 DOI: 10.46620/22-0042

A Leaky-Wave Interpretation of the Zenneck Wave

David R. Jackson, Francisco Mesa, Krzysztof A. Michalski, and Juan R. Mosig

Abstract – Here we establish a leaky-wave interpretation of the Zenneck wave, which is a wave that exists mathematically at the interface of air (or a dielectric in general) over a lossy dielectric such as the earth or the ocean. The Zenneck wave is seen to be an "upside-down" leaky wave that impinges on the lossy half-space at a complex Brewster angle. The impinging leaky wave is a physical leaky wave in the sense that the transverse wavenumber is a fast wave with respect to free space, and it is improper (exponentially growing in the vertical direction of power flow, i.e., the vertically downward direction.) A cylindrical version of the Zenneck wave may be created in a limited region of space by using a vertical leaky-wave antenna, which supports a backward leaky wave. Although the leaky wave that makes up the Zenneck wave in the air region is itself a physical wave, the overall Zenneck wave is "nonphysical" in the sense that a simple localized source such as a dipole will not excite it.

1. Introduction

A Zenneck wave is a wave that mathematically exists (satisfying Maxwell's equations and the boundary conditions) at an interface between two different half-spaces of material, usually one being lossless (such as the air) and the other one being lossy (e.g., the earth or the ocean) [1]. We assume here that we have air as the upper medium (called medium 0) and the lossy half-space as the lower medium (called medium 1). Figure 1 shows the geometry.

The lower region has a complex relative permittivity that is denoted as

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r''. \tag{1}$$

(We assume here a suppressed time harmonic convention of $\exp(+j\omega t)$). The loss can be due to conductivity or dielectric polarization loss or a combination of the two. The Zenneck wave is a TM_z surface wave with a propagation wavenumber given by [1]

$$k_{xp} = k_0\sqrt{\frac{\varepsilon_r}{\varepsilon_r + 1}} \tag{2}$$

The field variation of the Zenneck wave in the air and lossy regions is, for the vertical electric field,

$$E_{z0}(x,z) = Ae^{-jk_{xp}x}e^{-jk_{z0}z} \tag{3}$$

$$E_{z1}(x,z) = \frac{A}{\varepsilon_r}e^{-jk_{xp}x}e^{+jk_{z1}z} \tag{4}$$

for some constant A. The Zenneck wave has vertical wavenumbers in the air and lossy region that are given, respectively, as

$$k_{z0} = -j\sqrt{k_{xp}^2 - k_0^2} \tag{5}$$

$$k_{z1} = -j\sqrt{k_{xp}^2 - k_1^2} \tag{6}$$

where

$$k_1 = k_0\sqrt{\varepsilon_r} \tag{7}$$

with k_0 being the wavenumber of free space. The radical sign in the above equations denotes the principal square root, where the argument of the complex number z inside the square root is taken to lie in the range $-\pi < \arg(z) < \pi$. The principal square root has the property that the real part is always positive. This means that the vertical wavenumbers in (5) and (6) have an imaginary part that is negative. In leaky-wave terminology, this makes the field of the Zenneck wave "proper" in both the air and lossy regions. That is, the field is exponentially decreasing vertically in both regions away from the interface. The real part of the vertical wavenumber k_{z0} is negative, while the real part of the vertical wavenumber k_{z1} is positive. The time-average power flow is in the direction of the real part of the complex wavenumber vector, which for each respective region is given as

$$\mathbf{k}_0 = \hat{\mathbf{x}}k_{xp} + \hat{\mathbf{z}}k_{z0} \tag{8}$$

$$\mathbf{k}_1 = \hat{\mathbf{x}}k_{xp} - \hat{\mathbf{z}}k_{z1} \tag{9}$$

(The hat denotes unit vector here.) Therefore, the

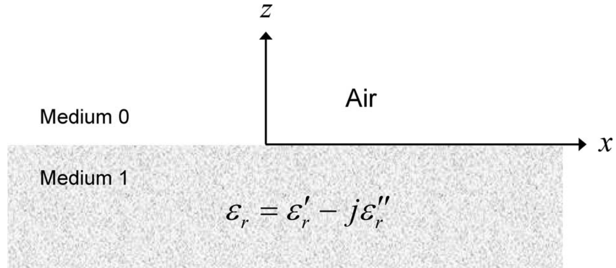

Figure 1. The geometry of the half-space problem.

