Junction of Two Coplanar Resistive Half-Planes: A Uniform Asymptotic Solution for the Plane Wave Diffraction and Validation Tests

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Abstract – The electromagnetic scattering by two coplanar resistive half-screens with different surface resistivity is considered in the case of plane waves at skew incidence with respect to the discontinuity. Reflection and transmission coefficients for parallel and perpendicular polarizations are determined accounting for the resistive boundary conditions in correspondence of the screens. Such coefficients are adopted for the calculation of the geometrical optics field and also used to apply the Uniform Asymptotic Physical Optics approach for the evaluation of the diffracted field. The corresponding total field is here validated by means of comparisons with data obtained by the radio-frequency module of a full-wave commercial tool.

1. Introduction

The Uniform Asymptotic Physical Optics (UAPO) approach has been recently applied in [1] to study the interaction between propagating plane waves and isolated lossy dielectric half-screens for which the resistive boundary conditions [2] hold. Such conditions characterize the behavior of electric and magnetic fields in correspondence of the screen and permit to substitute it by an electric equivalent polarization current if its thickness is smaller than the wavelength λ in the material (typically, it should not exceed one-tenth of λ). The resistive boundary conditions [2] have been also accounted in [1] for evaluating the reflection and transmission coefficients to be used for the electric field components parallel and perpendicular to the ordinary plane of incidence. The knowledge of such coefficients is also important for determining the electric equivalent polarization current under the physical optics approximation to be used in the scattering integral at the beginning of the UAPO approach. The resultant UAPO solution has been presented [1] in matrix form by adopting suitable ray-fixed reference systems in accordance with the Uniform Geometrical Theory of Diffraction (UTD) [3].

Other analytical methodologies have been proposed to solve the diffraction problem originated by the edge of a resistive screen, but most of them concern the two-dimensional case of normal incidence with respect to the edge. References [4–10] form a representative and nonexhaustive set of related articles.

In the case of structures with multiple surfaces interacting with the electromagnetic field, one of the advantages of the UAPO approach is to exploit the linearity of the scattering integral to attain the diffracted field as a linear combination of contributions associated to the edges of the involved surfaces [11–16]. The authors have recently exploited this feature to solve the problem tackled here and to obtain useful expressions and numerical preliminary results in [17], where the ability of the UAPO diffracted field to compensate the jump of the geometrical optics (GO) field at the reflection and transmission shadow boundaries has been demonstrated. This article is justified by the numerical validation of the UAPO approach for the evaluation of the plane wave diffraction by a planar junction of two resistive half-screens. They possess different values of the surface resistivity R_e.

The UAPO solution for the diffracted field [17] is reported in Section 2 for the reader's convenience, and the radio-frequency (RF) unit of Comsol Multiphysics is used in Section 3 to validate it. Concluding remarks in Section 4 close the article.

2. UAPO Diffracted Field

The geometry of the problem and the adopted reference systems are depicted in Figure 1. Let us denote by the uppercase letter A (B) the resistive half-screen for $x>0$ ($x<0$). The values of $R_e = -j\zeta_0/k_0 d(\varepsilon_r - 1)$ associated to the joining half-screens cause discontinuities of the reflected and transmitted fields in the surrounding free space. They change accounting for k_0 (the free-space propagation constant), ζ_0 (the free-

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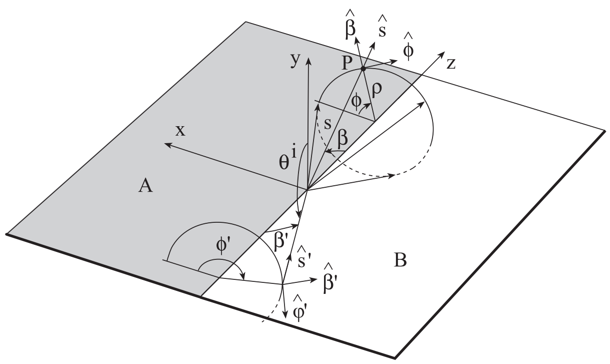

Figure 1. Convenient coordinate systems for the considered diffraction problem.

space impedance), d (the thickness of the screen), and ε_r (the complex relative permittivity of the screen).

