# Monte Carlo Augmented Channel Estimator

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*Abstract* – We have developed a novel Monte Carlo augmented channel estimator for millimeter-wave orthogonal frequency-division multiplexing systems to better estimate and correct a transmitted orthogonal frequency-division multiplexed signal by incorporating the predefined known responses of system hardware and their errors. We compare our estimator against current channel estimators for different phased-array configurations and propagation channels. By accounting for these known responses of system hardware, our estimator provides an order-of-magnitude reduction of the standard deviation of the error vector magnitude and a reduction of the mean and standard deviation of the bit error ratio.

## 1. Introduction

Millimeter-wave (mmWave) phased arrays are becoming more widely used with fifth-generation (5G) commercial systems. These communications systems, consisting of a transmitter, receiver, and propagation channel, leverage mmWave frequencies and bandwidths to enable increased throughput. Commercial hardware production of mmWave phased arrays requires tighter manufacturing tolerances and lower power consumption than microwave-frequency communications systems [1]. The shorter mmWave wavelengths also allow phased arrays with thousands of elements to be created in smaller form factors. Compared to microwave-frequency communications systems, the increased element count and new constraints can increase hardware errors (such as phase noise) and directly affect the accuracy of channel-estimator performance. To reach the higher throughput desired of 5G mmWave systems, channel estimators must be improved to decrease the errors in the received data due to the propagation channel and phased-array hardware components. Reduced received signal errors also reduce the bit errors and increase throughput in a communications system

Channel estimators calculate the effect of the physical layer of the communication system on a transmitted signal. This physical layer may include uncertainty effects from propagation channels and hardware components (e.g., frequency responses and noise). This estimate of the physical layer enables the correction of these effects within the received signal. Channel estimators in orthogonal frequency-division multiplexing (OFDM) communications systems use known pilot tones to estimate the frequency response to ensure the greatest reliability and throughput during operation. Two well-known channel estimators are the least-squares (LS) and the linear minimum mean squared error (LMMSE) estimators [2, 3]. These estimators are often suboptimal for large arrays. The LS estimator does not directly account for any hardware component or propagation-channel noise. The LMMSE estimator leverages correlations across frequencies to account for noise but requires a priori knowledge of the hardware and propagation channel for all system states.

Other novel channel estimators have also been designed and tested for 5G mmWave communications systems. Many of these account for the capabilities of 5G systems, such as using spatial diversity from the electrically large size of the phased arrays [4, 5]. Others use machine-learning methods to provide estimates [6]. Furthermore, many researchers have studied the efficacy of compressed sensing for evaluating subcarrier responses from pilot tones [7]. Previous authors have also developed methods and techniques to improve channel estimation in communications systems using Monte Carlo methods [8, 9]. However, neither of these efforts focuses on the hardware of the large arrays seen in 5G mmWave communications systems.

In this article, we devise a novel channel-estimation scheme augmented by Monte Carlo analysis, denoted the Monte Carlo channel estimator (MCCE). This new estimator is designed to reduce bit errors in 5G mmWave systems with electrically large phased arrays using a priori knowledge of the frequency responses and error characteristics of phased-array hardware components. Unlike the LMMSE estimator, the MCCE does not require a priori knowledge of the entire communications system. Therefore, unknown or unmodeled components of the communications system can exist. In this work, all transmit and receive hardware and any propagation channel are considered unknown except for amplifiers, phase shifters, and combiners. Frequency conversion and digitization is assumed to be perfect.

## 2. Monte Carlo Channel Estimation

The MCCE uses a multistep operation to provide a channel estimate given input signals $\mathbf{x}$ and output signals $\mathbf{y}$ at each subcarrier $f$. Estimated variables are denoted with hats (e.g., $\hat{x}$) and vectors are denoted in bold (e.g., $\mathbf{x}$). A visualization of the MCCE variables is seen in Figure 1. To our knowledge, this application in

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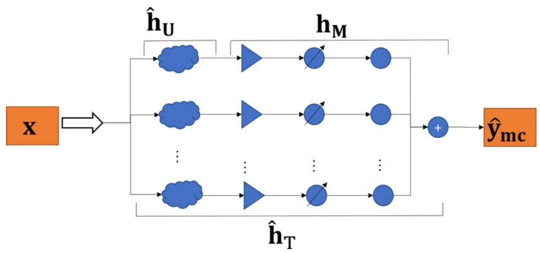


Figure 1. A depiction of the MCCE parameters. The input is given as $x$, the MCCE output is $\hat{\mathbf{y}}_\mathbf{mc}$, $\mathbf{h}_\mathbf{M}$ is the modeled response, $\hat{\mathbf{h}}_\mathbf{U}$ is the unmodeled response, and $\hat{\mathbf{h}}_\mathbf{T}$ is the total response.

the ways presented here has not been previously performed.