Manuscript received 28 December 2022.

David R. Jackson is with the University of Houston, Houston, Texas 77204-4005, USA; e-mail: djackson@uh.edu.

Francisco Mesa is with the University of Seville, Seville 41012, Spain; e-mail: mesa@us.es.

Krzysztof A. Michalski is with Texas A&M University, College Station, Texas 77843-3128, USA; email: k-michalski@tamu.edu.

Juan R. Mosig is with EPFL, Lausanne CH-1015, Switzerland; e-mail: juan.mosig@epfl.ch.

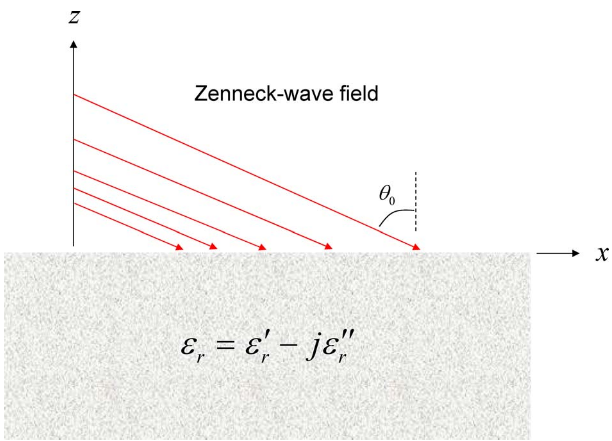

Figure 2a. The field of the Zenneck wave in the air region.

vertical component of the time-average power flow is in the downward direction in both regions.

The Zenneck wave is "nonphysical" in the sense that a simple source such as a dipole will never excite it because the Zenneck wave pole on the top sheet is not captured when the path of integration in a spectral-domain solution for the field from the source is deformed to the steepest-descent path [1, 2].

2. Leaky-Wave Interpretation

Consider now the simple substitutions

$$z' = -z, \quad k_{z0}' = -k_{z0} \tag{10}$$

The distance z' is measured downward from the interface. The field in the air region thus has the form

$$E_{z0}(x,z) = Ae^{-jk_{xp}x}e^{-jk_{z0}'z'} \tag{11}$$

The wavenumber k_{z0}' has a positive real part and a positive imaginary part. This corresponds to the wavenumber of a leaky wave, which is improper (exponentially increasing vertically in the direction of power flow). A leaky wave that radiates upward from the interface in the air region, due to a traveling-wave source on the interface (acting as a leaky-wave antenna) that is a fast wave with respect to free space, would have the form

$$E_{z0}(x,z) = Ae^{-jk_x^{\mathrm{LW}}x}e^{-jk_{z0}^{\mathrm{LW}}z} \tag{12}$$

where $0 < \mathrm{Re}\left(k_x^{\mathrm{LW}}\right) < k_0$ and $\mathrm{Im}\left(k_x^{\mathrm{LW}}\right) < 0$, along with $\mathrm{Re}\left(k_{z0}^{\mathrm{LW}}\right) > 0$ and $\mathrm{Im}\left(k_{z0}^{\mathrm{LW}}\right) > 0$.

Comparing (11) and (12), it is seen that the field of the Zenneck wave in the air region represents that of a leaky wave, except that the wave is "upside down," carrying power vertically downward instead of upward. Figure 2 compares the two scenarios: part (a) shows the field of the Zenneck wave in the air region, while part (b) shows the field of a typical leaky wave radiating in the air region away from a leaky-wave antenna at the interface. The angle θ_0 for the Zenneck wave and the leaky wave (LW) is given by

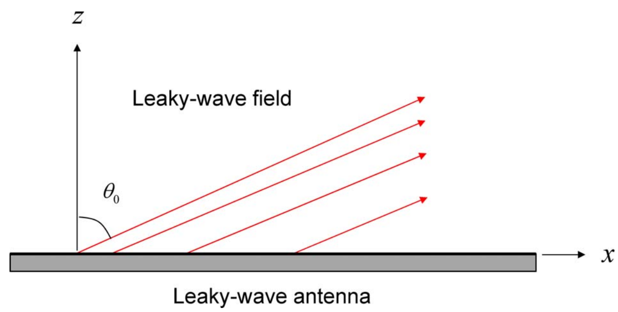

Figure 2b. The field of a leaky wave that is radiating from a leaky-wave antenna.

$$\tan\theta_0^{\mathrm{ZW}} = \frac{\mathrm{Re}\left(k_{xp}\right)}{\mathrm{Re}\left(k_{z0}'\right)} \quad , \quad \tan\theta_0^{\mathrm{LW}} = \frac{\mathrm{Re}\left(k_x^{\mathrm{LW}}\right)}{\mathrm{Re}\left(k_{z0}^{\mathrm{LW}}\right)} \tag{13}$$

In Figure 2, the spacing between the power flux lines signifies the magnitude of the power flow.

