Optimum Power Pattern Synthesis With Woodward–Lawson and Schelkunoff's Root Displacement Techniques

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Abstract – The Woodward–Lawson sampling technique is combined with Schelkunoff's root displacement method for power pattern synthesis of an antenna array. This hybrid method is shown to enhance the capability of the Woodward–Lawson technique to synthesize array patterns while optimizing the aperture distribution. An example of a cosec^2-type shaped power pattern is synthesized for implementation in a traveling-wave slot array.

1. Introduction

Orchard, Elliott, and Stern (OES) presented a method for synthesizing the power pattern of an antenna array [1]. Their method has since been used extensively in the literature for synthesizing sum patterns, difference patterns, and shaped patterns such as flat-top beams and cosec^2-shaped patterns. The method is based on perturbation of the roots of the Schelkunoff polynomial (SP) iteratively from a starting pattern until it converges to the desired pattern within some specified tolerance. In a 1985 article in the *IEEE Antennas and Propagation Society Newsletter*, Elliott was critical of the Woodward–Lawson (WL) technique in favor the OES method [2]. It was stated that the WL method is inefficient because it produces an antenna pattern that exhibits ripples at half the frequency and twice the amplitude of the OES method. The OES method has the advantage of producing many aperture distributions for a shaped power pattern, thereby allowing for optimization of the aperture distribution of the antenna array. However, for a large array, the OES method has the complexity of manipulating a large number of roots. The WL method synthesizes a given field pattern by sampling it at a number of points in the cosθ space, where θ is the angle measured from the array axis [3]. The samples are realized by orthogonal sinNx/sin$x$-type patterns. Thus, it is a simple technique that allows local control of the pattern at any angular region. Subsequent discussions between Elliott and Steyskal led to Elliott's statement that the WL method has some usefulness [4, 5].

Milne presented an excellent analysis of the capabilities of the WL method and demonstrated that the limitations of the WL method can be overcome [6]. By convolving the desired power pattern with a suitable scanning function, Milne was able to realize an optimum aperture distribution for the desired power pattern. In order to reduce the dynamic range of excitation, the WL technique was combined with the OES method in [7], and it was combined with the simulated annealing technique in [8]. The design procedure described in this article uses the WL method with an appropriate phase distribution for the array pattern. Subsequently, the roots in the shaped region of the pattern are displaced, thereby optimizing the aperture distribution to meet the specification. This procedure combines the advantages of the WL and OES methods, while also being more efficient and simpler than the OES method. Thus, the capability of this hybrid method is better than both the OES and WL methods. It will be demonstrated by synthesizing a cosec^2-type power pattern for implementation in a traveling-wave slot array application.

2. WL Synthesis

Figure 1 shows the desired power pattern to be realized by a traveling-wave slot array operating in the frequency band of 14.0–14.5 GHz in a ridge waveguide. The array consists of $N = 72$ resonant slots with alternating offsets and is fed by the dominant-mode traveling wave. The slot spacing d was chosen in the range $0.522\lambda_g$–$0.557\lambda_g$ in the frequency band, where λ_g is the guide wavelength. This corresponds to d/λ in the range 0.725λ–0.75λ, where λ is the free-space wavelength. Initially, the desired pattern was synthesized with uniform phase distribution. Based on the sampling

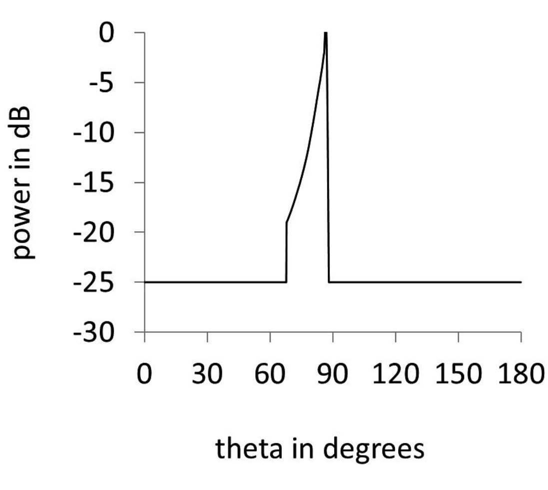

Figure 1. The desired far-field power pattern.

