Resonant Inductive WPT Link in MISO Configuration

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Abstract – This article considers an inductive resonant wireless power transfer link using a multiple input single output configuration. The general case of a link having N possibly coupled transmitters is solved. The analytical solution maximizing the efficiency is obtained from a generalized eigenvalue problem.

1. Introduction

Inductive resonant (IR) wireless power transfer (WPT) is an attractive solution for wirelessly providing power to electronic devices [1, 2]. Accordingly, in recent years, several configurations have been investigated in order to improve the performance in terms of one of the various figures of merit of interest. In particular, for applications where the position of the receiver is affected by uncertainty, the use of multiple transmitters (TXs) may be of interest.

In this regard, some useful results have been presented in the literature [3–5]. In [3], it is demonstrated that a nearly constant efficiency on a two-dimensional region can be obtained by using four transmitters. In [4], the problem of power maximization is solved for the case of three transmitters.

In [2, 5], the case of a link using two transmitters is considered. In more detail, in [5], the optimal load for both the maximum power and the maximum efficiency solutions is been presented. Interesting results are presented in [6], where a convex optimization algorithm to maximize the efficiency of a multiple input single output (MISO) link is presented. The algorithm allows obtaining both the maximum achievable efficiency of the link and the optimum load; the presented formulation allows calculating the maximum transfer efficiency of a given WPT link from its (unloaded) impedance matrix.

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A general analysis including all possible configurations (i.e., single input multiple output, MISO, and multiple input multiple output [MIMO]) is presented in [7, 8]. In particular, a very elegant approach has been presented in [7], where, referring to a generic WPT system in MIMO configuration, the solution for efficiency maximization is expressed as a generalized Rayleigh quotient problem. However, that article formulates only the solving equation for a generic WPT system; the analytical solution for the case of an inductive WPT link is not derived.

In [8], the case of an inductive WPT link is considered. The optimal loads are derived by imposing the zeroing of the first-order partial derivatives of the efficiency with respect to the input and output currents. However, the developed analysis assumes purely inductive couplings between the transmitters and the receiver, thus limiting the applicability of the approach, propagation channels having a negligible conductivity.

This article deepens the analysis presented in [9]; the general problem of a link using N transmitters is solved for efficiency maximization. The formulation of the problem is the same as that adopted in [7]: the solving equation of efficiency maximization is put in the form of a generalized eigenvalue problem. The specific solution for the case of an IR WPT link is derived and discussed. With respect to the analysis presented in [8], the one developed in this article is completely general; it requires only the impedance matrix of the network, no specific assumptions are made on the couplings, and it is required only that the network be passive and reciprocal. Analytical expressions of the optimal generators and load maximizing the efficiency are presented and discussed. It is demonstrated that the maximum realizable efficiency of the link does not depend on the coupling among the TXs. With respect to [9], new results are presented and discussed. In particular, the possibility of maximizing the power delivered to the load when the link operates at maximum efficiency is discussed. It is shown that the output power can be maximized by using power generators with a suitable impedance.

2. Optimal Analytical Solution

An IR WPT link using a MISO configuration is considered. Referring to the equivalent circuit of Figure 1, the analyzed link consists of $(N + 1)$ coupled inductors L_i, each one loaded by a suitable compensating capacitor C_i, realizing the resonance condition at the operating angular frequency (i.e., $\omega_0 = 1/\sqrt{L_i C_i}$). The resistors R_i are related to the quality factors of the

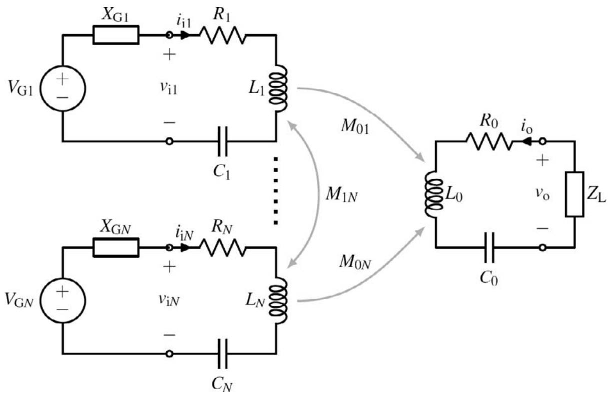

Figure 1. Inductively coupled WPT link with N transmitters.