In a spectral-domain solution for the field of a source, the propagation wavenumber k_{xp} of the Zenneck wave corresponds to a pole of the integrand in the complex spectral wavenumber k_x plane, located on the top sheet of the Riemann surface [1, 2]. The Riemann surface has four sheets since there are four possible combinations of square roots (two choices of the square root for each of the two vertical wavenumbers). The top sheet is taken as the one for which both vertical wavenumbers are proper. It is customary to use the Sommerfeld or hyperbolic branch cuts [1, 2]. The branch cuts become "escalators" or connections between the sheets on the Riemann surface, as shown in Figure 3. In Figure 3 the (–,–) notation for the top sheet denotes that the wavenumber k_{z0} has a negative imaginary part (the first – sign) and that the wavenumber k_{z1} also has a negative imaginary part (the second – sign). A similar notation is used for the other three sheets.

The path of integration in the spectral-domain solution stays on the top sheet of the Riemann surface [1]. The wavenumber k_{xp} corresponds to a pole of the reflection coefficient on the (–,–) and (+,+) sheets and a zero on the other two sheets [3]. This follows from the fact that the TM reflection coefficient as a function of k_x of the incident wave in the air region is given by

$$\Gamma_{\mathrm{TM}}(k_x) = \frac{Z_1^{\mathrm{TM}} - Z_0^{\mathrm{TM}}}{Z_1^{\mathrm{TM}} + Z_0^{\mathrm{TM}}} = \frac{\frac{k_{z1}}{\omega\varepsilon_0\varepsilon_r} - \frac{k_{z0}}{\omega\varepsilon_0}}{\frac{k_{z1}}{\omega\varepsilon_0\varepsilon_r} + \frac{k_{z0}}{\omega\varepsilon_0}} \tag{14}$$

If either vertical wavenumber is replaced by its negative, the reflection coefficient gets replaced by its reciprocal. The fact that there is a pole on the top sheet is consistent with the physical interpretation of a leaky wave impinging on the lossy lower half-space that gets perfectly absorbed (transmitted) with no reflection. This is because the pole at k_{xp} on the top sheet corresponds to a zero at k_{xp} on the (+,–) sheet (the sheet immediately below the top sheet). This follows since on the (+,–) sheet, the wavenumber k_{z0} for the pole on the top sheet

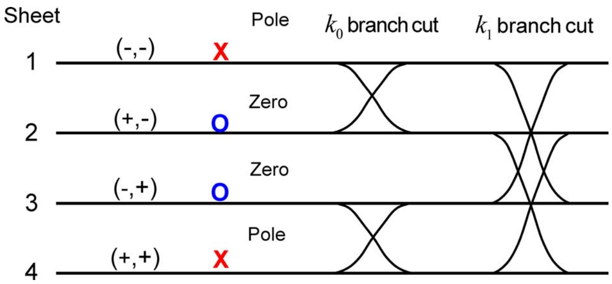

Figure 3. Side view of the Riemann surface, showing the four sheets along with the pole and zero locations on the different sheets.

has been replaced by its negative. There is thus a zero of the reflection coefficient for an impinging field of the form shown in (11).