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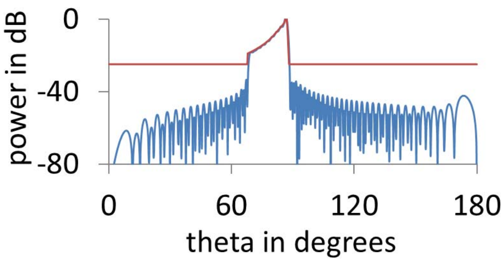

Figure 2. Synthesized uniform phase power pattern (blue) and mask (red).

theorem, the pattern specified in Figure 1 is sampled in the cosθ domain at intervals of $\lambda/Nd = 0.0189$. Thus, the total number of samples in the visible space ($-1 < \cos\theta < 1$) is $M = 2/0.0189 + 1 = 107$. In the shaped region, $68° < \theta < 88°$, the amplitude is given by the mask in Figure 1. Just outside the shaped region, the sample amplitudes were chosen starting at –40 dB below the peak and gradually tapering down. The excitation coefficient of the nth element of the array is

$$a_n = \sum_{m=1}^{M} f(c_m) e^{-jkndc_m} \qquad (1)$$

where c_m is the mth sample of the pattern in cosθ space.

The element pattern of a half-wavelength resonant slot along the axis of the waveguide is multiplied by the array factor synthesized by the WL method. The synthesized array pattern and the mask are shown in Figure 2. The amplitude and phase of the aperture distribution are illustrated in Figures 3 and 4, respectively. In order to assess the suitability of the design for implementation in a traveling-wave slot array, the phase delay of the traveling wave and the 180° phase difference between successive alternating offset slots have been subtracted from the phase distribution shown in Figure 4.

For a traveling-wave array, the elements near the feed should have relatively large amplitudes. Otherwise, these elements will have small coupling from the large incident-wave power and will be sensitive to tolerances. In addition, the dynamic range and element-to-element amplitude and phase variations should be small. The phase variations can be implemented by slightly detuning the slots and including a lens in front. The realized aperture distribution, shown in Figures 3 and 4,

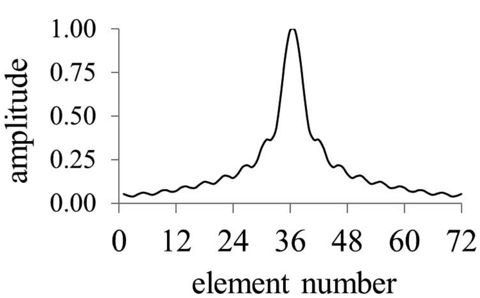

Figure 3. Aperture amplitude distribution.

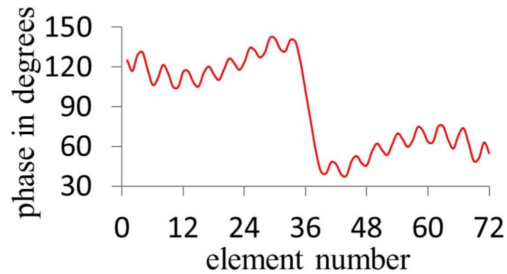

Figure 4. Aperture phase distribution.

may be implemented in an array using a standing-wave feed or a corporate feed. However, it is not suitable for implementation in a traveling-wave array, since the excitation amplitude is high near the center of the aperture but very low for elements near the feed end. To realize the required aperture distribution with large excitations near the feed end, Schelkunoff's root displacement technique was combined with the WL method.

