Time Estimation of a Miniature Atomic Clock Using Instantaneous Frequency Information

Yuichiro Yano, Motoaki Hara, and Tetsuya Ido

Abstract – We propose a time estimation method utilizing data assimilation for the time synchronization of wireless networks. This method utilizes a state-space model of a miniature atomic clock and the instantaneous frequency information obtained through wireless communication. The time can be maintained with high accuracy by estimating the time with maximum likelihood using a Kalman filter. In this study, the effectiveness of the proposed method is verified by numerical calculations on frequency stability in the case where instantaneous frequency information is available intermittently.

1. Introduction

The number of wireless terminals connected to the Internet is expected to increase enormously in the future, and the information collected by these terminals is expected to be utilized as big data, leading to a smart society [1]. It is important that the information to be collected is not only sensor information but also time information that is added along with the sensor information. For example, it is known that acceleration sensors used in bridge safety inspections require accurate time synchronization due to the need for vibration analysis.

Although the most commonly used conventional method for time synchronization is Network Time Protocol (NTP), in wireless sensor networks, there are cases where NTP cannot be applied due to limitations such as non-IP, communication bandwidth, network topology, and asymmetric communication. Therefore, there is a need for a highly accurate timekeeping method that can be applied in such environments, and various methods have been proposed [2, 3].

We propose a time estimation method utilizing data assimilation for timekeeping that can be applied to sensor networks. This method uses the state-space model of a miniature atomic clock and the instantaneous frequency obtained from wireless communication as the observation value. With these, the time of the atomic clock is precisely estimated using a Kalman filter in a maximum likelihood manner. Unlike quartz clocks, which are commonly used today, miniature atomic clocks can be treated as a mathematical model with high predictability, with white-frequency noise as the dominant factor for short-term frequency stability and frequency random walk as the dominant factor for the long-term frequency stability [4]. Miniature atomic clocks have made progress in recent years in terms of micro-fabrication and low power consumption and are expected to be mounted on Internet of Things devices in the future [5, 6]. However, in the conventional state-space model of atomic clocks, only the epoch time or time difference can be given as observable information from the definition of the internal state, and the instantaneous frequency cannot be treated as observable information. Moreover, in wireless communication, the radio-frequency band cannot be occupied from the effective use of radio wave resources, and instantaneous frequency information cannot always be obtained. Therefore, because communication exchanges occur intermittently, instantaneous frequency information can also be obtained only intermittently.

In this study, we extended the state-space model of atomic clocks so that the instantaneous frequency can be taken into account and the timekeeping accuracy can be investigated numerically when instantaneous frequency information is obtained intermittently. The results show that when instantaneous frequency information is obtained intermittently, the short-term frequency stability is determined with respect to the time rate of the intermittent information. It is found that the longer the interval during which instantaneous frequency information is not available, the greater the degradation in long-term stability.

2. Extended State-Space Model of Atomic Clocks for Using Instantaneous Frequency Information

A mathematical oscillator model of an atomic clock can be described by a state-space model [4]. This model is widely used for the ensemble averaging of atomic clocks in the generation of national standard time and for position and time estimation in the Global Navigation Satellite System. The conventional model allows epoch time or time differences between clocks to be input as observed values but does not allow instantaneous frequencies to be used as observed values. This is because the model has no time information of the previous time as internal time, although the instantaneous frequency is defined by the time derivative of the phase, that is, the time difference from the previous time.

We extend the state-space model of atomic clocks so that instantaneous frequency information can be treated as observed values by adding the time information of previous time. Here, a cesium atomic clock model is employed in which white-frequency

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Yuichiro Yano, Motoaki Hara, and Tetsuya Ido are with the National Institute of Information and Communications Technology (NICT), 4-2-1 Nukui-kitamachi, Koganeishi, Tokyo, Japan; e-mail: y-yano@nict.go.jp, hara.motoaki@nict.go.jp, mailto:ido@nict.go.jp.

noise and frequency random walks dominate. It is a second-order state-space model with second-order integration properties.

