On Interactions Between Spherical Waves in Multiple Scattering by an All-Dielectric Cluster

Andreas Kalogeropoulos and Nikolaos L. Tsitsas

Abstract – A magnetic dipole enclosed in a dielectric host medium stimulates a cluster of dielectric scatterers. Energy conservation laws are derived with the help of the complex form of the time-averaged Poynting vector. Calculation formulas resembling the optical theorem are obtained for the primary power flux and the power flux induced by the interaction between the primary and secondary fields.

1. Introduction

The process enabling the energy transfer from the origin of excitation to the far field is a key factor in the physical explanation of scattering phenomena. When multiple sources of radiation operate in the same volume, the various interactions between the generated fields produce additional energy flux. These extra terms in the energy functionals, especially in the Poynting vectors, have been associated with the power flow induced due to the interactions between the participating fields [1–5].

The nonlinear nature of the energy functionals poses significant limitations in the quantification of fields interactions [6]. Thus, the exploration of the different energy exchanges with respect to the quadratic behavior of the involved energy quantities has much to offer in the understanding of the energy processes and the implementation of quantification procedures [7–9]. In particular, when a dipole excites an all-dielectric cluster, such interactions occur in abundance and diversity, especially when the scatterers are distributed in a contested manner, something that finds significant applications in the design of nanoantennas [10–14].

In this work, we analyze a cluster of dielectric scatterers excited by a dipole located inside a host medium at close proximity to the cluster. By adopting the complex form of the time-averaged Poynting vector, we derive an energy conservation law that describes the energy travel from the host medium through the common exterior of the cluster to the far field. In addition, the role of the secondary fields in this energy transfer process is revealed by relating them with the reactive power. Analytical expressions for the involved power fluxes are also extracted.

2. Mathematical Formulation

Let V be a cluster of N homogeneous, dielectric scatterers identified as the union of N disjoint open subsets V_n of $\mathbb{R}^3$, where V_n each of the cluster's scatterers (with $n = 1,\ldots,N$ and $V_n \cap V_m = \emptyset$, for $n \neq m$). Each scatterer has a smooth $\mathcal{C}^1$ boundary S_n, oriented by the outward normal unit vector $\hat{\mathbf{n}}$ and is characterized by its wavenumber k_n, dielectric permittivity ε_n, and magnetic permeability μ_n. The cluster's exterior $V_0 = \mathbb{R}^3 \setminus V$ is an unbounded, homogeneous, and isotropic medium with wavenumber k_0, permittivity ε_0, and permeability μ_0.

The scatterers V_n are excited by a magnetic dipole located at $\mathbf{a}$ with dipole moment $A\hat{\mathbf{p}}$. The dipole is enclosed in an isotropic dielectric host medium V_h characterized by wavenumber k_h, dielectric permittivity ε_h, and magnetic permeability μ_h. The host medium is at close proximity to the cluster so that the spherical wave emitted from the dipole in V_h does not reduce to a plane wave. Therefore, the elicited secondary fields interact with both the cluster and the host medium. The geometrical setting of the multiple scattering problem is depicted in Figure 1.

The magnetic dipole at $\mathbf{a} \in V_h$ emits a monochromatic spherical wave of angular frequency ω with its corresponding *primary electric field* and *primary magnetic field*, given by [under $\exp(-\mathrm{i}\omega t)$ time dependence]

$$\mathbf{E}^{\mathrm{pr}}(\mathbf{r}) = A\nabla \times \left(\frac{\exp(\mathrm{i}k_h|\mathbf{r}-\mathbf{a}|)}{|\mathbf{r}-\mathbf{a}|} \hat{\mathbf{p}} \right) \qquad (1)$$

$$\mathbf{H}^{\mathrm{pr}}(\mathbf{r}) = \frac{1}{\mathrm{i}\omega\mu_h} \nabla \times \mathbf{E}^{\mathrm{pr}}(\mathbf{r}) \qquad (2)$$

The primary field interacts with the surfaces S_n for $n = 1,\ldots,N$ of the scatterers and the surface S_h of the host medium. Thus, secondary fields are elicited in both the exterior V_0 and the bounded dielectric regions V_n and V_h. Each of the secondary fields in V_0 acts as an excitation source for the scatterers of the cluster. Hence, each scatterer elicits scattered fields as a result of the excitation from the secondary external field of the host medium V_h and the rest of the cluster's scatterers. Under the close proximity assumption, none of these spherical waves reduce to plane waves. Therefore, all generated fields accumulate to an *overall electric field,* including all the fields elicited from the various interactions.

