Distance Characteristics of Power Absorption Ratio in a Semi-Infinite Flat Plate Model Using Sommerfeld Theory

Ken Sato, Kenta Someya, Chongsengchang Khamneexay, and Yoshitsugu Kamimura

Abstract – The power absorption ratio from the millimeter-wave band antenna to the human body can be obtained analytically using the Sommerfeld semi-spatial problem, which discusses the power absorption from the antenna to Earth. In this study, we applied this model to the case where a minute electric dipole was placed vertically and horizontally to the surface of the human body and theoretically analyzed the power absorption ratio in a three-layer flat plate model. As a result, with the horizontal antenna, the power absorption ratio was suppressed to approximately 20% when the separation distance of the antenna was one-fourth of a wavelength or more; on the other hand, with the vertical antenna, the interactions between the antenna and the human body increased non-negligibly when the separation distance was one wavelength or less. Furthermore, we compared the three-layer model with the one-layer model, which has a simpler structure for computational analysis. As a result, although a clear difference between the one-layer model and the three-layer model was confirmed at 6 GHz to 10 GHz, the one-layer model can be sufficiently approximated at 20 GHz and above. We have also confirmed that the absorption ratio approaches 100% when the antenna is closer to the skin surface by numerical simulations using the finite difference time domain method.

1. Introduction

In recent years, research on the body area network using wearable and implanted wireless devices has been actively conducted in medical communication technology. The situation of being exposed to millimeter waves will increase; thus, it is important how the power absorption ratio to the human body changes depending on the distance between the antenna and the human body. For wireless devices with a frequency of 6 GHz or higher that are used within 10 cm of the human body, regulating methods by power density has been proposed [1–3].

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Ken Sato is with the National Institute of Technology, Hachinohe College, 16-1, Uwanotai, Tamonoki, Hachinohe 039-1192, Japan; e-mail: satok@m.ieice.org.

Kenta Someya is with TKC Corporation, SEC, Development Standard Promotion Department, Japan, 1758 Tsurutamachi, Utsunomiya-shi, Tochigi 320-8644, Japan.

Chongsengchang Khamneexay is with UXO clearance at JMAS Organization, Laos.

Yoshitsugu Kamimura is with the Graduate School of Engineering, Utsunimoya University, 7-1-2, Yoto, Utsunomiya, Tochigi 321-8585, Japan; e-mail: gami@is.utsunomiya-u.ac.jp.

However, in the case of wearable and implanted devices, it is difficult to measure the specific absorption rate accurately because the distance is less than a few millimeters or is embedded in the body. In addition, although the formula for calculating the distribution of absorbed power has been obtained by an analytical method [4], the formula for calculating the total power amount has not been derived. Therefore, if a formula for analytically calculating the total amount of electric power can be derived, an exact solution of the total amount of electric power can be obtained. Since the wavelength in the millimeter-wave band can be regarded as sufficiently smaller than the radius of curvature of the body surface, the theoretical formula by Sommerfeld and Renner, which discusses the power absorption of the antenna into the ground, can be applied [4]. By using the theoretical formula, the distance dependence of the power absorption rate of the minute antenna in the millimeter-wave band is calculated, and the interaction between the antenna and the human body is quantitatively evaluated. We derive the theoretical formulas of a three-layer semi-infinite flat plate model that considers tissues such as fat and muscle under the skin to obtain the power absorption ratio in the millimeter-wave band at 6 GHz to 100 GHz. From these results, the interaction between the antenna and the human body can be estimated, and it is possible to obtain a basis for determining the separation distance required for exposure evaluation based on the power density.

2. The Theoretical Formula of the Half-Space Model

We derive the theoretical formula for the semi-infinite flat plate model that extends Sommerfeld's formula to three layers simulating the human body. The theoretical formulas of the magnetic field and electric field when the minute electric dipole is arranged in the vertical and horizontal directions with respect to the human body are obtained. Following is the case of a vertical direction antenna.