coupled resonators: $Q_n = (\omega_0 L_n)/R_n$, $(n = 0, \ldots, N)$. The magnetic coupling is described by the coupling factors $k_{nm} = M_{nm}/\sqrt{L_n L_m}$, $(n, m = 0, \ldots, N;\ n \neq m)$. The WPT system is represented as an $(N+1)$-port strictly passive reciprocal network, described by the normalized impedance matrix $\mathbf{Z}$ expressed in [9, eq. (3)]. Two column vectors are defined for the currents: the vector $\mathbf{i}_\mathrm{i}$ of the input currents with elements $i_{\mathrm{i}j}$, $(j = 1, \ldots, N)$, and the vector $\mathbf{i}$ of all the currents with elements $[i_0, i_{ij} \ldots, i_{iN}]$.

According to the demonstration reported in [9], the following expression can be obtained for the efficiency:

$$\eta = \frac{P_\mathrm{o}}{P_\mathrm{i,TOT}} = -\frac{\mathbf{i}^\mathrm{H}\mathbf{A}\mathbf{i}}{\mathbf{i}^\mathrm{H}\mathbf{B}\mathbf{i}} \tag{1}$$

where H denotes Hermitian transpose and the matrices $\mathbf{A}$ and $\mathbf{B}$ are defined as follows:

$$\mathbf{A} = \begin{bmatrix} z_\mathrm{o} + z_\mathrm{o}^* & \mathbf{z}_\mathrm{c}^\mathrm{T} \\ \mathbf{z}_\mathrm{c}^* & \mathbf{0} \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 & \mathbf{z}_\mathrm{c}^\mathrm{H} \\ \mathbf{z}_\mathrm{c} & \mathbf{z}_\mathrm{i} + \mathbf{z}_\mathrm{i}^\mathrm{H} \end{bmatrix} \tag{2}$$

From (1), it is evident that the efficiency is a function of the normalized port currents, whose stationary points can be determined by requiring the first-order necessary condition $\delta\eta = 0$ on the currents; this implies solving the following generalized eigenvalue problem:

$$-\mathbf{A}\mathbf{i} = \eta\mathbf{B}\mathbf{i} \tag{3}$$

The eigenvalues can be determined by requiring that the determinant of $\mathbf{A} + \eta\mathbf{B}$, referred to as *Det*, vanishes. It is possible to prove that

$$Det = -\eta^{N-1} \det(\mathbf{z}_\mathrm{i} + \mathbf{z}_\mathrm{i}^\mathrm{H})\left[\gamma\eta^2 - 2\,\mu\,\eta + \gamma\right] \tag{4}$$

where

$$\gamma = (\alpha^2 - 1), \mu = (\alpha^2 + 1), \alpha = \sqrt{1 + Q_0 \sum_{n=1}^{N} k_{0n}^2 Q_n}$$

Equation (4) shows that there are always $(N-1)$ null eigenvalues, corresponding to sets of currents at the input ports that nullify the output current, and that the remaining eigenvalues are

$$\eta_1 = \frac{\alpha - 1}{\alpha + 1}, \qquad \eta_2 = \frac{\alpha + 1}{\alpha - 1}$$

From (5), it can be seen that $\eta_2 > 1$, so that it does not represent a physically acceptable solution, and the maximum efficiency is given by $\eta_\mathrm{max} = \eta_1$. By choosing an arbitrary value for i_o acting as normalization constant, the following eigenvectors can be obtained:

$$\mathbf{i}_{\mathrm{i}n} = j\,\frac{k_{0n}Q_n}{\alpha - 1}\,i_\mathrm{o} \qquad (n = 1, \ldots, N) \tag{5}$$