With an atomic clock with the state and Gaussian white noise $\boldsymbol{v}$, $\boldsymbol{w}$, the extended dynamics of an atomic clock dominated by white-frequency noise and frequency random walk is given by

$$\begin{aligned} \boldsymbol{x}[k+1] &= \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{v}[k] \\ \boldsymbol{y}[k] &= \boldsymbol{C}\boldsymbol{x}[k] + \boldsymbol{w}[k], \qquad k = 0, 1, \ldots \end{aligned} \tag{1}$$

where $\boldsymbol{x}$ and $\boldsymbol{y}$ represent the state vector of the atomic clock and the observation value of instantaneous frequency, respectively, and $\boldsymbol{x}$ has three values x_1, x_2, x_3 that correspond to the clock reading deviation, the clock frequency, and the time information of the previous time, respectively. The matrices $\boldsymbol{A}$ and $\boldsymbol{C}$ are defined by

$$\boldsymbol{A} := \begin{bmatrix} 1 & \Delta t & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad \boldsymbol{C} := [\,1 \quad 0 \quad -1\,] \tag{2}$$

where Δt is the time interval of a sample and $\boldsymbol{v}$ and $\boldsymbol{w}$ represent the system noise and observation noise, respectively. The covariance matrix $\boldsymbol{Q}$, $\boldsymbol{R}$ is derived as

$$\boldsymbol{Q} = \begin{bmatrix} \Delta t \sigma_1^2 + \frac{\Delta t^3}{3}\sigma_2^2 & \frac{\Delta t^2}{2}\sigma_2^2 & 0 \\ \frac{\Delta t^2}{2}\sigma_2^2 & \Delta t \sigma_2^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \boldsymbol{R} = \mathrm{diag}(\sigma_0^2, 0, 0) \tag{3}$$

where σ_1 and σ_2 correspond to the white-frequency noise and random walk frequency levels of the atomic clock, respectively, and σ_0 corresponds to a comparison noise level of instantaneous frequency.

3. Calculation Conditions and Estimation Methods

The state-space model of the atomic clock is used to estimate the time when the instantaneous frequency is available. The state-space model is modeled on the miniature atomic clock of Microchip CSAC SA.45s; σ_1 and σ_2 are set to account for the clock performance. The observed noise σ_0 was set to be 1.5×10^{-11} for 1 s integration, which is a floor noise level when using carrier phase comparison in sub-GHz wireless communication [7]. A Kalman filter, which was used for time estimation, allows us to obtain a maximum likelihood estimate of the time from the observed information.

In this evaluation, estimated values are calculated from the pseudo-generated observation value and compared with reference values to evaluate the estimation accuracy. The time fluctuation data of atomic clocks are numerically generated from the above state-space model and used as a reference value. Instantaneous frequency data for maximum likelihood estimation are also pseudo-generated numerically. If the difference between the estimated value and the reference value is small and stable, high estimation accuracy is obtained. Therefore, the stability of the estimation accuracy is evaluated by calculating the Allan deviation of the difference between estimation and reference.

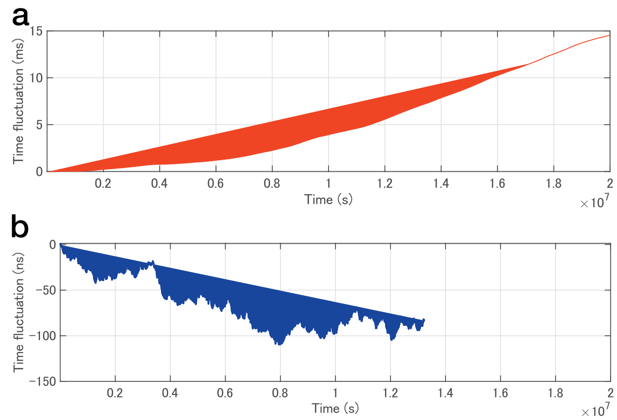

Figure 1. Time fluctuations of a miniature atomic clock.

For verifying the effect of instantaneous frequency information, the calculations were based on the following assumptions:

- The clock performance and its model are known.
- The time difference in the initial state is zero.
- Observation noise is white noise.
- Doppler frequency shift is not considered.