Hence, we conclude that a physical interpretation of the Zenneck wave is a leaky wave that is traveling downward in the air region, impinging on the lossy half-space at a complex Brewster angle where the reflection coefficient is zero. The complex Brewster angle θ_B (measured from vertical in the air region) is given by

$$\tan\theta_B = \frac{k_0 \sin\theta_B}{k_0 \cos\theta_B} = \frac{k_{xp}}{k'_{z0}} = -\frac{k_{xp}}{k_{z0}} = -j\frac{k_{xp}}{\sqrt{k_{xp}^2 - k_0^2}} \tag{15}$$

Because k_{z0} has a real part that is negative, we can also write

$$\tan\theta_B = \frac{k_{xp}}{\sqrt{k_0^2 - k_{xp}^2}} \tag{16}$$

Finally, from (2) and the fact that $\tan(\theta_B)$ must have a negative imaginary part (which follows from the second equality in (15)), we have

$$\tan\theta_B = \sqrt{\varepsilon_r} \tag{17}$$

The branch of the inverse tangent is chosen so that $\cos(\theta_B)$ (and hence k'_{z0}) has a positive imaginary part. As the loss in the lower half-space approaches zero (ε_r becomes real), both vertical wavenumbers then become real (with k_{z0} being negative), and the angle θ_B becomes real and is located in the first quadrant, $0 < \theta_B < 90°$. This situation corresponds to the classic textbook case of a homogeneous plane wave impinging on a lossless half-space at the (real) Brewster angle [4]. The complex Brewster angle is thus the analytic continuation of the real Brewster angle off of the real axis as loss is added.

Table 1 gives the value of the complex Brewster angle for the case of a lossy half-space having $\varepsilon'_r = 10$ as ε''_r varies. As the loss tends to zero, the complex Brewster angle tends to the real value of $\tan^{-1}\sqrt{10} = 1.264519$ radians. For high losses, that is, when ε''_r becomes large, some analysis reveals that the complex Brewster angle tends to the limit

Table 1. Complex Brewster angle for $\varepsilon'_r = 10$

ε''_r	θ_B (radians)
0.001	$1.264519 - j(1.4374 \times 10^{-5})$
0.01	$1.264519 - j(1.4374 \times 10^{-4})$
0.1	$1.264529 - j(1.4373 \times 10^{-3})$
1	$1.265525 - j(1.4295 \times 10^{-2})$
10	$1.327612 - j(0.096211)$
100	$1.49664 - j(0.066667)$
1000	$1.548317 - j(0.02224)$
10,000	$1.567321- j(0.007067)$
100,000	$1.56856 - j(0.002236)$
1,000,000	$1.57009 - j(0.000707)$

$$\theta_B \to \frac{\pi}{2} + \frac{1-j}{\sqrt{2}}\frac{1}{\sqrt{\varepsilon''_r}} \tag{18}$$

3. Excitation of the Zenneck Wave by a Leaky-Wave Antenna

Since the Zenneck wave consists of an "upside-down" leaky wave in the air region, it is natural to consider if the Zenneck wave can be excited by a leaky-wave antenna (LWA) located in the air region. For this, we consider a cylindrical version of the Zenneck wave whose fields are given as

$$E_{z0}(\rho, z) = AH_0^{(2)}(k_{\rho p}\rho)e^{-jk_{z0}z} \tag{19}$$

$$E_{z1}(\rho, z) = \frac{A}{\varepsilon_r}H_0^{(2)}(k_{\rho p}\rho)e^{+jk_{z1}z} \tag{20}$$

where $k_{\rho p} = k_{xp}$.

The field in the air region is in the form of a downward-propagating leaky wave with a conical shape, as depicted in Figure 4. This type of field can be launched by a one-dimensional vertical azimuthally omnidirectional LWA in the air region, which is fed at $z = 0$ (the interface) and extends up to a height h. The

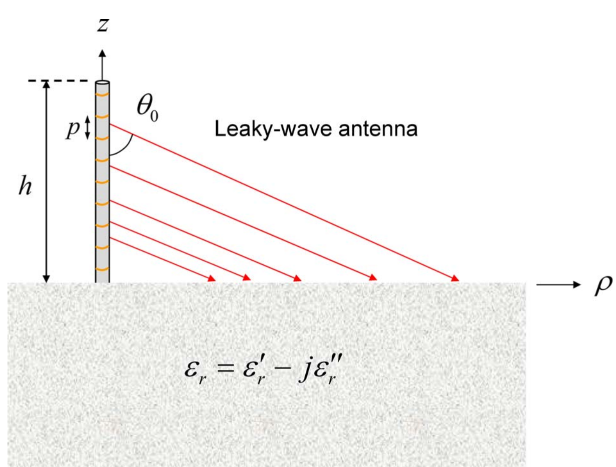

Figure 4. A vertical periodic LWA producing a near field that matches that of the Zenneck wave in a conical region (rotationally symmetric about the z-axis).

height h will directly influence the size of the conical region where the fields resemble that of the pure cylindrical Zenneck wave in the air region. The LWA is shown in Figure 4. (The conical downward radiation is rotationally symmetric about the z-axis, but radiation is shown on only one side of the source for simplicity.)