The currents expressed in (5) are the optimal normalized input currents to be used for maximizing the efficiency of the link. The normalized output voltage v_o and input voltages $\boldsymbol{v}_{\mathrm{i}n}$ for $(n = 1, \ldots, N)$ are:

$$v_\mathrm{o} = -\frac{\alpha}{Q_0}\,i_\mathrm{o} \tag{6}$$

$$\mathbf{v}_{\mathrm{i}n} = -\frac{1}{\alpha - 1}\left[\sum_{\substack{m=1\\ m\neq n}}^{N} k_{0m}k_{nm}Q_m - j\,\alpha k_{0n}\right] i_\mathrm{o} \tag{7}$$

Accordingly, the optimal load impedance is given by:

$$Z_\mathrm{L} = \omega_0\,L_0\,z_\mathrm{L} = R_0\,\alpha \tag{8}$$

It can also be noted that when the network operates in maximum power transfer conditions, from the input side it is equivalent to a set of uncoupled passive impedances:

$$z_n = \frac{\alpha}{Q_n} + j\,\frac{1}{k_{0n}Q_n}\sum_{\substack{m=1\\ m\neq n}}^{N} k_{0m}k_{nm}Q_m \qquad (n = 1, \ldots, N) \tag{9}$$

This shows that the optimal input currents can be obtained by means of a set of N independent generators operating in maximum power transfer conditions. The internal impedances of these generators are:

$$Z_{\mathrm{G}n} = R_{\mathrm{G}n} + j\,X_{\mathrm{G}n} = z_n^*$$

$$R_{\mathrm{G}n} = R_i\,\alpha, \qquad X_{\mathrm{G}n} = -\frac{R_i}{k_{0n}}\sum_{\substack{m=1\\ m\neq n}}^{N} k_{0m}k_{nm}Q_m \tag{10}$$

where $(n = 1, \ldots, N)$; their normalized voltages are:

$$v_{\mathrm{G}n} = \mathbf{v}_{\mathrm{i}n} + z_{\mathrm{G}n}\mathbf{i}_{\mathrm{i}n} = 2\,j\,k_{0n}\,\frac{\alpha}{\alpha - \mathbf{1}}\,i_\mathrm{o} \qquad (n = 1, \ldots, N) \tag{11}$$

Making use of (1), the input and output power are:

$$P_\mathrm{o} = \frac{\alpha}{2Q_0}\left|i_\mathrm{o}\right|^2 \tag{12}$$

$$P_{in} = \frac{1}{2}\,\frac{\alpha}{(\alpha-1)^2}\,k_{0n}^2\,Q_n\,|i_o|^2 \qquad (13)$$

$$P_{i,TOT} = \frac{\alpha}{2Q_0}\,\frac{\alpha+1}{\alpha-1}\,|i_o|^2 = \frac{P_o}{\eta_{max}} \qquad (14)$$

Accordingly, the efficiency of the link can be maximized by using as sources one of the following alternatives:

- Current sources providing the currents expressed by (5), that is, $i_{Gn} = \mathbf{i}_{in}$, ($n = 1, \ldots, N$). More generally, according to (5), the efficiency is maximized when the input currents satisfy the relation:

$$\frac{i_{Gi}}{i_{Gj}} = \frac{k_{0i}}{k_{0j}}\,\frac{Q_i}{Q_j} \qquad (i,j = 1,\ldots,N) \qquad (15)$$

  Accordingly, it is possible to set one of the input currents at an arbitrary value and the other ones so as to satisfy (15).