The MCCE first generates a combined modeled response distribution $\mathbf{h}_{\mathbf{M}_{\mathbf{f,n}}}$ at each array element $n$ and frequency $f$ by randomly sampling and cascading the responses and distributions of each known hardware component $k$ $(k = [1, \ldots, K])$ times. These known hardware-component models are defined by either a mathematical formulation or measured $S$-parameter values. These models are the a priori knowledge provided to the MCCE. Currently, models used in the MCCE include a phase shifter and amplifier behind each antenna element $n$, with a combiner at the output of the phased array.

The MCCE assumes that all other system hardware and any propagation channel response are unmodeled. This combined unmodeled response is denoted as $\hat{h}_{\mathbf{U_f}}$. Using $\mathbf{h}_{\mathbf{M_f}}$ and our received values $y_f$, we can create an overdetermined system of equations to solve for $\hat{\mathbf{h}}_{\mathbf{U_f}}$ as

$$\begin{bmatrix} x_f h_{\mathrm{M}_{f,1,1}} & \cdots & x_f h_{\mathrm{M}_{f,N,1}} \\ \vdots & \ddots & \vdots \\ x_f h_{\mathrm{M}_{f,1,\mathrm{K}}} & \cdots & x_f h_{\mathrm{M}_{f,N,\mathrm{K}}} \end{bmatrix} \begin{bmatrix} \hat{h}_{\mathrm{U}_{f,1}} \\ \hat{h}_{\mathrm{U}_{f,2}} \\ \vdots \\ \hat{h}_{\mathrm{U}_{f,N}} \end{bmatrix} = \begin{bmatrix} y_f \\ y_f \\ \vdots \\ y_f \end{bmatrix} \quad (1)$$

where $y_f$ is our received pilot value and $h_{\mathrm{M}_{f,n,k}}$ is the $k$th sample of $\mathbf{h}_{\mathbf{M}_{\mathbf{f,n}}}$ at element $n$ $(n = [1, \ldots, N])$ and subcarrier frequency $f$. The unmodeled response $\hat{h}_{\mathrm{U}_{f,n}}$ is then calculated using a least-squares approximation of the overdetermined system of equations.

Using the known inputs $x_f$, $j$ $(j = [1, \ldots, J])$ Monte Carlo iterations are performed to estimate the output $\hat{\mathrm{y}}_{\mathrm{mc}_{f,j}}$ of both the modeled portion $\mathbf{h}_{\mathbf{M}_{\mathbf{f,n}}}$ and the unmodeled portion $\hat{\mathbf{h}}_{\mathbf{U_f}}$ of the channel. The relationship between $\mathrm{x}_f$, $\mathrm{y}_{\mathrm{mc}_{f,j}}$, and the total estimated response $\hat{\mathbf{h}}_{T_f}$ is given as

$$\begin{bmatrix} \frac{\hat{y}_{mc_{1,1}}}{x_1} & \cdots & \frac{\hat{y}_{mc_{F,1}}}{x_F} \\ \vdots & \ddots & \vdots \\ \frac{\hat{y}_{mc_{1,J}}}{x_1} & \cdots & \frac{\hat{y}_{mc_{F,J}}}{x_F} \end{bmatrix} = \begin{bmatrix} \hat{h}_{T_{1,1}} & \cdots & \hat{h}_{T_{F,1}} \\ \vdots & \ddots & \vdots \\ \hat{h}_{T_{1,J}} & \cdots & \hat{h}_{T_{F,J}} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{h}}_{\mathrm{T}_1} \\ \vdots \\ \hat{\mathbf{h}}_{\mathrm{T_f}} \end{bmatrix} \quad (2)$$

where $\hat{h}_{T_{f,j}}$ is the relation between $x_f$ and $\hat{y}_{mc_{f,j}}$ for subcarrier $f = [1, \ldots, F]$. The values of $\hat{\mathbf{h}}_{\mathrm{T_f}}$ are used to calculate the final channel estimate $\hat{\mathbf{h}}_{\mathrm{MCCE_f}}$ at subcarrier $f$ as

$$\hat{\mathbf{h}}_{\mathbf{MCCE_f}} = E\left[\hat{\mathbf{h}}_{\mathbf{T_f}}\right] = \frac{1}{J}\sum_{j=1}^{J}\frac{\hat{y}_{mc_{f,j}}}{x_f} \quad (3)$$

This provides an estimate of the total response between the measurement points of the estimator inputs $\mathbf{x}$ and $\mathbf{y}$. It is important to note that in this article we focus solely on estimating the response from the phased-array hardware and the propagation channel. Other components such as analog-to-digital (ADC) and digital-to-analog converters (DAC) are assumed to be captured by $\hat{\mathbf{h}}_{\mathbf{U_f}}$. A better estimation can likely be obtained by future addition of more modeled hardware components.