The overall field $\mathbf{E}^h$ inside the host medium V_h incorporates the contributions due to the primary excitation, as well as due to the secondary fields of the rest of the scatterers V_n. The overall external field $\mathbf{E}^0$, the internal fields $\mathbf{E}^n$ in V_n, and the secondary field $\mathbf{E}^{\mathrm{sec}}$ in V_h satisfy the vector Helmholtz equations in V_0, V_n, and V_h, respectively, i.e., it holds

Manuscript received 26 December 2021.

Andreas Kalogeropoulos and Nikolaos L. Tsitsas are with the School of Informatics, Aristotle University of Thessaloniki, Aristotle University Campus, 54124, Thessaloniki, Greece; e-mail: akaloger@csd.auth.gr, ntsitsas@csd.auth.gr.

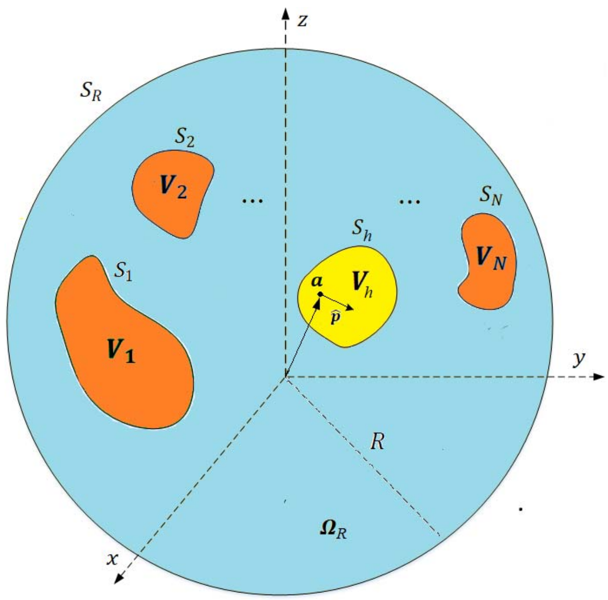

Figure 1. A sphere of radius R surrounding a cluster V of dielectric scatterers that are excited by a magnetic dipole located at $\mathbf{a}$ inside a host medium V_h.

$$\nabla^2\mathbf{E}^0(\mathbf{r}) + k_0^2\mathbf{E}^0(\mathbf{r}) = \mathbf{0}, \quad \mathbf{r} \in V_0 \tag{3}$$

$$\nabla^2\mathbf{E}^n(\mathbf{r}) + k_n^2\mathbf{E}^n(\mathbf{r}) = \mathbf{0}, \quad \mathbf{r} \in V_n \tag{4}$$

$$\nabla^2\mathbf{E}^{\text{sec}}(\mathbf{r}) + k_h^2\mathbf{E}^{\text{sec}}(\mathbf{r}) = \mathbf{0}, \quad \mathbf{r} \in V_h \tag{5}$$

whereas the primary electric field $\mathbf{E}^{\text{pr}}$ satisfies

$$\nabla^2\mathbf{E}^{\text{pr}}(\mathbf{r}) + k_h^2\mathbf{E}^{\text{pr}}(\mathbf{r}) = \mathbf{0}, \quad \mathbf{r} \in V_h \setminus \{\mathbf{a}\} \tag{6}$$

On the surfaces S_n of the cluster's scatterers, the transmission conditions hold

$$\hat{\mathbf{n}} \times \mathbf{E}^0(\mathbf{r}) = \hat{\mathbf{n}} \times \mathbf{E}^n(\mathbf{r}), \quad \mathbf{r} \in S_n \tag{7}$$

$$\hat{\mathbf{n}} \times \mathbf{H}^0(\mathbf{r}) = \hat{\mathbf{n}} \times \mathbf{H}^n(\mathbf{r}), \quad \mathbf{r} \in S_n \tag{8}$$

Similar conditions hold for the fields $\mathbf{E}^0$ and $\mathbf{E}^h$ on the surface S_h of the host medium.