- Voltage sources providing the voltages

$$v_{Gn} = j\,k_{0n}\,\frac{\alpha}{\alpha-1}\,i_o \qquad (n = 1,\ldots,N) \qquad (16)$$

  and having in series to the input ports the reactances X_{Gn} expressed in (10). According to (16), the maximum efficiency can be obtained by setting one of the input voltages at an arbitrary value and the other ones so as to satisfy the relation:

$$\frac{v_{Gi}}{v_{Gj}} = \frac{k_{0i}}{k_{0j}} \qquad (i,j = 1,\ldots,N) \qquad (17)$$

  The reactances given in (10) are necessary in the case of coupled TXs. In fact, according to (7), the expressions of the optimal input voltages have a real part that depends on the couplings between the TXs. This means that, in order to maximize the efficiency, for the case of coupled TXs the network has to be powered with input voltages having a phase shift that depends on the couplings between the TXs. The reactances given in (10) play the role of compensating reactances that introduce the required phase shift.

- Power sources providing the powers expressed in (13), that is, $P_{Gn} = P_{in}$ ($n = 1, \ldots, N$), and having the internal impedances expressed in (10). As in the case of voltage generators, the imaginary parts of Z_{Gn} play the role of compensating reactances and are required for coupled TXs. The real parts of Z_{Gn} do not affect the efficiency; they allow obtaining the maximum power transfer condition at the input ports, thus maximizing the output power corresponding to the maximum efficiency solution. Similar to the previous cases, it is possible to set one of the input powers at an arbitrary value and the other ones so as to satisfy the relation:

$$\frac{P_{Gi}}{P_{Gj}} = \left(\frac{k_{0i}}{k_{0j}}\right)^2 \frac{Q_i}{Q_j} \qquad (i,j = 1,\ldots,N) \qquad (18)$$

Table 1. Parameters of the analyzed example

Parameter	Value
L_0 (μH)	4.59
L_1 (μH)	4.13
L_2 (μH)	3.67
L_3 (μH)	3.21
Q	270.27
f_0 (μH)	6.78
k_{01}	0.15
k_{02}	0.13
k_{03}	0.11
k_{12}	0.09
k_{13}	0.07
k_{23}	0.05

2.1. Discussion of the Results

According to the theory developed in this section, some important conclusions can be drawn. First, it can be noticed that the maximum efficiency of the link does not depend on possible couplings between the TXs, this implies that these couplings can always be compensated. Second, the optimal currents do not depend on the couplings between the TXs; are always in phase, and have an amplitude that depends on the couplings between the TXs and the multiple receivers. Conversely, the optimal input voltages depend on the couplings between the TXs, are in phase for uncoupled TXs, and have an amplitude that depends on both the couplings between the TXs and the multiple receivers and the couplings between the TXs. By adding the series reactances expressed in (10), the optimal input voltages are in phase and independent of the coupling between the TXs. Finally, it can be seen that the expression of the optimal load does not depend on the coupling between the TXs provided that the network is powered by the optimal currents expressed in (5).

3. Validation of the Theory

In order to validate the theory, a numerical example has been analyzed through circuital simulations; an IR WPT link with three transmitters has been analyzed. The parameters of the analyzed link are given in Table 1. Each inductor is loaded by a series compensating capacitor $C_i = 1/[(2\,\pi f_0)^2\,L_i]$.

According to (5), the maximum realizable efficiency for the analyzed link is about 0.97, and it is independent of the coupling among the TXs. The optimal value of the load is given by (8) and is equal to 44.37 Ω. As per the optimal generators, they can be obtained from (15)–(18). In particular, in the case of voltage generators, the values provided by the theory

Table 2. Optimal load and generators for the example of Table 1. The voltages are in volts, the powers in milliwatts, the impedances in ohms, and the capacitances in nanofarads.