2.1 Theoretical Formula of the Three-Layer Model

As shown in Figure 1, the Hertz vector of the incident wave of each medium is presented only in the z-direction, as shown in (1)–(4), where $P = (I\ell)/(4\pi\omega\varepsilon_0)$ [Vm^2], $\alpha_n = \sqrt{u^2 - k_n^2}$ (if $u < k_n$, then $-i\sqrt{k_n^2 - u^2}$), $R_z^{(n)}$ is the reflection coefficient, and $T_z^{(n)}$ is the transmission coefficient:

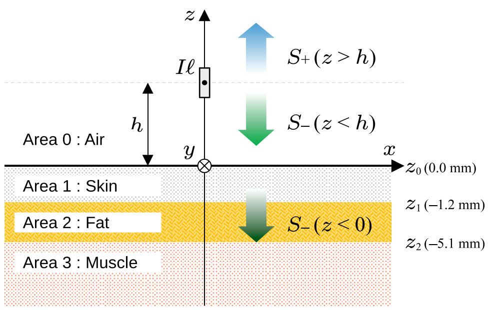

Figure 1. Three-layer human model and electric power flow (vertical).

$$\Pi_z^{(0)} = P\int_0^{\infty} \frac{u}{\alpha_0}\left(e^{-\alpha_0|z-h|} + R_z^{(0)} e^{-\alpha_0(z+h)}\right) J_0(u\rho)du \quad (1)$$

$$\Pi_z^{(1)} = P\int_0^{\infty} \frac{u}{\alpha_1}\left(T_z^{(1)} e^{\alpha_1 z-\alpha_0 h} + R_z^{(1)} e^{-\alpha_1 z-\alpha_0 h}\right) J_0(u\rho)du \quad (2)$$

$$\Pi_z^{(2)} = P\int_0^{\infty} \frac{u}{\alpha_2}\left(T_z^{(2)} e^{\alpha_2 z-\alpha_0 h} + R_z^{(2)} e^{-\alpha_2 z-\alpha_0 h}\right) J_0(u\rho)du \quad (3)$$

$$\Pi_z^{(3)} = P\int_0^{\infty} \frac{u}{\alpha_3} T_z^{(3)} e^{\alpha_3 z-\alpha_0 h} J_0(u\rho)du \quad (4)$$

Using the boundary conditions at $z = z_n$ $(n = 0,\ 1,\ 2)$,

$$\frac{\partial \Pi_z^{(n)}}{\partial z} = \frac{\partial \Pi_z^{(n+1)}}{\partial z},\ k_n^2 \Pi_z^{(n)} = k_{n+1}^2 \Pi_z^{(n+1)}$$

Then the linear simultaneous equations $Ax = b$ can be defined, and the reflection coefficient $R_z^{(n)}$ and the transmission coefficient $T_z^{(n)}$ can be calculated by solving the equations. Here, because $z_0 < h$, matrices A, x, and b are presented as (5),

$$A = \begin{pmatrix} -a_{00}^- & -a_{10}^+ & a_{10}^- & 0 & 0 & 0 \\ a_{00}^- b_0 & -a_{10}^+ b_1 & -a_{10}^- b_1 & 0 & 0 & 0 \\ 0 & a_{11}^+ & -a_{11}^- & -a_{21}^+ & a_{21}^- & 0 \\ 0 & a_{11}^+ b_1 & a_{11}^- b_1 & -a_{21}^+ b_2 & -a_{21}^- b_2 & 0 \\ 0 & 0 & 0 & a_{22}^+ & -a_{22}^- & -a_{32}^+ \\ 0 & 0 & 0 & a_{22}^+ b_2 & a_{22}^- b_2 & -a_{32}^+ b_3 \end{pmatrix},$$

$$x = \begin{pmatrix} R_z^{(0)} \\ T_z^{(1)} \\ R_z^{(1)} \\ T_z^{(2)} \\ R_z^{(2)} \\ T_z^{(3)} \end{pmatrix},\ b = \begin{pmatrix} -a_{00}^+ \\ -a_{00}^+ b_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (5)$$

where $a_{nm}^{\pm} = e^{\pm\alpha_n z_m}, b_n = k_n^2/\alpha_n$. In free space, the electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ are calculated using the Hertz vector Π as follows:

$$\mathbf{E} = k^2\Pi + \nabla(\nabla\cdot\Pi), \mathbf{H} = \frac{k}{iZ}\nabla\times\Pi$$