The overall external field satisfies the Silver–Müller radiation condition

$$\lim_{r\to\infty}\left(Z_0\mathbf{r} \times \mathbf{H}^0(\mathbf{r}) + r\mathbf{E}^0(\mathbf{r})\right) = \mathbf{0} \tag{9}$$

with $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ the impedance of V_0. The overall electric far field $\mathbf{g}(\hat{\mathbf{r}})$ and the overall scattering cross section σ are defined, respectively, by [15]

$$\mathbf{E}^0(\mathbf{r}) = \mathbf{g}(\hat{\mathbf{r}})h_0(k_0 r) + \mathcal{O}(r^{-2}), \quad r \to \infty \tag{10}$$

$$\sigma = \frac{1}{k_0^2}\int_{S^2} |\mathbf{g}(\hat{\mathbf{r}})|^2 \mathrm{d}s(\hat{\mathbf{r}}) \tag{11}$$

where S^2 denotes the unit sphere of $\mathbb{R}^3$.

3. Energy Functionals

The time-averaged Poynting vector $\mathbf{S}^h$ of V_h that measures the rate of power flow is defined as follows:

$$\mathbf{S}^h(\mathbf{r}) = \mathbf{E}^h(\mathbf{r}) \times \overline{\mathbf{H}^h}(\mathbf{r}) \tag{12}$$

with the overbar denoting complex conjugation.

From this point on, we use the term *power flux* for the real part of the Poynting vector and the term *reactive power* for its imaginary part. We note that the reactive power is sometimes called *alternating flow* [16].

Two other energy functionals that play a significant role in the energy transfer process are the *electric energy density* W_E^h and the *magnetic energy density* W_H^h of V_h that are defined as follows:

$$W_E^h(\mathbf{r}) = \frac{\varepsilon_h}{2}|\mathbf{E}^h(\mathbf{r})|^2 \tag{13}$$

$$W_H^h(\mathbf{r}) = \frac{\mu_h}{2}|\mathbf{H}^h(\mathbf{r})|^2 \tag{14}$$

The energy densities W_E^0, W_H^0 of V_0 and W_E^n, W_H^n of the scatterers V_n are defined accordingly. Another key quantity in conservation laws is the *Lagrangian density* $L(\mathbf{r})$, which in electromagnetics coincides with the difference between the magnetic and electric energy densities [17]. In particular, the Lagrangian density in V_j is defined as

$$L^j(\mathbf{r}) = W_H^j(\mathbf{r}) - W_E^j(\mathbf{r}) \tag{15}$$

with $j = 0$ for the exterior V_0, $j = 1, \ldots, N$ for the cluster's scatterers V_n, and $j = h$ for the host medium V_h. The *primary energy densities* and the *secondary energy densities* in V_h are defined similarly.

Inside the host medium V_h, there exist three types of fields: the primary field, the scattered field induced by the interaction of the primary field with the host medium's surface, and the scattered field induced by the interaction of the overall external field with the host medium's surface. Hence, the Poynting vector in V_h has the decomposition

$$\mathbf{S}^h(\mathbf{r}) = \mathbf{S}^{\text{pr}}(\mathbf{r}) + \mathbf{S}^{\text{sec}}(\mathbf{r}) + \mathbf{S}^{\text{ext}}(\mathbf{r}) \tag{16}$$

with $\mathbf{S}^{\text{pr}}$ the primary Poynting vector, $\mathbf{S}^{\text{sec}}$ the secondary Poynting vector, and $\mathbf{S}^{\text{ext}}$ the interaction Poynting vector, the latter due to the interaction of the primary and secondary fields in V_h. These partial Poynting vectors are given by

$$\mathbf{S}^{\text{pr}}(\mathbf{r}) = \mathbf{E}^{\text{pr}}(\mathbf{r}) \times \overline{\mathbf{H}^{\text{pr}}}(\mathbf{r}) \tag{17}$$