In accordance with the analytical results in [17], the UAPO field diffracted by the junction writes as

$$\begin{pmatrix} E^d_\beta \\ E^d_\phi \end{pmatrix} = \left[I^d_A \, \underline{\underline{A}} + I^d_B \, \underline{\underline{B}} \right] \begin{pmatrix} E^i_{\beta'} \\ E^i_{\phi'} \end{pmatrix} = \underline{\underline{D}} \, \frac{\exp(-jk_0 s)}{\sqrt{s}} \begin{pmatrix} E^i_{\beta'} \\ E^i_{\phi'} \end{pmatrix} \tag{1}$$

where s is the distance from the diffraction point to the observation point P. The functions $I^d_{A,B}$ depend on the incidence and diffraction directions given by the pairs of angles β', ϕ' and $\beta = \beta', \phi$, respectively, and contain the UTD transition function F [3], that is,

$$I^d_{A,B} = \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}} \, \frac{\exp(-jk_0 s)}{\sqrt{s}} \cdot \cdot \frac{F\left(2k_0 s \, \sin^2\beta' \cos^2\left(\frac{\phi_{A,B} \pm \phi'_{A,B}}{2}\right)\right)}{\sin^2\beta' (\cos\phi_{A,B} + \cos\phi'_{A,B})} \tag{2}$$

where $\phi'_A = \phi'$, $\phi'_B = \pi - \phi'$, $\phi_A = \phi$, and $\phi_B = \pi - \phi$. The sign + (–) is needed to be used if $0 < \phi_{A,B} < \pi$ $(\pi < \phi_{A,B} < 2\pi)$. The matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$ are given by

$$\underline{\underline{A}} = \underline{\underline{A}}_1 \underline{\underline{A}}_2 \underline{\underline{A}}_3 \underline{\underline{A}}_4 \underline{\underline{A}}_5 \tag{3}$$

$$\underline{\underline{B}} = \underline{\underline{A}}_1 \underline{\underline{B}}_2 \underline{\underline{B}}_3 \underline{\underline{B}}_4 \underline{\underline{A}}_5 \tag{4}$$

with

$$\underline{\underline{A}}_1 = \begin{pmatrix} \cos\beta' \cos\phi & \cos\beta' \sin\phi & -\sin\beta' \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \tag{5}$$

$$\underline{\underline{A}}_2 = \begin{pmatrix} 1 - \sin^2\beta' \cos^2\phi & -\sin\beta' \cos\beta' \cos\phi \\ -\sin^2\beta' \sin\phi \cos\phi & -\sin\beta' \cos\beta' \sin\phi \\ -\sin\beta' \cos\beta' \cos\phi & \sin^2\beta' \end{pmatrix} \tag{6}$$

$$\underline{\underline{A}}_3 = \frac{1}{G(\beta', \phi')} \begin{pmatrix} -\cos\beta' & -\sin\beta' \cos\phi' \\ -\sin\beta' \cos\phi' & \cos\beta' \end{pmatrix} \tag{7}$$

$$\underline{\underline{A}}_4 = \begin{pmatrix} 0 & (1 - R_{\perp_A} - T_{\perp_A}) \sin\beta' \sin\phi' \\ 1 + R_{\parallel_A} - T_{\parallel_A} & 0 \end{pmatrix} \tag{8}$$

$$\underline{\underline{A}}_5 = \frac{1}{G(\beta', \phi')} \begin{pmatrix} \cos\beta' \sin\phi' & \cos\phi' \\ -\cos\phi' & \cos\beta' \sin\phi' \end{pmatrix} \tag{9}$$

$$\underline{\underline{B}}_2 = \begin{pmatrix} B_{2_{11}}(\beta', \phi) & B_{2_{12}}(\beta', \phi) & B_{2_{13}}(\beta', \phi) \\ B_{2_{12}}(\beta', \phi) & B_{2_{22}}(\beta', \phi) & B_{2_{23}}(\beta', \phi) \\ B_{2_{13}}(\beta', \phi) & B_{2_{23}}(\beta', \phi) & B_{2_{33}}(\beta', \phi) \end{pmatrix} \tag{10}$$

$$\underline{\underline{B}}_3 = \frac{1}{G(\beta', \phi')} \begin{pmatrix} -\cos\beta' & -\sin\beta' \cos\phi' \\ 0 & 0 \\ -\sin\beta' \cos\phi' & \cos\beta' \end{pmatrix} \tag{11}$$

$$\underline{\underline{B}}_4 = \begin{pmatrix} 0 & (1 - R_{\perp_B} - T_{\perp_B}) \sin\beta' \sin\phi' \\ 1 + R_{\parallel_B} - T_{\parallel_B} & 0 \end{pmatrix} \tag{12}$$

where $G(\beta', \phi') = \sqrt{1 - \sin^2\beta' \sin^2\phi'}$. The elements of $\underline{\underline{B}}_2$ are