Here, because $\Pi = (0,\ 0,\ \Pi_z)$ has only the z-direction element, it does not depend on φ:

$$E_\rho = \frac{\partial^2 \Pi_z}{\partial\rho\partial z},\quad E_z = k^2\Pi_z + \frac{\partial^2\Pi_z}{\partial z^2} \quad (6)$$

$$H_\varphi = -\frac{k}{iZ}\frac{\partial\Pi_z}{\partial\rho},\ \text{where } Z = \sqrt{\mu/\varepsilon} \quad (7)$$

The theoretical formula of the horizontal antenna can be also derived by same process. The electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ for the horizontal antenna are shown as follows:

$$E_\rho = k^2\Pi_\rho + \frac{\partial}{\partial\rho}(\nabla\cdot\Pi), H_\rho = \frac{k}{iZ}\left(\frac{1}{\rho}\frac{\partial\Pi_z}{\partial\varphi} - \frac{\partial\Pi_\varphi}{\partial z}\right) \quad (8)$$

$$E_\varphi = k^2\Pi_\varphi + \frac{1}{\rho}\frac{\partial}{\partial\varphi}(\nabla\cdot\Pi),\ H_\varphi = \frac{k}{iZ}\left(\frac{\partial\Pi_\rho}{\partial z} - \frac{\partial\Pi_z}{\partial\rho}\right)$$

$$E_z = k^2\Pi_z + \frac{\partial}{\partial z}(\nabla\cdot\Pi), H_z = \frac{k}{iZ}\frac{1}{\rho}\left(\frac{\partial}{\partial\rho}(\rho\Pi_\varphi) - \frac{\partial\Pi_\rho}{\partial\varphi}\right) \quad (9)$$

2.2 Theoretical Formula of the One-Layer Model

The theoretical formula of the one-layer flat plate model can be derived from the three-layer model. On the vertical electric dipole in Figure 1, when the boundary positions z_0, z_1, and z_2 are equal to 0, $\alpha_2 = \alpha_3 = \alpha_1$, $k_2 = k_3 = k_1$, $R_z^{(1)} = R_z^{(2)} = 0$, $T_z^{(2)} = T_z^{(3)} = 1$ in (5); therefore, the reflection coefficient R_z and transmission coefficient T_z are obtained as follows:

$$R_z = 1 - T_z = \frac{k_1^2\alpha_0 - k_0^2\alpha_1}{k_1^2\alpha_0 + k_0^2\alpha_1}, T_z = \frac{2k_0^2\alpha_1}{k_1^2\alpha_0 + k_0^2\alpha_1}$$

In the case of a model with four or more layers, it can be derived by formally increasing the number of intermediate layers in the three-layer model.

3. Derivation of the Power Absorption Ratio

To obtain the power absorption ratio to the human body, the radiant power S_+, the absorbed power S_-, and the supplied power W are derived from the magnetic field and electric field. To calculate the power, $\mathbf{S} = \mathbf{E}\times\mathbf{H}$ is used. $\mathbf{S}$(W/m^2) is the Poynting vector, which represents the flow of the power density. The power integrated on a horizontal plane in free space is defined as follows:

$$S_\pm = \int_{S_\infty}^{\pm} S_z ds = \int_{S_\infty} (E_\rho^\pm H_\varphi - E_\varphi H_\rho^\pm) ds. \quad (10)$$

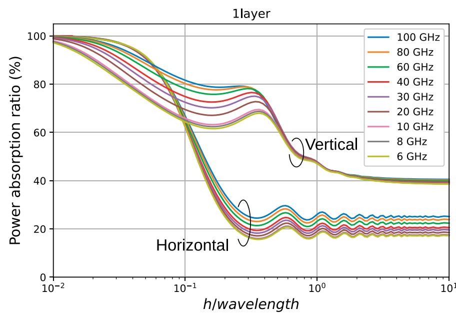

Figure 2. Theoretical power absorption ratio of the one-layer model.