$$\mathbf{S}^{\text{sec}}(\mathbf{r}) = \mathbf{E}^{\text{sec}}(\mathbf{r}) \times \overline{\mathbf{H}^{\text{sec}}}(\mathbf{r}) \tag{18}$$

$$\mathbf{S}^{\text{ext}}(\mathbf{r}) = \mathbf{E}^{\text{pr}}(\mathbf{r}) \times \overline{\mathbf{H}^{\text{sec}}}(\mathbf{r}) + \mathbf{E}^{\text{sec}}(\mathbf{r}) \times \overline{\mathbf{H}^{\text{pr}}}(\mathbf{r}) \tag{19}$$

with $\mathbf{S}^{\text{pr}}(\mathbf{r})$ and $\mathbf{S}^{\text{ext}}(\mathbf{r})$ defined for $\mathbf{r} \in V_h \setminus \{\mathbf{a}\}$, while $\mathbf{S}^{\text{sec}}(\mathbf{r})$ for $\mathbf{r} \in V_h$.

4. Conservation of Energy and the Optical Theorem

In this section, we prove relations that elaborate the way energy is transferred from the exciting scatterer, through the exterior of the cluster, to the far-field zone. The following theorem gives the complex form of the energy conservation law describing the energy process for the examined problem.

Theorem 1

The overall scattering cross section, the Lagrangian densities in all regions, and the primary and interaction Poynting vectors are connected as follows:

$$\frac{\sigma}{Z_0} = 2i\omega\left(\sum_{n=0}^{N}\int_{V_n} L^n(\mathbf{r})\mathrm{d}v(\mathbf{r}) + \int_{V_h} L_h^{\mathrm{sec}}(\mathbf{r})\mathrm{d}v(\mathbf{r})\right) + \int_{S_h} \hat{\mathbf{n}}\cdot(\mathbf{S}^{\mathrm{pr}}(\mathbf{r}) + \mathbf{S}^{\mathrm{ext}}(\mathbf{r}))\mathrm{d}s(\mathbf{r}) \tag{20}$$

where L_h^{sec} is the secondary Lagrangian density in V_h.

Proof: Let B_R be a ball of radius R enclosing the cluster V and V_h, and $\Omega_R = B_R \setminus \{V \cup V_h\}$; see Figure 1. By Green's first vector identity for $\mathbf{S}^0 = \mathbf{E}^0 \times \overline{\mathbf{H}^0}$ in Ω_R and using Ampere's and Faraday's laws, we arrive at

$$\int_{\partial\Omega_R} \hat{\mathbf{n}}\cdot\mathbf{S}^0(\mathbf{r})\mathrm{d}s(\mathbf{r}) = i\omega\int_{\Omega_R}\left(\mu_0|\mathbf{H}^0(\mathbf{r})|^2 - \varepsilon_0|\mathbf{E}^0(\mathbf{r})|^2\right)\mathrm{d}v(\mathbf{r}) \tag{21}$$

For the left-hand side of (21), it holds

$$\int_{\partial\Omega_R} \hat{\mathbf{n}}\cdot\mathbf{S}^0(\mathbf{r})\mathrm{d}s(\mathbf{r}) = \int_{\partial B_R} \hat{\mathbf{n}}\cdot\mathbf{S}^0(\mathbf{r})\mathrm{d}s(\mathbf{r}) - \sum_{n=1}^{N}\int_{S_n} \hat{\mathbf{n}}\cdot\mathbf{S}^0(\mathbf{r})\mathrm{d}s(\mathbf{r}) - \int_{S_h} \hat{\mathbf{n}}\cdot\mathbf{S}^0(\mathbf{r})\mathrm{d}s(\mathbf{r}) \tag{22}$$

Imposing the boundary conditions on the surfaces S_n of the cluster's scatterers, we obtain for $n = 1,\ldots,N$ that

$$\int_{S_n} \hat{\mathbf{n}}\cdot\mathbf{S}^0(\mathbf{r})\mathrm{d}s(\mathbf{r}) = \int_{S_n} \hat{\mathbf{n}}\cdot\mathbf{S}^n(\mathbf{r})\mathrm{d}s(\mathbf{r}) \tag{23}$$