3. Schelkunoff's Root Displacement Combined With the WL Method

For an N-element array, the SP of order $N-1$ is

$$f(w) = a_1 + a_2 w + a_3 w^2 + \cdots + a_N w^{N-1} \qquad (2)$$

where $w = \exp(jkd\cos\theta)$, k is the wave number in free space, and $a_1, a_2, \ldots, a_N$ are the complex coefficients given in (1). In the synthesis of shaped patterns, if there are P roots of the SP off the unit circle, each root may be moved to its reciprocal radial value at the same angular location, without changing the power pattern. This yields 2^P distinct aperture distributions, all yielding the same power patterns [2]. One may find an optimum aperture distribution from among these. Such a possibility was examined in the WL technique. For simplicity, it was decided to displace the roots in the shaped region only.

The roots in the shaped region of the pattern shown in Figure 1 were obtained by a numerical search technique for a range of radial values between 0.8 and 1.3 and angular values in the w-space corresponding to the shaped region. A function minimization technique was used by evaluating the polynomial at a given value of $w = w_0$ in the search space until the function value went down by more than 10 orders of magnitude. Horner's method was used to find the value of the polynomial efficiently, starting from the right-most term in (3), in the root-finding process [9]:

$$f(w_0) = a_1 + w_0(a_2 + w_0(a_3 + \ldots + w_0(a_{N-1} + a_N w_0)\ldots)) \qquad (3)$$

This process yielded eight pairs of roots w_n and $1/w_n^*$ in the complex w-plane in the angular range between 20° and 90°, approximately corresponding to the shaped

Table 1. Roots of the Schelkunoff polynomial in the shaped pattern region for a pattern with uniform phase

Root pair number	Radial values	Angular value (°)
1	0.916, 1.092	21.81
2	0.903, 1.107	32.26
3	0.900, 1.111	42.39
4	0.898, 1.114	52.43
5	0.895, 1.118	62.36
6	0.891, 1.123	72.16
7	0.886, 1.129	81.45
8	0.899, 1.112	89.93

pattern region. Table 1 shows these roots. It has been observed previously that the WL synthesis of a pattern with uniform phase yields pairs of roots with reciprocal complex conjugates [2, 6].

There are three possible placements of each pair of roots: both inside the unit circle, both outside the unit circle, and one inside and the other outside. All three cases will yield the same power pattern but different aperture distributions. Thus, there are 3^8 or 6561 distinct aperture distributions. When a root is displaced from w_a to w_b, the SP is divided by a factor $(w - w_a)$ and multiplied by $(w - w_b)$. Horner's method is used to divide the SP by the factor $(w - w_a)$ efficiently, as shown by the following recursive equations:

$$\begin{aligned} b_N &= a_N \\ b_{N-1} &= a_{N-1} + w_a a_N \\ b_{N-2} &= a_{N-2} + w_a(a_{N-1} + w_a a_N) = a_{N-2} + w_a b_{N-1} \\ &\cdot \\ &\cdot \\ b_1 &= a_1 + w_a b_2 \end{aligned} \tag{4}$$

Since w_a is a root, the remainder b_1 is zero. The quotient is given by the polynomial

$$f(w)/(w - w_a) = b_2 + b_3 w + b_4 w^2 + \cdots + b_N w^{N-2} \tag{5}$$

Table 2. Roots of the Schelkunoff polynomial in the shaped pattern region for a pattern with linear phase variation

Root number	Radial value	Angular value (°)
1	1.014	9.05
2	1.062	20.48
3	0.903	24.40
4	1.072	28.96
5	0.893	37.65
6	1.075	37.29
7	1.076	45.55
8	0.892	50.52
9	1.076	53.80
10	1.077	62.02
11	0.892	63.22
12	1.078	70.28
13	0.893	75.79
14	1.080	78.61
15	1.088	87.09
16	0.901	87.97

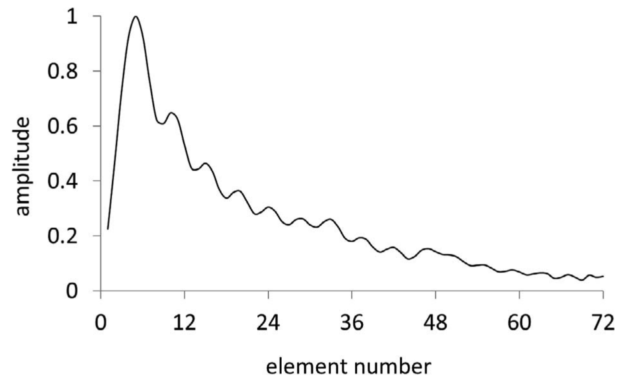

Figure 5. Optimum aperture amplitude distribution.