$$B_{2_{11}}(\beta', \phi) = 1 - \sin^2\beta' \cos^2\phi \tag{13a}$$

$$B_{2_{12}}(\beta', \phi) = -\sin^2\beta' \sin\phi \cos\phi \tag{13b}$$

$$B_{2_{13}}(\beta', \phi) = -\sin\beta' \cos\beta' \cos\phi \tag{13c}$$

$$B_{2_{22}}(\beta', \phi) = 1 - \sin^2\beta' \sin^2\phi \tag{13d}$$

$$B_{2_{23}}(\beta', \phi) = -\sin\beta' \cos\beta' \sin\phi \tag{13e}$$

$$B_{2_{33}}(\beta', \phi) = \sin^2\beta' \tag{13f}$$

It must be stressed that only $\underline{\underline{A}}_4$ and $\underline{\underline{B}}_4$ contain information about the half-screens by means of the reflection (R) and transmission (T) coefficients for parallel ($||$) and perpendicular ($\perp$) field components. Such coefficients are determined by applying the resistive boundary conditions [1, 17]

$$R_{\parallel_{A,B}} = \frac{\cos\theta^i}{\gamma_{A,B} + \cos\theta^i}; \; T_{\parallel_{A,B}} = \frac{\gamma_{A,B}}{\gamma_{A,B} + \cos\theta^i} \tag{14}$$

$$R_{\perp_{A,B}} = -\frac{1}{1 + \gamma_{A,B} \cos\theta^i}; \; T_{\perp_{A,B}} = \frac{\gamma_{A,B} \cos\theta^i}{1 + \gamma_{A,B} \cos\theta^i} \tag{15}$$

where $\gamma_{A,B} = 2R_{e_{A,B}}/\zeta_0$ and θ^i is the standard incidence angle (see Figure 1).

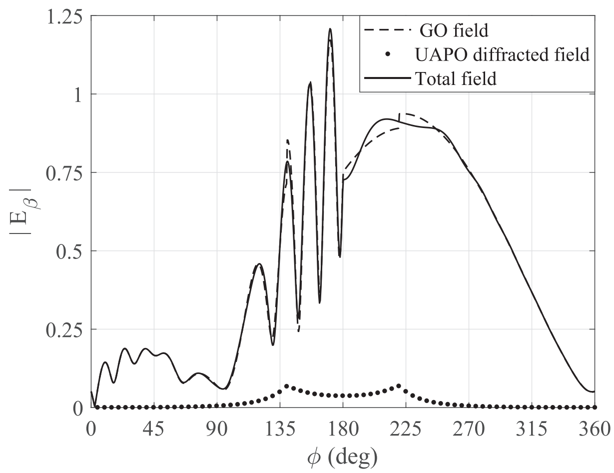

Figure 2. Magnitude of the β-component when $\beta' = 40^\circ, \phi' = 40^\circ$ and $E^i_{\beta'} = 1,\ E^i_{\phi'} = 0$.

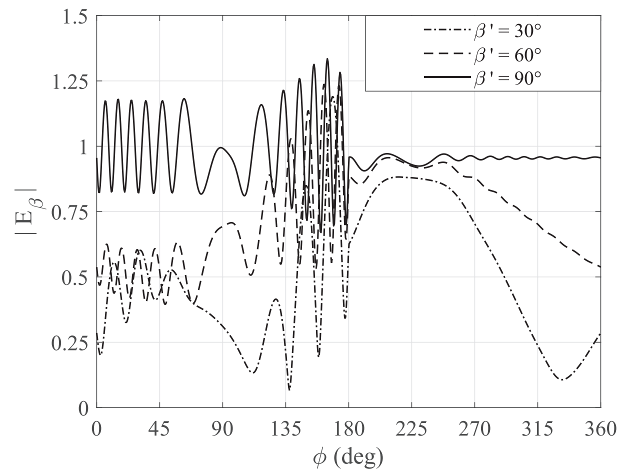

Figure 4. Magnitude of the β-component when $\phi' = 40^\circ$ and $E^i_{\beta'} = 1,\ E^i_{\phi'} = 0$.

3. Numerical Validation

The results reported in this section are collected on a circular path with radius equal to $5\lambda_0$, where λ_0 is the free-space wavelength, and refer to resistive half-screens with $\varepsilon_{r_A} = 2.5 - j0.25,\ \varepsilon_{r_B} = 3.7 - j0.16$, and $d = 0.025\lambda_0$. It must be stressed that no limitations exist on acceptable values of the relative permittivity.