Applying the Green's first vector identity in V_n for $n = 1,\ldots,N$, we find

$$\int_{S_n} \hat{\mathbf{n}}\cdot\mathbf{S}^n(\mathbf{r})\mathrm{d}s(\mathbf{r}) = i\omega\int_{V_n}\left(\mu_n|\mathbf{H}^n(\mathbf{r})|^2 - \varepsilon_n|\mathbf{E}^n(\mathbf{r})|^2\right)\mathrm{d}v(\mathbf{r}) \tag{24}$$

By letting $R \to \infty$, we are transferred in the far-field region, where the Silver–Müller radiation condition holds. Because $\hat{\mathbf{n}} = \hat{\mathbf{r}}$ on the spherical surface ∂B_R, we have

$$\hat{\mathbf{r}}\cdot\mathbf{S}^0(\mathbf{r}) = \frac{1}{Z_0 k_0^2 R^2}|\mathbf{g}(\hat{\mathbf{r}})|^2 + \mathcal{O}(R^{-3}) \tag{25}$$

Integrating (25) on the boundary of B_R for $R \to \infty$ and omitting the terms of order $\mathcal{O}(R^{-3})$ leads to

$$\lim_{R\to\infty}\int_{\partial B_R} \hat{\mathbf{n}}\cdot\mathbf{S}^0(\mathbf{r})\mathrm{d}s(\mathbf{r}) = \frac{\sigma}{Z_0} \tag{26}$$

By imposing the transmission conditions on the host medium's surface and considering (16), we obtain

$$\int_{S_h} \hat{\mathbf{n}}\cdot\mathbf{S}^0(\mathbf{r})\mathrm{d}s(\mathbf{r}) = \int_{S_h} \hat{\mathbf{n}}\cdot\mathbf{S}^h(\mathbf{r})\mathrm{d}s(\mathbf{r}) = \int_{S_h} \hat{\mathbf{n}}\cdot(\mathbf{S}^{\mathrm{pr}}(\mathbf{r}) + \mathbf{S}^{\mathrm{sec}}(\mathbf{r}) + \mathbf{S}^{\mathrm{ext}}(\mathbf{r}))\mathrm{d}s(\mathbf{r}) \tag{27}$$

Applying Green's first identity in V_h for $\mathbf{S}^{\mathrm{sec}}$ gives

$$\int_{S_h} \hat{\mathbf{n}}\cdot\mathbf{S}^{\mathrm{sec}}(\mathbf{r})\mathrm{d}s(\mathbf{r}) = i\omega\int_{V_h}\left(\mu_h|\mathbf{H}_h^{\mathrm{sec}}(\mathbf{r})|^2 - \varepsilon_h|\mathbf{E}_h^{\mathrm{sec}}(\mathbf{r})|^2\right)\mathrm{d}v(\mathbf{r}) \tag{28}$$

where $\mathbf{H}_h^{\mathrm{sec}}$ and $\mathbf{E}_h^{\mathrm{sec}}$ are, respectively, the secondary magnetic and electric fields in V_h. Relation (20) is derived by substituting (22)–(24) and (26)–(28) to (21) and taking into account definition (15).

Remark: Equation (24) has two physical implications about a (dipole-free) dielectric scatterer V_n. First, it states that the Poynting vector coincides with the reactive power, and additionally by means of definition (15), it relates the reactive power through S_n with the Lagrangian density in V_n.

Taking the real parts of (20) yields the following corollary relating the scattering cross section with the power flux induced in the host medium.

Corollary 2

The overall scattering cross section equals the power flux directed out of the exciting scatterer, i.e.,

$$\sigma = Z_0\mathrm{Re}\left[\int_{S_h} \hat{\mathbf{n}}\cdot(\mathbf{S}^{\mathrm{pr}}(\mathbf{r}) + \mathbf{S}^{\mathrm{ext}}(\mathbf{r}))\mathrm{d}s(\mathbf{r})\right] \tag{29}$$

Equation (29) implies that the energy flux radiated in the far-field results from the power flux induced by the interactions with the primary field.

On the other hand, another useful relation is derived by taking the imaginary parts of (20).