Multiplying a polynomial by the factor $(w - w_b)$ is a straightforward process.

A search of the 6561 aperture distributions to meet the specifications for implementation in a traveling-wave slot array showed no satisfactory results—that is, large excitations were not near the feed end. Therefore, different phase distributions were examined in the pattern function. A linearly progressive phase distribution in the pattern function (~40° per pattern sample) with lagging phase values toward decreasing values of $\cos\theta$ produced interesting results. It moved the peak of the aperture distribution close to the feed end. In addition, the roots of the SP had different angular locations, instead of occurring in pairs of reciprocal complex conjugates, thereby reducing the amount of ripple in the shaped region. Table 2 shows 16 roots in the shaped region corresponding to the eight pairs in Table 1. Displacement of each root to a reciprocal radial value at the same angular location provided $2^{16} = 65536$ distinct aperture distributions, all yielding the same power pattern.

A computer search of all 65536 solutions yielded an optimum aperture distribution with a relatively low dynamic range, small element-to-element phase variations, and an amplitude peak close to the feed. Figure 5 shows the amplitude of the optimum aperture distribution. It has a peak value at element 5 near the feed end and has a dynamic range of 28 dB. Since the elements near the feed have large amplitudes, the power incident at elements near the load will be significantly lower, and hence it is easy to realize low values of excitations. Figure 6 shows the optimized aperture phase distribution. The phase ripples are relatively small. They may be implemented by slightly detuning the resonant slots and including a lens. Figure 7 shows the synthesized array pattern at the center frequency and the band edges. The ripple levels in the shaped region are small, and small variations in the pattern amplitude with frequency are expected.

4. Conclusion

This work has demonstrated that the Woodward–Lawson method can be improved substantially for the

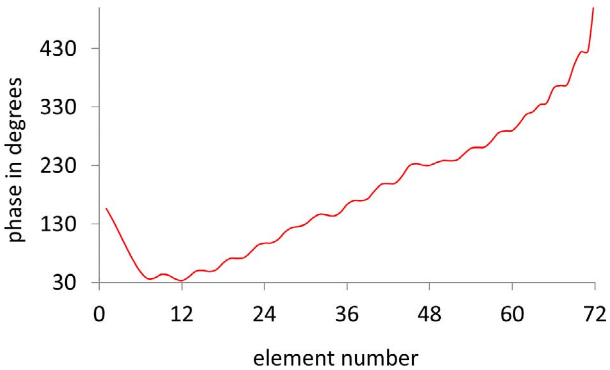

Figure 6. Optimum aperture phase distribution.

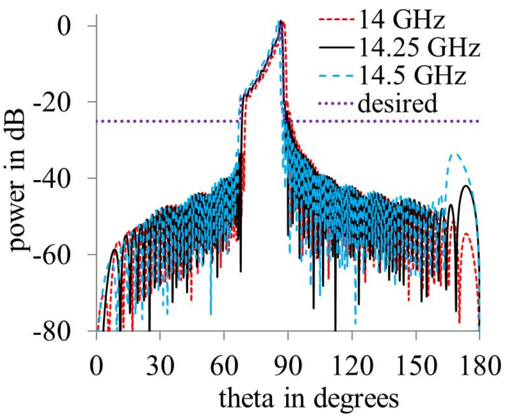

Figure 7. Optimized synthesis of the array power pattern.

synthesis of power patterns by combining it with Schelkunoff's root displacement method and choosing an appropriate phase distribution in the pattern to be synthesized. Thus, it has the simplicity of direct synthesis and subsequent optimization.

5. Acknowledgments

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6. References

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