Figures 2 and 3 show the amplitudes of the co-polar and cross-polar components, respectively, when $E^i_{\beta'} = 1$, $E^i_{\phi'} = 0$ and the incidence direction is given by $\beta' = 40^\circ, \phi' = 40^\circ$. The GO field possesses jumps at the reflection and transmission boundaries, whereas the UAPO diffracted field shows its peaks in correspondence of these directions as expected, thus obtaining the continuity of the total field and demonstrating the ability of the UAPO solution to counterbalance the GO field jumps.

The subsequent figure contains the amplitudes of the co-polar component when $\phi' = 40^\circ$ and the angle β' assumes several increasing values. The curve relevant to $\beta' = 90^\circ$ (normal incidence with respect to the junction) is also shown. As expected, the values of the co-polar component increase when approaching the normal incidence.

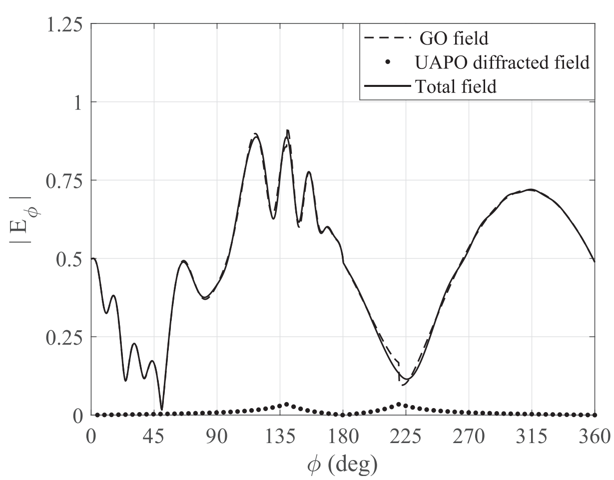

Figure 3. Magnitude of the ϕ-component when $\beta' = 40^\circ, \phi' = 40^\circ$ and $E^i_{\beta'} = 1,\ E^i_{\phi'} = 0$.

In order to reduce the needed computational effort, the case of normal incidence is also considered for the numerical validation of the UAPO approach by means of comparisons with the data obtained by running the RF unit of Comsol Multiphysics.

Amplitude and phase of the total field β-component are shown in Figures 5a and 5b, respectively, when $\beta' = 90^\circ, \phi' = 65^\circ$ and $E^i_{\beta'} = 1,\ E^i_{\phi'} = 0$. A very good accordance is evident in the whole observation range, thus endorsing the reliability of the proposed solution. A further assessment results from the comparison in Figure 6, where the amplitude of the total field ϕ-component is shown when $E^i_{\beta'} = 0,\ E^i_{\phi'} = 1$.

Figures 7 and 8 refer to an incidence direction lying in the second ϕ'-quadrant ($\phi' = 115^\circ$) and confirm the effectiveness of the proposed approach also in this case.

Note that the resistive boundary conditions [2] work well if d is very thin with respect to the wavelength, and, as already studied and pointed out in [1], inaccurate results must be expected if d increases.

4. Conclusions

The validation of the UAPO approach to solve the scattering problem involving a planar junction of resistive screens has been challenged. The presented UAPO solution exploits the linearity of the scattering integral to obtain the diffracted field as a linear combination of contributions related to the involved screens, and it is formulated in closed form according to the UTD framework. Such a solution is easy to use, and its trustworthiness has been proved by means of comparisons with Comsol Multiphysics data.