Corollary 3

The Lagrangian density in the exterior of the cluster and the reactive intensity directed into the exciting scatterer are connected as follows:

$$2\omega \sum_{n=0}^{N} \int_{V_n} L^n(\mathbf{r}) \mathrm{d}v(\mathbf{r}) = -\mathrm{Im}\left[\int_{S_h} \hat{\mathbf{n}} \cdot \mathbf{S}^0(\mathbf{r}) \mathrm{d}s(\mathbf{r})\right] \quad (30)$$

Equation (30) reveals that the reactive power is manifested as the accumulated difference between the magnetic and electric energies stored in the cluster and its exterior.

Now, we prove an optical theorem connecting the overall scattering cross section with the secondary magnetic field in the host medium V_h. Relations connecting the overall scattering cross sections with magnetic or electric fields inside a volume have been investigated by several authors [1, 18–21].

Theorem 4

The overall scattering cross section is connected to the secondary magnetic field at the dipole's position as follows:

$$\sigma = 4\pi Z_0 \left(\mathrm{Re}[\bar{A}\mathbf{H}^{\mathrm{sec}}(\mathbf{a}) \cdot \hat{\mathbf{p}}] + |A|^2 \frac{k_h^2}{Z_h} \right) \quad (31)$$

Proof: Let B_δ be a ball of radius δ centered at the dipole's position $\mathbf{a}$ and enclosed by the host medium V_h, while $\Omega_\delta = V_h \setminus B_\delta$. Applying Green's second identity for $\mathbf{E}^{\mathrm{pr}}, \overline{\mathbf{E}^{\mathrm{sec}}}$ in Ω_δ, and using Ampere's law yields

$$\int_{S_h} \hat{\mathbf{n}} \cdot \left(\mathbf{E}^{\mathrm{pr}}(\mathbf{r}) \times \overline{\mathbf{H}^{\mathrm{sec}}}(\mathbf{r}) + \overline{\mathbf{E}^{\mathrm{sec}}}(\mathbf{r}) \times \mathbf{H}^{\mathrm{pr}}(\mathbf{r})\right) \mathrm{d}s(\mathbf{r}) = \int_{\partial B_\delta} \hat{\mathbf{n}} \cdot \left(\mathbf{E}^{\mathrm{pr}}(\mathbf{r}) \times \overline{\mathbf{H}^{\mathrm{sec}}}(\mathbf{r}) + \overline{\mathbf{E}^{\mathrm{sec}}}(\mathbf{r}) \times \mathbf{H}^{\mathrm{pr}}(\mathbf{r})\right) \mathrm{d}s(\mathbf{r}) \quad (32)$$

Substituting (1) and (2) into the right-hand side of (32), we arrive after lengthy calculations at

$$\begin{aligned} &\int_{S_h} \hat{\mathbf{n}} \cdot \left(\mathbf{E}^{\mathrm{pr}}(\mathbf{r}) \times \overline{\mathbf{H}^{\mathrm{sec}}}(\mathbf{r}) + \overline{\mathbf{E}^{\mathrm{sec}}}(\mathbf{r}) \times \mathbf{H}^{\mathrm{pr}}(\mathbf{r})\right) \mathrm{d}s(\mathbf{r}) \\ &= A\Bigg[\int_{\partial B_\delta} \left(\mathrm{i}k_h + \frac{1}{\delta}\right) \frac{e^{\mathrm{i}k_h\delta}}{\delta} \overline{\mathbf{H}^{\mathrm{sec}}}(\mathbf{r}) \cdot \hat{\mathbf{p}} \, \mathrm{d}s(\mathbf{r}) \\ &\quad + \mathrm{i}k_h \int_{\partial B_\delta} \hat{\mathbf{n}} \cdot \left(\overline{\mathbf{E}^{\mathrm{sec}}}(\mathbf{r}) \times \hat{\mathbf{p}}\right) \frac{e^{\mathrm{i}k_h\delta}}{\delta} \mathrm{d}s(\mathbf{r}) \\ &\quad + \int_{\partial B_\delta} \hat{\mathbf{n}} \cdot \nabla \times \left(\nabla \frac{e^{\mathrm{i}k_h\delta}}{\delta} \cdot \hat{\mathbf{p}}\right) \overline{\mathbf{E}^{\mathrm{sec}}}(\mathbf{r}) \mathrm{d}s(\mathbf{r})\Bigg] \end{aligned} \quad (33)$$