The power presented in (10) is defined as S_+ (radiant power) or $S_–$ (absorbed power), corresponding to the case where the horizontal plane is placed above ($z > h$) or below ($z < h$) the dipole antenna. The power supply can be obtained as $W = S_+ - S_–$. $S_–$ is defined as the positive z-direction similar to S_+; therefore, the active power that penetrates in the negative direction of z is –S–. In (10), $E_\varphi = 0$, $H_\rho = 0$ with the vertical dipole, the time average of S is expressed as

$$S_\pm = \pi \mathrm{Re}\left\{ \int_0^\infty E_\rho^{*\pm} H_\varphi \rho d\rho \right\}$$

E_ρ^* and H_φ can be derived using (6) and (7). Then S_+, S–, and W are defined by

$$S_+ = \frac{2\pi k_0^4 P^2}{Z_0}\left(\frac{2}{3} + K_{(0,k_0)}(h) + K_1\right)$$

$$S_- = \frac{2\pi k_0^4 P^2}{Z_0}\left(-K_{(k_0,\infty)}(h) + K_1\right)$$

$$W = \frac{2\pi k_0^4 P^2}{Z_0}\left(\frac{2}{3} + K_{(0,k_0)}(h) + K_{(k_0,\infty)}(h)\right) \quad (11)$$

where $K_{(\mathrm{a,b})}(h)$ and K_1 are defined as

$$K_{(a,b)}(h) = \frac{1}{k_0^3}\mathrm{Re}\left\{-i\int_a^b R_z^{(0)}(u)e^{-2\alpha_0(u)h}\frac{u^3}{\alpha_0(u)}du\right\} \quad (12)$$

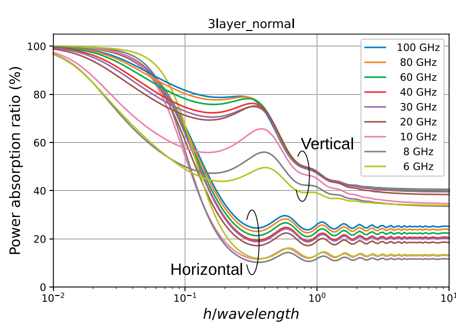

Figure 3. Theoretical power absorption ratio of the three-layer model.

$$K_1 = \frac{1}{k_0^3}\mathrm{Re}\left\{i\int_0^{k_0}\frac{1}{2}\left(1 - |R_z^{(0)}(u)|^2\right)\frac{u^3}{\alpha_0(u)}du\right\} \quad (13)$$

In (10), with the horizontal dipole, the time average of S is expressed as

$$S_\pm = \pi \mathrm{Re}\left\{\int_0^\infty [E_\rho H_\varphi^{\pm *} - E_\varphi H_\rho^{\pm *}]\rho d\rho\right\}$$

$E_\rho, E_\varphi, H_\rho^{(\pm *)}$, and $H_\varphi^{(\pm *)}$ can be derived using (8) and (9). Then S_+, $S_–$, and W are defined by

$$S_+ = \frac{2\pi k_0^4 P^2}{Z_0}\left(\frac{2}{3} + L_{(0,k_0)}(h) + L_1 + L_2\right)$$

$$S_- = \frac{2\pi k_0^4 P^2}{Z_0}\left(-L_{(k_0,\infty)}(h) + L_1 + L_2\right)$$

$$W = \frac{2\pi k_0^4 P^2}{Z_0}\left(\frac{2}{3} + L_{(0,k_0)}(h) + L_{(k_0,\infty)}(h)\right)$$

where $L_{(\mathrm{a,b})}(h)$, L_1, and L_2 are defined as

$$L_{(a,b)}(h) = \frac{1}{k_0^3}\mathrm{Re}\left\{i\int_a^b \frac{1}{2}\left[(u^2 - 2k_0^2)R_x^{(0)}(u) + \frac{\alpha_0(u)^2 u^2}{k_0^2}R_z^{(0)}(u)\right]e^{-2\alpha_0(u)h}\frac{u}{\alpha_0(u)}du\right\} \quad (14)$$

$$L_1 = \frac{1}{k_0^3}\int_0^{k_0}\frac{1}{4}\left(|R_x^{(0)}(u)|^2 - 1\right)\frac{2k_0^2 - u^2}{\sqrt{k_0^2 - u^2}}du \quad (15)$$

$$L_2 = \frac{1}{k_0^3}\int_0^{k_0}\frac{1}{4}\left(2\mathrm{Re}\frac{R_x^{(0)}(u)}{R_z^{(0)}(u)} + \frac{u^2}{k_0^2}\right)|R_z^{(0)}(u)|^2\frac{\sqrt{k_0^2 - u^2}}{k_0^2}u^3 du \quad (16)$$

Figures 2 and 3 show the theoretical power absorption ratio of the one-layer and three-layer human models. The three-layer model consists of skin (1.2 mm), fat (3.9 mm), and muscle (semi-infinite), and the one-layer model consists of only skin (semi-infinite). The electrical constants of the skin, subcutaneous fat, and muscle were derived from the 4-Cole–Cole model [5].