Along the axis of the LWA, the field of the radiating LWA aperture is chosen to match that of the vertical profile of the Zenneck wave. That is, the LWA has a complex wavenumber denoted as

$$k_z^{\mathrm{LWA}} = \beta - j\alpha \tag{21}$$

where

$$\beta = \mathrm{Re}(k_{z0}) < 0, \quad \alpha = -\mathrm{Im}(k_{z0}) > 0 \tag{22}$$

The LWA is thus seen to be radiating in the backward direction relative to the positive z-axis (since $\beta < 0$). This could be achieved by using a periodic type of LWA that radiates from the $n = -1$ space harmonic (Floquet mode), where the phase constant β in (21) would be chosen as β_{-1}, where [5]

$$\beta_{-1} = \beta_0 - \frac{2\pi}{p} \tag{23}$$

where β_0 is the fundamental phase constant (zero harmonic) of the LWA and p is the periodicity of the radiating elements that make up the LWA. An idealized omnidirectional LWA that launches a TM_z polarized wave can be represented as a vertical electric line current source of the form

$$I(z) = I_0 e^{-jk_z^{\mathrm{LWA}}}, \quad 0 < z < h \tag{24}$$

Such an LWA radiates a backward conical beam in the far field (when used as an antenna), but in the near field, it will produce a conical-shaped region of fields, as shown in Figure 4. More details about the field produced by this type of source, along with numerical results, may be found in [6]. The region in which the near field of the LWA matches that of the Zenneck wave depends on the height of the LWA and the wavenumber of the leaky wave. A simple geometrical optics reasoning would indicate that a conical region of Zenneck wave fields will be set up in the air region, extending from the top of the LWA to the interface, as illustrated in Figure 4. However, in actuality, there will be a transition distance along the interface at which the fields lose resemblance to that of the Zenneck wave, and this transition distance will occur before the boundary of the conical region is reached [6].

4. Conclusions

The Zenneck wave is a wave that can exist in a half-space problem, consisting of air over a lossy medium such as the earth or the ocean. It is shown here that a Zenneck wave can be interpreted as an “upside-down” leaky wave that is impinging on the earth at a complex Brewster angle. A cylindrical coordinate version of the Zenneck wave can be created in the near field by a vertical omnidirectional periodic leaky-wave antenna that radiates in the backward direction. In this case, the aperture field of the leaky-wave antenna matches that of the Zenneck wave. The Zenneck wave is a “nonphysical” type of wave that cannot be excited by a simple source such as a dipole. However, in a near-field region, a vertical periodic leaky-wave antenna radiating in the backwards direction can create a conical region in which the fields resemble that of the Zenneck wave.

5. References

1. K. A. Michalski and J. R. Mosig, “The Sommerfeld Half Space Problem Redux: Alternative Field Representations, Role of Zenneck and Surface Plasmon Waves,” *IEEE Transactions on Antennas and Propagation*, **63**, 12, December 2015, pp. 5777-5790.
2. D. R. Jackson, F. Mesa, K. A. Michalski, and J. R. Mosig, “Reflections on the Zenneck Wave,” IEEE AP-S International Symposium on Antennas and Propagation, Denver, CO, USA, July 2022.
3. K. A. Michalski and J. R. Mosig, “Sommerfeld Pole or Brewster Zero,” *IEEE Antennas and Propagation Society Magazine*, December 2022, pp. 70-80.
4. W. H. Hayt and J. A. Buck, *Engineering Electromagnetics*, 9th Ed., New York, McGraw-Hill, 2019.
5. D. R. Jackson and A. A. Oliner, “Surface-Wave Antennas and Leaky-Wave Antennas,” in *Antenna Engineering Handbook*, J. L. Volakis (ed.), New York, McGraw-Hill, 2018, Chapter 15.
6. F. Mesa and D. R. Jackson, “Excitation of the Zenneck Wave by a Tapered Line Source Above the Earth or Ocean,” *IEEE Transactions on Antennas and Propagation*, **68**, 6, June 2020, pp. 4848-4859.