with $\mathbf{d} = \mathbf{r} - \mathbf{a}$, $\delta = |\mathbf{d}|$, $\hat{\mathbf{d}} = \mathbf{d}/\delta$. By means of the Stokes theorem, the third integral in the right-hand side of (33) vanishes. For the first and second integrals, we use the mean value theorem and let $\delta \to 0$ to obtain

$$\int_{S_h} \hat{\mathbf{n}} \cdot \left(\mathbf{E}^{\mathrm{pr}}(\mathbf{r}) \times \overline{\mathbf{H}^{\mathrm{sec}}}(\mathbf{r}) + \overline{\mathbf{E}^{\mathrm{sec}}}(\mathbf{r}) \times \mathbf{H}^{\mathrm{pr}}(\mathbf{r})\right) \mathrm{d}s(\mathbf{r}) = 4\pi A \overline{\mathbf{H}^{\mathrm{sec}}}(\mathbf{a}) \cdot \hat{\mathbf{p}} \quad (34)$$

Adding (34) to its complex conjugate and taking into account definition (19), leads to

$$\mathrm{Re}\left[\int_{S_h} \hat{\mathbf{n}} \cdot \mathbf{S}^{\mathrm{ext}}(\mathbf{r}) \mathrm{d}s(\mathbf{r})\right] = 4\pi \mathrm{Re}[\bar{A}\mathbf{H}^{\mathrm{sec}}(\mathbf{a}) \cdot \hat{\mathbf{p}}] \quad (35)$$

On the other hand, using expressions (1) and (2) and after lengthy calculations, we arrive at the following formula for the primary power flux in V_h

$$\mathbf{S}^{\mathrm{pr}}(\mathbf{r}) = \frac{k_h|A|^2}{Z_h\delta^4} \left(k_h\delta^2 - \mathrm{i}(3\delta - 1) - \frac{3\mathrm{i}}{\delta k_h^2}\right) \hat{\mathbf{p}} \times \left(\hat{\mathbf{p}} \times \hat{\mathbf{d}}\right) \quad (36)$$

while Z_h denotes the impedance of the host medium. Application of mean value theorem, while letting $\delta \to 0$, yields

$$\mathrm{Re}\left[\int_{B_\delta} \mathbf{n} \cdot \mathbf{S}^{\mathrm{pr}}(\mathbf{r}) ds(\mathbf{r})\right] = \frac{4\pi k_h^2 |A|^2}{Z_h} \quad (37)$$

Thus, (31) follows by combining (35) and (37) with (29).

Remark: Equation (35) demonstrates the connection between the interaction power flux and the secondary magnetic field inside the host medium V_h.

5. Conclusions

Energy conservation laws for the excitation of an all-dielectric cluster by a magnetic dipole were derived. Such laws are vital for understanding the energy transfer process. Exact expressions for the primary power flux and the interaction power flux induced inside the host medium were established. These expressions lead to an optical theorem for the overall scattering cross section.

The importance of the Lagrangian density in the energy transfer process was highlighted, and the connection between the reactive power and the secondary fields was revealed. Specifically, the fact that reactive power results from the interaction between participating fields was demonstrated, in contrast with the overall scattering cross section, which stems directly from the power flux directed out of the host medium.

6. References

1. M. I. Mishchenko, "The Electromagnetic Optical Theorem Revisited," *Journal of Quantitative Spectroscopy and Radiative Transfer*, **101**, 3, October 2006, pp. 404-410.
2. M. I. Mishchenko and M. A. Yurkin, "Additivity of Integral Optical Cross Sections for a Fixed Tenuous Multi-Particle Group," *Optics Letters*, **44**, 2, January 2019, pp. 419-422.
3. I. Liberal, I. Ederra, R. Gonzalo, and R. W. Ziolkowski, "Superbackscattering Antenna Arrays," *IEEE Transac-*

tions on Antennas and Propagation, **63**, 5, May 2015, pp. 2011-2021.

