned} \tag{32}$$

$$\begin{aligned} N_\pm(\alpha) &= i\left(\frac{\sin kb}{k}\right)^{1/2}\exp\left[\frac{i\gamma b}{\pi}\ln\left(\frac{\pm\alpha-\gamma}{k}\right)\right] \\ &\cdot\exp\left\{\pm\frac{i\alpha b}{\pi}\left[1-C+\ln\left(\frac{2\pi}{kb}\right)+i\frac{\pi}{2}\right]\right\} \\ &\cdot(k\pm\alpha)\prod_{\substack{n=2\\ \text{even}}}^{\infty}\left[\left(1\pm\frac{\alpha}{i\gamma_n}\right)\exp\left(\pm\frac{2i\alpha b}{n\pi}\right)\right] \end{aligned} \tag{33}$$

with $C(= 0.57721566)$ being the Euler constant and $\gamma_n = [(n\pi/2b)^2 - k^2]^{1/2}$. Multiplying both sides of (24) by $e^{\mp i\alpha a}N_\pm(\alpha)$ and decomposing the resultant equations via the use of the edge condition, we arrive at

$$\begin{aligned} V_{(+)}^{s,d}(\alpha) &= \frac{1}{N_+(\alpha)}\left[-\frac{A_1N_+(k\cos\theta_0)}{\alpha-k\cos\theta_0}\right. \\ &\left.\mp\frac{A_2N_-(k\cos\theta_0)}{\alpha+k\cos\theta_0}\pm v_{s,d}(\alpha)\right] \end{aligned} \tag{34}$$

where

$$\begin{aligned} v_{s,d}(\alpha) &= (\pi i)^{-1} \\ &\cdot\int_k^{k+i\infty}T_+(\beta)\frac{(\beta-k)^{3/2}}{\beta+\alpha}V_{(+)}^{s,d}(\beta)e^{2i\beta a}d\beta \end{aligned} \tag{35}$$

with

$$T_+(\beta) = -\frac{\sinh^2(\gamma b)(\beta+k)^{1/2}}{N_+(\beta)(\beta-k)} \tag{36}$$

$$V_{(+)}^{s,d}(\alpha) = V_{(+)}(\alpha) \pm V_-(-\alpha) \tag{37}$$

Thus, (34) is the exact solution of the Wiener–Hopf equation (24), but it is formal because the infinite integrals with unknown functions, as given by (35), are involved. In the following, we derive a high-frequency solution by using a rigorous asymptotic method developed by [8, 9].

Note that (34) can be rewritten in the form

$$J_+^d(\alpha) \pm J_-^d(-\alpha) = \left[A_1Q_1(\alpha) \pm A_2Q_2(\alpha) \pm v_{s,d}(\alpha)\right]/N_+(\alpha) \tag{38}$$

where

$$Q_{1,2}(\alpha) = \frac{N_+(\alpha) - N_\pm(k\cos\theta_0)}{\alpha \mp k\cos\theta_0} \tag{39}$$

In a similar way, the functions $v_{s,d}(\alpha)$ defined by (35) can be rearranged as in

$$\begin{aligned} v_{s,d}(\alpha) &= A_1\eta_{t2}(\alpha) \pm A_2\eta_{t1}(\alpha) + (\pi i)^{-1} \\ &\cdot\int_k^{k+i\infty}T_+(\beta)\frac{J_+^d(\beta)\pm J_-^d(-\beta)}{\beta+\alpha}(\beta-k)^{3/2}e^{2i\beta a}d\beta \end{aligned} \tag{40}$$

where

$$\eta_{t1,t2}(\alpha) = \frac{\xi_{01}^t(\alpha) - \xi_{01}^t(\pm k\cos\theta_0)}{\alpha \mp k\cos\theta_0} \tag{41}$$

$$\begin{aligned} \xi_{pn}^t(\alpha) &= \frac{e^{2ika}}{\pi}(-1)^p p!\frac{i^{n-p-1/2}}{(2a)^{n-p+1/2}} \\ &\cdot\Gamma_{1+p}^t\left[\frac{3}{2}+n, -2i(\alpha+k)a\right] \end{aligned} \tag{42}$$

$$\Gamma_m^t(u,v) = \int_0^\infty\frac{t^{u-1}e^{-t}}{(t+v)^m}T_+\left(k+\frac{i}{2a}t\right)dt \tag{43}$$