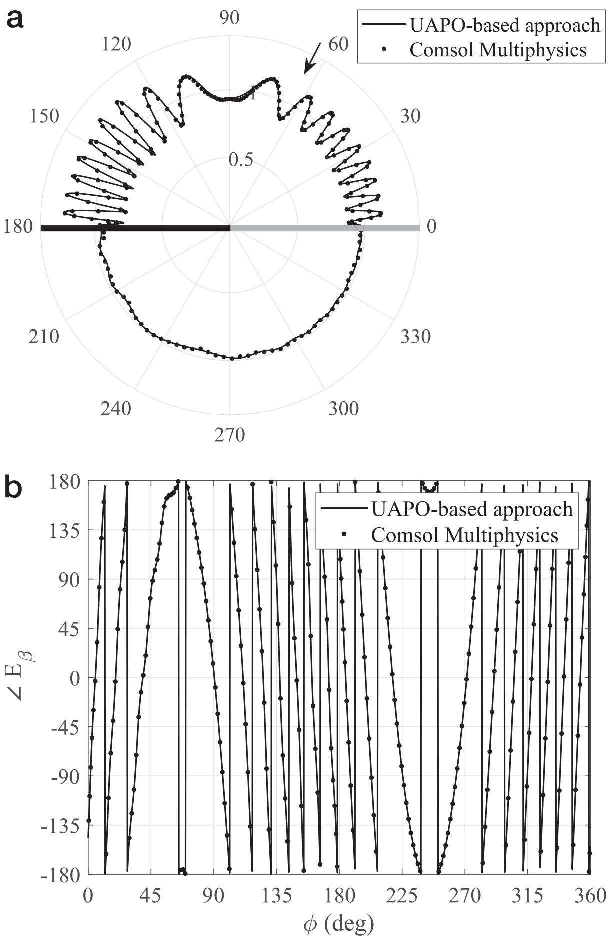

Figure 5. Magnitude and phase of the β-component when $\beta' = 90^{\circ}$, $\phi' = 65^{\circ}$ and $E^i_{\beta'} = 1,\ E^i_{\phi'} = 0$. (a) Amplitude of the total field β-component. (b) Phase of the total field β-component.

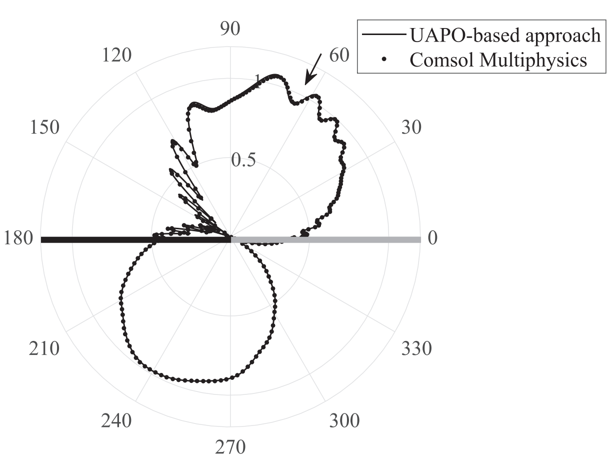

Figure 6. Magnitude of the ϕ-component when $\beta' = 90^{\circ}, \phi' = 65^{\circ}$ and $E^i_{\beta'} = 0,\ E^i_{\phi'} = 1$.

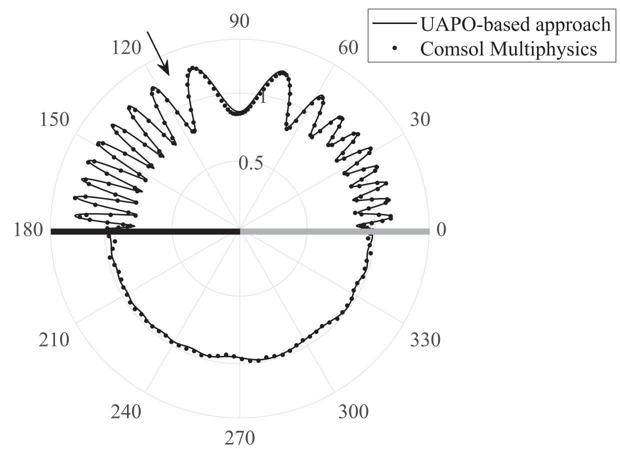

Figure 7. Magnitude of the β-component when $\beta' = 90^{\circ}, \phi' = 115^{\circ}$ and $E^i_{\beta'} = 1,\ E^i_{\phi'} = 0$.

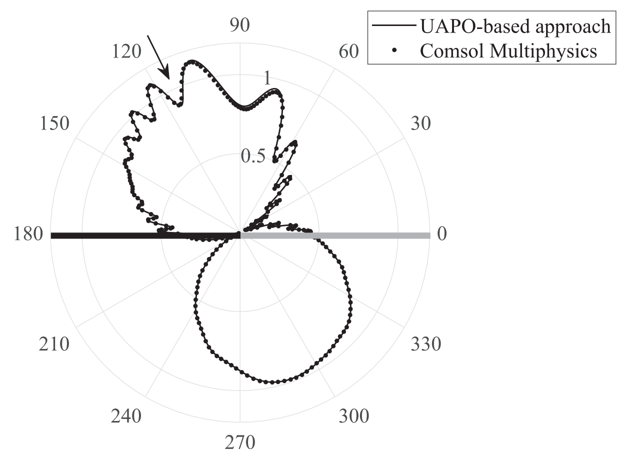

Figure 8. Magnitude of the ϕ-component when $\beta' = 90^{\circ}, \phi' = 115^{\circ}$ and $E^i_{\beta'} = 0,\ E^i_{\phi'} = 1$.

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