## 3. Monte Carlo Channel Estimator Evaluation Configuration

The efficacy of the LS, LMMSE, and MCCE channel estimators were evaluated in a simulated 5G mmWave system. These simulations used similar modeled hardware components to those provided to the MCCE. Each MCCE estimation used $K = 20,000$ and $J = 2500$. These values were found through testing to provide a reasonable trade-off of accuracy and time complexity. These simulations tested multiple phased-array hardware configurations with varying signal-to-noise ratios (SNRs) to evaluate the efficacy of each channel estimator. The mean square error (MSE), error vector magnitude (EVM), and bit error ratio (BER) are the metrics of interest.

Each estimator was tested using quadrature shift phase keying and 16-, 64-, 256-, 1024-, and 4096-quadrature amplitude modulations (QAM). Each test performed 1000 repeat simulations with 100 subcarriers spaced at 15 kHz, starting at 27 GHz, with $2^{19}$ total transmitted bits. Pilot tones were sent with each OFDM symbol evenly spaced at intervals of 10 subcarriers. Testing showed that the pilot-tone density had minimal effect on the results. The responses of non-pilot subcarriers were calculated from each pilot tone using a two-dimensional linear interpolation and nearest-neighbor extrapolation.

The results tested a propagation channel with a single incident plane wave at boresight (i.e., $\theta_i = 0$, $\phi_i = 0$) with a uniform frequency response. This propagation channel was coupled with simulated phased arrays consisting of isotropic antennas, ideal (lossless) combiners, unity gain amplifiers, and non-discretized phase shifters. Phase and magnitude errors were then added onto each of these components to produce phased arrays with varying SNRs. Each phased array was an $8 \times 8$ element rectangular planar array with 5.25 mm between elements. Each estimator was provided five OFDM symbols for estimation. It is assumed that the total system response does not vary from symbol to symbol, and therefore the noise is the

Table 1. Channel estimator metrics—mean (SD) across repeat estimations—with 256-QAM, SNR $\approx$ 38.6 dB.

| Metric | LS | LMMSE | MCCE |
|---|---|---|---|
| MSE | 0.000298 ($1.6 \times 10^{-5}$) | 0.000298 ($1.6 \times 10^{-5}$) | 0.000292 ($1.3 \times 10^{-6}$) |
| EVM | 1.7269 (0.04) | 1.727 (0.04) | 1.714 (0.004) |
| BER | $5.379 \times 10^{-7}$ ($1.1 \times 10^{-6}$) | $5.379 \times 10^{-7}$ ($1.1 \times 10^{-6}$) | $2.346 \times 10^{-7}$ ($6.6 \times 10^{-7}$) |

only difference in the values of subcarriers between symbols. The LS and LMMSE estimators calculate the mean at each subcarrier across the symbols. The MCCE uses the same mean to calculate the unmodeled response $\hat{\mathbf{h}}_{\mathbf{U_f}}$.

## 4. Monte Carlo Channel Estimator Evaluation Results

Table 1 shows the resulting mean (SD) of metrics for the LS, LMMSE, and MCCE estimators for an SNR of approximately 38.6 dB with 256-QAM modulation. The MCCE outperforms the other estimators on all metrics for this noise level. It also has a standard deviation that is a factor of 10 lower than those of the other channel estimators. The distribution in MCCE estimation is tighter and has fewer outlier values that are incorrectly demodulated for a sufficiently low modulation scheme and SNR.

The BER as a function of SNR is seen in Figure 2, and the percent decrease in the mean BER for the MCCE over the LS estimator $\Delta_{LS}$ is given in Figure 3. This figure further highlights the improvements that are realized using the MCCE method. These plots quantify what constitutes a sufficient modulation and SNR by showing at what SNR value the MCCE outperforms other estimators for a given modulation scheme.

## 5. Analysis and Applications

The MCCE provides a reduced BER by lowering the standard deviation in an estimator output. This relation is due to the nonlinear nature of demodulating based on set thresholds for different constellation points. Figure 4 helps visualize this phenomenon; it provides the distributions of possible corrected constellation values from the MCCE and LS estimators for a 64-QAM modulation. The shaded areas along the edge denote values outside the boundaries of the constellation-point thresholds. Values in these shaded areas are incorrectly demodulated and increase the BER. While each distribution has an almost identical EVM, more of the MCCE distribution lies within the threshold boundaries than the LS distribution. Therefore, more symbols will be correctly demodulated with the MCCE and provide a lower BER. The reduced BER could allow a communications system to maintain a higher-order modulation for a given EVM than in specifications of EVM versus modulation like [10]. As either SNR increases or the threshold boundaries are moved in (i.e., as we move to a higher modulation scheme), the tight distribution of the MCCE begins to underperform the LS estimator with a wider distribution.