Here, $\Gamma_m^t(u,v)$ is the generalized gamma function introduced by [9, 10]. This special function rigorously accounts for the multiple-edge diffraction. By using the method developed in [9], we can derive a high-frequency asymptotic representation of (34) for large $|k|a$ with the result that

$$\left.\begin{array}{l} V_{(+)}(\alpha) \\ V_{-}(\alpha) \end{array}\right\} \sim \left[\mp \frac{A_{1,2}N_{\pm}(k\cos\theta_0)}{\alpha - k\cos\theta_0} + A_{2,1}\eta_{t1,t2}(\pm\alpha) + \sum_{n=0}^{\infty} \frac{f_n^{vs} \mp f_n^{vd}}{2} \xi_{0(n+1)}^{t}(\pm\alpha)\right] \Big/ N_{\pm}(\alpha) \tag{44}$$

where

$$f_n^{vs,vd} = \frac{1}{n!}\frac{d^n}{d\beta^n}\left[J_+^d(\beta) \pm J_-^d(-\beta)\right]\Big|_{\beta=k} \tag{45}$$

with $f_n^{vs,vd}$ being unknowns numerically determined by solving the matrix equation

$$f_m^{vs,vd} \mp \sum_{n=0}^{\infty} A_{mn}^{v} f_n^{vs,vd} \sim B_m^{vs,vd}, \quad m = 1,2,3,\ldots \tag{46}$$

where

$$A_{mn}^{v} = \sum_{p=0}^{m} \frac{\xi_{p(n+1)}^{t}(k)}{p!(m-p)!} \frac{d^{m-p}[1/N_+(\alpha)]}{d\alpha^{m-p}}\bigg|_{\alpha=k} \tag{47}$$

$$B_m^{vs,vd} = \sum_{p=0}^{m} \left\{ \frac{1}{p!(m-p)!} \cdot \frac{d^{m-p}[1/N_+(\alpha)]}{d\alpha^{m-p}} \frac{d^p \chi_{vs,vd}(\alpha)}{d\alpha^p}\bigg|_{\alpha=k} \right\} \tag{48}$$

$$\chi_{vs,vd}(\alpha) = A_1[Q_1(\alpha) \pm \eta_{t2}(\alpha)] + A_2[\eta_{t1}(\alpha) \pm Q_2(\alpha)] \tag{49}$$

This completes the derivation of a rigorous high-frequency solution of the Wiener–Hopf equation (24), which contains all the higher order diffraction terms. Similarly, (23) can be solved, and the details are omitted.

4. Scattered Field

The scattered field in real space is obtained by taking the inverse Fourier transform of $\Phi(x,\alpha)$, given by (14), according to the formula:

$$\phi(x,z) = (2\pi)^{-1/2} \int_{-\infty+ic}^{\infty+ic} \Phi(x,\alpha) e^{-i\alpha z} d\alpha, \quad -k_2\cos\theta_0 < c < k_2 \tag{50}$$

First, we consider the region $|x| < b$ and derive the scattered field inside the waveguide. We substitute the scattered field representation for $|x| < b$, namely,

$$\Phi(x,\alpha) = \cosh[\gamma(x+b)]\frac{D_1(\alpha)+S_1(\alpha)}{2\gamma\sinh(2\gamma b)} + \cosh[\gamma(x-b)]\frac{D_1(\alpha)-S_1(\alpha)}{2\gamma\sinh(2\gamma b)} \tag{51}$$

into (50) and evaluate the integral by computing the residue contributions due to an infinite number of simple poles at $\alpha = \pm i\gamma_n, n = 0,1,2,3,\ldots$ with $\gamma_0 = -ik$. This yields

$$\phi(x,z) = \pm(\pi/2)^{1/2}\sum_{n=0}^{\infty}(-1)^n\frac{1}{\gamma_n b} \cdot \left\{\Phi'_1(b,\pm i\gamma_n)\cos\left[\frac{n\pi}{2b}(x+b)\right] - \Phi'_1(-b,\pm i\gamma_n)\cos\left[\frac{n\pi}{2b}(x-b)\right]\right\}e^{\pm\gamma_n z}, \quad z \lessgtr \mp a \tag{52}$$

We now consider the region $|x| > b$ and derive the scattered far field. In view of (14), the scattered field in the Fourier transform domain is written as

$$\Phi(x,\alpha) = \tilde{\Phi}(\alpha)e^{-\gamma|x|} \tag{53}$$

where

$$\tilde{\Phi}(\alpha) = \sinh(\gamma b)\left[e^{i\alpha a}V_{(+)}(\alpha) + e^{-i\alpha a}V_-(\alpha)\right]/2 \pm \cosh(\gamma b)\left[e^{i\alpha a}U_{(+)}(\alpha) + e^{-i\alpha a}U_-(\alpha)\right]/2 \tag{54}$$

Substituting (53) into (50) and evaluating the resulting integral using the saddle point method, we derive that

$$\phi(\rho,\theta) \sim \tilde{\Phi}(-k\cos\theta)k\sin|\theta|e^{i(k\rho-\pi/4)}(k\rho)^{-1/2} \tag{55}$$

as $|k|\rho \to \infty$, where (ρ,θ) is the cylindrical coordinate defined by $x = \rho\sin\theta, z = \rho\cos\theta$ $(-\pi < \theta < \pi)$.