4. A. Kalogeropoulos and N. L. Tsitsas, “Electromagnetic Interactions of Dipole Distributions With a Stratified Medium: Power fluxes and Scattering Cross Sections,” *Studies in Applied Mathematics*, **148**, 3, April 2022, pp. 1040-1068
5. V. Giannini, A. I. Fernandez-Domínguez, S. C. Heck, and S. A. Maier, “Plasmonic Nanoantennas: Fundamentals and Their Use in Controlling the Radiative Properties of Nanoemitters,” *Chemical Reviews*, **111**, 6, March 2011, pp. 3888-3912.
6. J. C. Auger and B. Stout, “A Recursive Centered T-Matrix Algorithm to Solve the Multiple Scattering Equation: Numerical Validation,” *Journal of Quantitative Spectroscopy and Radiative Transfer*, **79–80**, 1, June-September 2003, pp. 533-547.
7. P. A. Martin, “Quadratic Quantities in Acoustics: Scattering Cross-Section and Radiation Force,” *Wave Motion*, **86**, March 2019, pp. 63-78.
8. A. Bernland, M. Gustafsson, and S. Nordebo “Physical Limitations on the Scattering of Electromagnetic Vector Spherical Waves,” *Journal of Physics A*, **44**, 14, March 2011, p. 21.
9. A. Kalogeropoulos and N. L. Tsitsas, “Analysis of Interaction Scattering Cross Sections and their Physical Bounds for Multiple-Dipole Stimulation of a Three-Dimensional Layered Mediums,” *IEEE Open Journal of Antennas and Propagation*, **2**, March 2021, pp. 506-520.
10. A. E. Krasnok, A. E. Miroshnichenko, P. A. Belov, and Y. S. Kivshar, “All-dielectric optical nanoantennas,” *Optics Express*, **20**, 18, August, 2012, pp. 20599-20604.
11. H. Fischer and O. J. F. Martin, “Engineering the Optical Response of Plasmonic Nanoantennas,” *Optics Express*, **16**, 12, June 2008, pp. 9144-9154.
12. Y. Liu, S. Palomba, Y. Park, T. Zentgraf, X. Yin, et al., “Compact Magnetic Antennas for Directional Excitation of Surface Plasmons,” *Nano Letters*, **12**, 9, July 2012, pp. 4853-4858.
13. S. J. Norton and T. Vo-Dinh, “Optical Response of Linear Chains of Metal Nanospheres and Nanospheroids,” *Journal of Optical Society of America A*, **25**, 11, November 2008, pp. 2767-2775.
14. G. P. Zouros, G. D. Kolezas, G. Fikioris, and N. L. Tsitsas “End-Fire All-Anisotropic Transition Metal Dichalcogenide Nanoantennas,” *Physical Review B*, **104**, 24, December 2021, 245432.
15. G. Dassios and R. Kleinmann, *Low Frequency Scattering*, Oxford, Clarendon Press, 2000.
16. J. D. Jackson, *Classical Electrodynamics*, New York, Wiley, 1998.
17. C. Civelek and T. F. Bechteler, “Lagrangian Formulation of Electromagnetic Fields in Nondispersive Medium by Means of the Extended Euler-Lagrange Differential Equation,” *International Journal of Engineering Science*, **46**, 12, December 2008, pp. 1218-1227.
18. Y. A. Eremin, “Generalization of the Optical Theorem for a Multipole Based on Integral Transforms,” *Differential Equations*, **53**, 9, October 2017, pp. 1121-1126.
19. Y. A. Eremin and A. G. Sveshnikov, “Generalization of the Optical Theorem to Multipole Sources in the Scattering Theory of Electromagnetic Waves,” *Computational Mathematics and Mathematical Physics*, **57**, 7, July 2017, pp. 1173-1180.
20. C. Athanasiadis, P. A. Martin, A. Spyropoulos, and I. G. Stratis, “Scattering Relations for Point Sources: Acoustic and Electromagnetic Waves,” *Journal of Mathematical Physics*, **43**, 11, October 2002, pp. 5683-5697.
21. C. Athanasiadis and N. L. Tsitsas, “Electromagnetic Scattering Theorems for Interior Dipole Excitation of a Layered Obstacle,” *Mathematical Methods in the Applied Sciences*, **30**, 12, August 2007, pp. 1467-1482.