4. Results

The calculation results for the case where instantaneous frequencies are always available are shown in Figure 1. The vertical axis shows the time fluctuation from the ideal time, and the horizontal axis shows the elapsed time. Figure 1a shows the pseudo-generated time fluctuation when the atomic clock is operated in free-run mode. Under these conditions, a miniature atomic clock would show a time difference of about 15 ms from the ideal time after 2×10^7 s. The difference between the pseudo-generated time fluctuation and the estimated value is shown in Figure 1b. The instantaneous frequency information allows accurate time estimation, and the difference is kept below 150 ns. The time fluctuation is improved by five orders of magnitude, indicating that more stable time can be maintained by using the instantaneous frequency information.

The Allan standard deviation in Figure 1 is shown in Figure 2. The red line shows the frequency stability of the miniature atomic clock; the Allan deviation at 1 s is set to 1.55×10^{-10}. The stability of the atomic clock at averaging time $\tau < 100$ s has a trend of $1/\sqrt{\tau}$, indicating that white-frequency noise is dominant. At $\tau > 1000$ s, the Allan deviation increases at $\sqrt{\tau}$, and

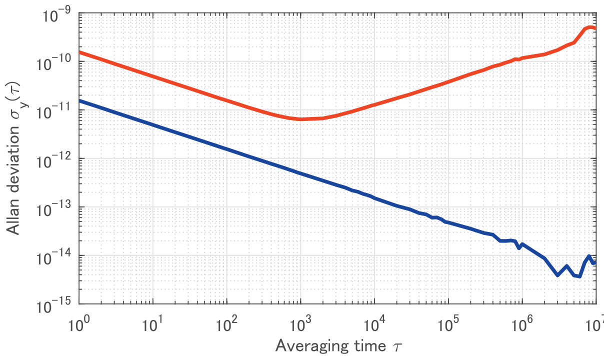

Figure 2. Allan standard deviation in Figure 1. Red: Time fluctuations of free-running miniature atomic clocks. Blue: Time fluctuations corrected on the basis of instantaneous frequency information.

frequency random walk becomes dominant, indicating that the trend of the atomic clock is well reflected. The blue line shows the estimation accuracy when instantaneous frequency information is obtained. Its frequency stability is monotonically improved with a trend of $1/\sqrt{\tau}$, which is a trend of white-frequency noise.

Next, Figure 3 shows the results of the calculation in the case where instantaneous frequency information is obtained intermittently. Here, T_A is the time when intermittent instantaneous frequency information is available, and T_{NA} is the time when it is not available. These time slots are set to appear alternately. It can be seen that in all cases, the long-term stability is improved by using the instantaneous frequency rather than operating in free-run mode. It can also be seen that the improvement is greater when T_{NA} is shorter. When the averaging time τ is longer than the time interval T_{NA}, the frequency stability tends to decrease by $1/\sqrt{\tau}$. The averaging time, which is the starting point of the decrease by $1/\sqrt{\tau}$, lengthens as T_{NA} becomes longer.

In most cases, the use of instantaneous frequency improves the frequency stability, but for averaging times τ from 10^3 s to 10^5 s, the estimation accuracy is at most about $\sqrt{2}$ times worse than for free-running atomic clocks. This degradation is particularly notable for conditions $T_{NA} > 1000$ s. This is because the dominant noise in miniature atomic clocks with an averaging time of $\tau > 1000$ s is frequency random walk, and the accurate estimation of the frequency of the atomic clocks is not carried out during T_{NA}, resulting in the degradation of the estimation accuracy for averaging times τ between 10^3 s and 10^5 s. Therefore, to obtain good frequency stability in all cases, T_{NA} should be shorter than the averaging time τ, where the random walk frequency dominates.