As shown in Figure 2, in the one-layer model, the power absorption ratio by the horizontal antenna converges to about 20% at a distance greater than one-quarter of a wavelength. On the other hand, in the case of a vertical antenna, the power absorption increases to a non-negligible level when the antenna is closer than one wavelength. In the case of both antennas, the power absorption rate approaches 100% when the antenna is closer to the skin. Comparing the three-layer model shown in Figure 3 with the one-layer model, there is a clear difference from 6 GHz to 10 GHz. This indicates that the millimeter-wave band radio waves reached the subcutaneous fat layer without attenuation in the skin layer. From 20 GHz to 100

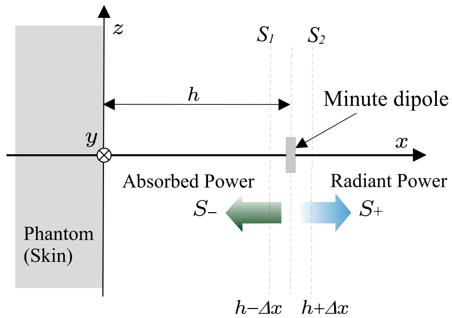

Figure 4. Numerical analysis model of the horizontal dipole.

GHz, the results were similar between the one-layer and the three-layer models.

4. Numerical Analysis of the Power Absorption Ratio Near the Skin

As shown in Figures 2 and 3, the power absorption ratio approaches 100% when the antenna is closer to the skin. To confirm these tendencies, numerical analysis using the finite difference time domain method was performed. Figure 4 shows the analysis model. Assuming that the human body surface exists vertically, the x-axis was set in the direction of the separation distance. A minute electric dipole is placed at a distance h from the surface of the phantom where the electrical constants of the skin are set. The direction of the current was horizontal or vertical to the surface of the phantom. The separation distance is changed in the range of $h/\lambda = 10^{-2}$ to 10^{0}, normalized by the wavelength. The power absorption rate was obtained from the absorbed power S_{-} in the direction from the dipole to the skin, and the radiated power S_{+} radiated in the opposite direction. S_{-} and S_{+} are obtained by time averaging of the Poynting vectors in planes S_1 and S_2 at a small distance from the antenna. Figure 5 shows the numerical analysis result at the frequency 100 GHz. As with the results of the theoretical equation, the numerical simulation results of absorption ratio also approach 100% when the antenna is closer to the skin.

5. Conclusion

In this study, we derived a new theoretical formula for the power absorption characteristics of the vertical and horizontal electric dipoles based on the analytical solution and calculated the power absorption ratio in the millimeter-wave band for the combination of the three-layer flat plate model and the micro dipole. The power absorption ratio in the millimeter-wave band was calculated using a one-layer flat plate model that assumes skin tissue and a three-layer model that assumes skin subcutaneous fat muscle. As a result, a

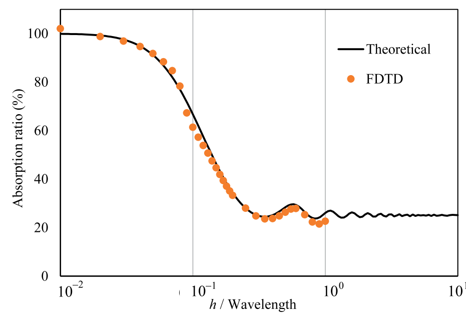

Figure 5. Numerical analysis result of the horizontal dipole. The source frequency is 100 GHz.

difference was confirmed between the one-layer and three-layer models at 6 GHz to 10 GHz. It is considered that this is because the radio waves in the millimeter-wave band reached the subcutaneous fat layer without being attenuated by the skin layer because of characteristics such as the straightness of the radio waves and attenuation by water. On the other hand, in the case of 20 GHz to 100 GHz, the radio waves were attenuated in the skin tissue layer, so it is considered that the results of the three-layer model were similar to those of the one-layer model. From these results, it is considered that the separation distance can be reduced for the horizontal antenna compared to the vertical antenna. Future tasks include expanding the model to four layers, deriving the theoretical formula when the antenna is embedded in the body, and calculating the power absorption ratio in that case.

6. References

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