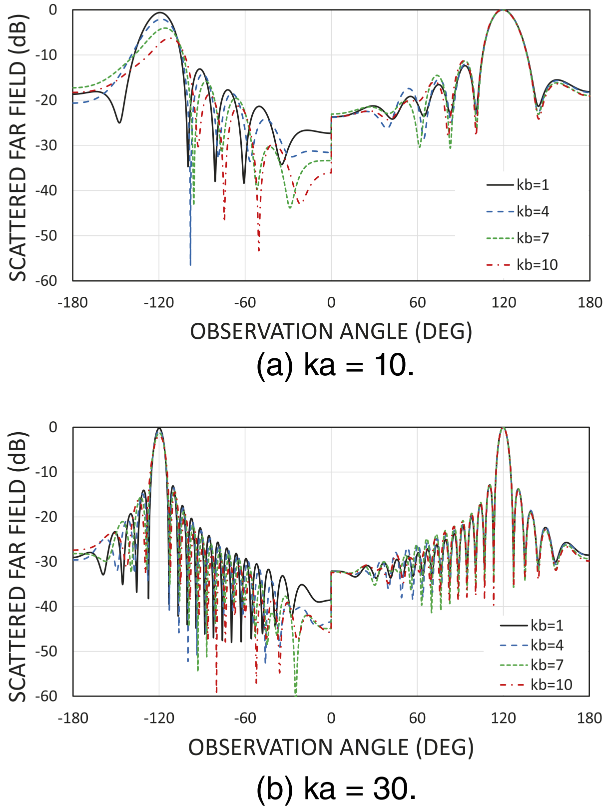
The complementary diffraction problem related to the problem considered in this article is the problem of a finite parallel-plate waveguide, which was solved by [11] for E polarization and by [12] for H polarization.

5. Numerical Results and Discussion

We now present numerical examples of the scattered far field for H polarization and discuss the far-field scattering characteristics. The normalized far-field intensity is defined by

$$|\phi(\rho,\theta)|[\mathrm{dB}] = 20\log_{10}\left[\frac{\lim\limits_{\rho\to\infty}\left|(k\rho)^{1/2}\phi(\rho,\theta)\right|}{\max\limits_{|\theta|\le\pi}\lim\limits_{\rho\to\infty}\left|(k\rho)^{1/2}\phi(\rho,\theta)\right|}\right] \tag{56}$$

Figure 2 shows the normalized far-field intensity as a function of observation angle θ, where the slit width and the waveguide spacing are chosen as $ka = 10$, 30 and $kb = 1$, 4, 7, 10, respectively, and the incidence angle is fixed as $\theta_0 = 60°$. From the figure, for all chosen values of ka and kb, the far-field intensity shows noticeable peaks along the reflection boundary at $\theta = 120°$, as expected. On the other hand, note that for negative θ, the peak location gradually shifts from



(a) ka = 10.

(b) ka = 30.

Figure 2. Scattered far-field $|\phi(\rho,\theta)|$ (decibels) for H polarization and $\theta_0 = 60°$.

the incident shadow boundary at $-120°$, with an increase of kb. This feature is particularly seen for smaller ka and larger kb. Comparing the results for a different kb with fixed ka, the curves for all kb show close features for $0° < \theta < 180°$, whereas those for $-180° < \theta < 0°$ are somewhat different, depending on kb, and the far-field intensity for negative θ becomes smaller with an increase of kb. In addition, we see that the results are discontinuous across $\theta = 0°$ and $\pm 180°$ for all kb, and these gaps are noticeable for larger kb.

Figure 3 shows comparison of the results for $kb = 0.01$ and $kb = 0$ (a slit in an infinite, perfectly conducting plane [13, 14]) for H polarization $ka = 10$, $\theta_0 = 60°$. We observe that the two curves become very close to each other, as expected.

6. Conclusion

In this article, we solved the H-polarized plane-wave diffraction by a slit formed by two semi-infinite parallel-plate waveguides using the Wiener−Hopf technique. Applying a rigorous asymptotics, a high-frequency solution for large slit width was obtained. The scattered field inside and outside the waveguides was evaluated. Some numerical examples on the far-field intensity were presented, and the far-field scattering characteristics of the slit were discussed.

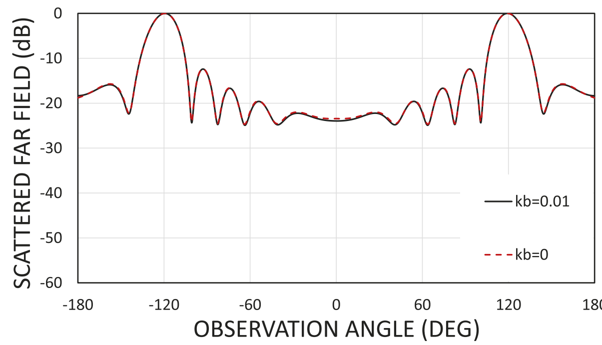


Figure 3. Scattered far-field = $|\phi(\rho,\theta)|$ (decibels) for H polarization $\theta_0 = 60°$, $ka = 10$, $kb = 0.01, 0$.

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