In the short-term stability at an averaging time of 1 s, the longer the available time T_A, the higher the short-term stability. This is because time fluctuations are suppressed by the estimation during the available time. That is, it is expected that the instantaneous frequency information will be used during the available time T_A and that the free run of an atomic clock will be

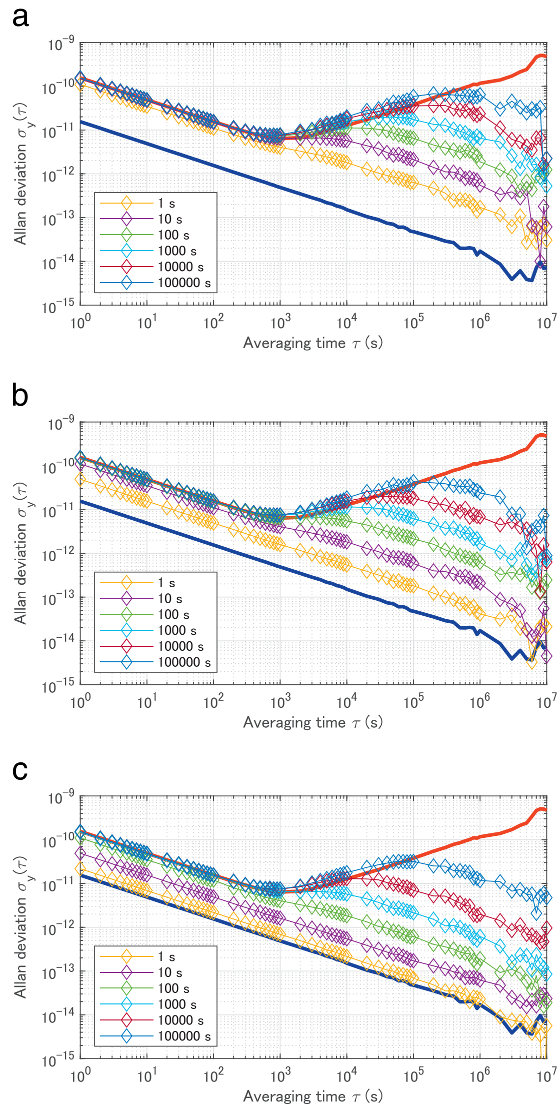

Figure 3. Allan standard deviation for different T_{NA}.

used during the non-available time T_{NA}. The short-term frequency stability is considered to be their time average, and the short-term stability in this proposal can be described as follows:

$$\sigma_y(\tau = 1s) = \sqrt{\frac{T_A}{T_A + T_{NA}}\sigma_0^2 + \frac{T_{NA}}{T_A + T_{NA}}\sigma_1^2} \qquad (4)$$

Figure 4 shows the relationship between T_A and T_{NA} for short-term stability at 1 s. The numerical results are plotted as dots, and the short-term frequency stability predicted using (4) is plotted as a solid line. The solid line and dots almost overlap, indicating that the short-term stability can be predicted using (4). Note, however, that this prediction equation is an expected value based on all time averages, and in actual forecasting, it is necessary to consider the stability according to the available or non-available status of instantaneous frequency information.

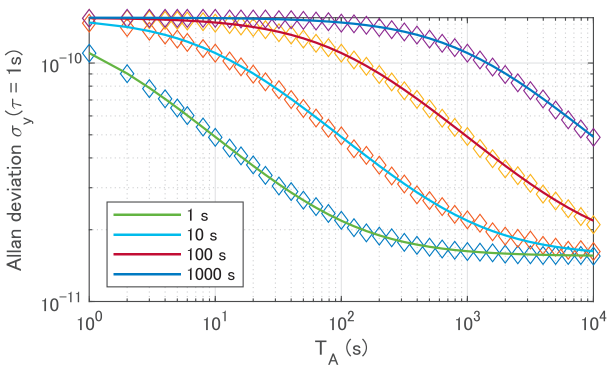

Figure 4. Short-term frequency stability at 1 s as a function of T_A for different T_{NA}.

5. Conclusion

Assuming time synchronization in wireless communication, we proposed a time estimation method that extends the state-space model of atomic clocks so that instantaneous frequencies can be taken into account. The accuracy of timekeeping is numerically investigated when instantaneous frequency information is obtained continuously and intermittently. When instantaneous frequency information is obtained intermittently, the average short-term stability is determined with respect to the time rate of the intermittent information. It was also found that the longer the interval during which information is not available, the more the estimation accuracy deteriorates in the long term, affecting the long-term stability.

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7. References

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