The decreased BER of the MCCE comes at the cost of computational complexity. The MCCE requires many Monte Carlo simulations to calculate the unmodeled response and the channel estimate. It also assumes that the total response is slowly varying such that multiple symbols can be used to estimate the error distribution (e.g., $\sim 66\ \mu s \times 5$ symbols $\approx 330\ \mu s$). Therefore, the MCCE is best in communications systems that can offload processing to a computational unit such as a coprocessor connected to a base station, ensure a slowly varying channel for millisecond timeframes, and provide responses and uncertainties for hardware components in use.

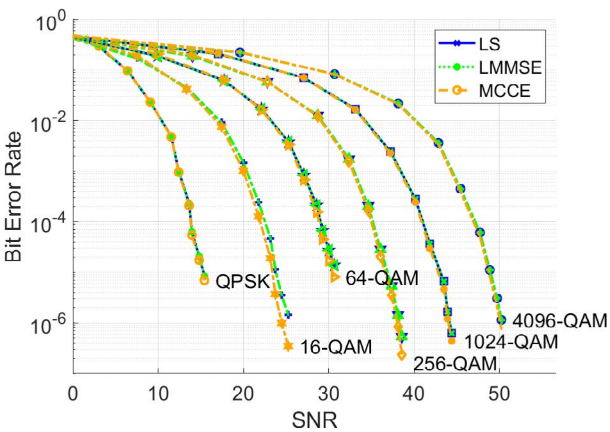


Figure 2. BER versus SNR for varying modulation schemes using the MCCE, LMMSE, and LS channel estimation techniques.

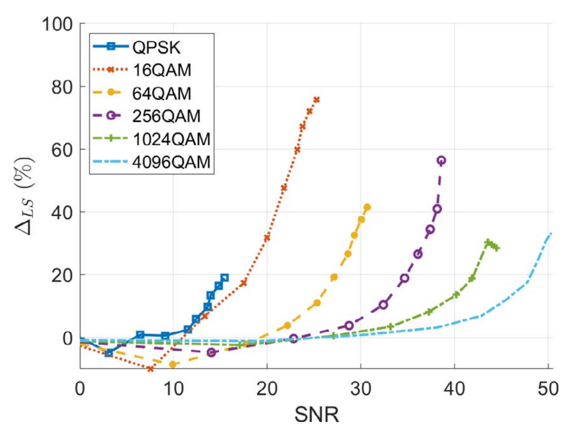


Figure 3. Percent decrease in BER over the LS estimator when using the MCCE for varying modulation schemes.

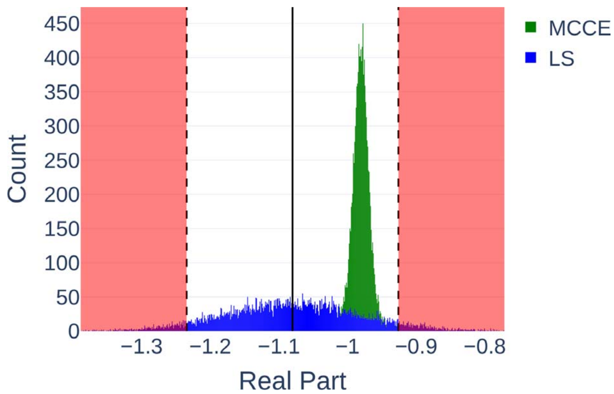


Figure 4. Example distributions of corrected values with a 64-QAM modulation. The real part of the value is shown centered around a single constellation point. Threshold boundaries are shaded in red, denoting areas of incorrect demodulation.

## 6. Conclusions and Future Work

Here we have presented a novel channel estimator that leverages Monte Carlo simulations to reduce bit errors by leveraging known hardware responses and distributions. This estimator can provide over a tenfold reduction in the standard deviation of EVM and MSE compared to the LS and LMMSE estimators. This reduced standard deviation of the EVM provides a reduction of the mean and standard deviation of the BER when compared to classical estimation schemes. Therefore, this estimator can help improve the throughput of 5G mmWave communications systems in situations with ample computational power, knowledge of system hardware, and SNR.

This estimator can be further developed and validated by including more modeled hardware components to better capture the modeled response of a system. This includes modeling imperfect up and down converters and DACs and ADCs. The estimator could also be implemented in a real-life communications system to more accurately test its capabilities in non-virtual environments.

## 7. References


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