Parameter	Value
$\eta_{\max}$	0.97
$R_{L,\mathrm{opt}}$	44.37
V_{G1}	1
V_{G2}	$k_{02}/k_{01} = 0.867$
V_{G3}	$k_{03}/k_{01} = 0.733$
P_{G1}	1
P_{G2}	0.751
P_{G3}	0.538
R_{G1}	39.9
R_{G2}	35.5
R_{G3}	31.1
Reactances to be added for coupled TXs	
X_{G1} (C_{G1})	−22.8 (1.03)
X_{G2} (C_{G2})	−22.9 (1.03)
X_{G3} (C_{G3})	−21.2 (1.11)

are summarized in Table 2. The maximum efficiency can be obtained by using voltage generators with amplitudes satisfying (17) and having the series compensating reactances expressed in (10). The values calculated for the analyzed example correspond to series capacitances and are given in Table 2.

These values have been validated through circuital simulations; the results are summarized in Figure 2. The figure shows the efficiency as function of the load impedance in the case of voltage sources. Simulations have been performed for different values of the couplings among the transmitting coils for two different cases: 1) voltage sources having the same amplitudes (i.e., $V_{\mathrm{G}n} = 1$ V, $n = 1, 2, 3$) and no compensating reactances and 2) voltage sources having the amplitudes and the series compensating capacitances reported in Table 2. In case 2, optimized voltage generators, the same results were obtained for all the analyzed cases; this confirms that the compensating reactances make the efficiency independent of the couplings between the TXs. The obtained results also confirm the optimal value provided by the theory for R_L. As per case 1 (generators not optimized), it can be seen that suboptimal results that depend on the values assumed for the couplings among the TXs have been obtained. In particular, the worst results have been obtained for the case of uncoupled TXs, thus confirming the importance of using suitable amplitudes for the voltage generators. In order to also investigate the behavior of the output power, the case of power sources has been also analyzed. According to the theory, the optimal values for $P_{\mathrm{G}n}$ and $R_{\mathrm{G}n}$ are given in Table 2. Simulations have been performed by varying a generator resistance $R_{\mathrm{G}n}$ at a time while the other two generator impedances and the load are set to their optimum value. The results obtained for the output power P_{o} normalized with respect to $P_{\mathrm{G}} = \left(\sum_{m=1}^{N} P_{\mathrm{G}n}\right)$ are given in Figure 3. It can be seen that the resistances $R_{\mathrm{G}n}$ play a key role in maximizing the output power corresponding to the maximum efficiency solution; it can also be seen that the values of $R_{\mathrm{G}n}$ that maximize P_{o} are those provided by the theory. A possible experimental validation of the presented formulas will be performed in a future work. In this regard, it is worth observing that the application of the proposed theory requires only the impedance matrix of the $(N + 1)$-port network. Accordingly, the derived formulas can be experimentally validated simply starting from the measured impedance parameters of the link.

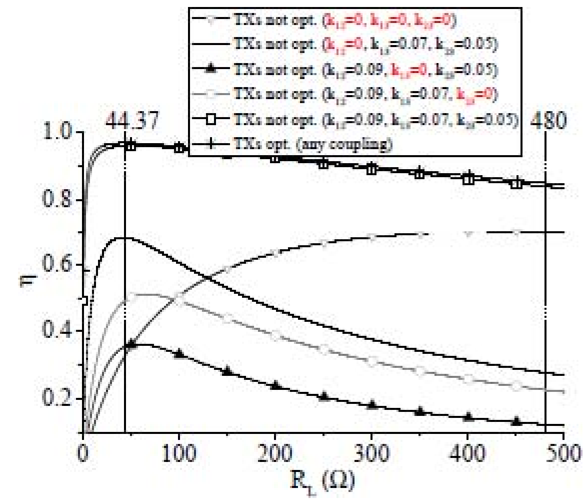

Figure 2. Results obtained for the analyzed example when the TXs are coupled as indicated by the labels. The figure compares the results obtained by using: 1) in-phase unitary voltage generators (TXs not opt.) and 2) voltage generators with amplitudes satisfying (17) and series compensating reactances given by (10) (TXs opt.).

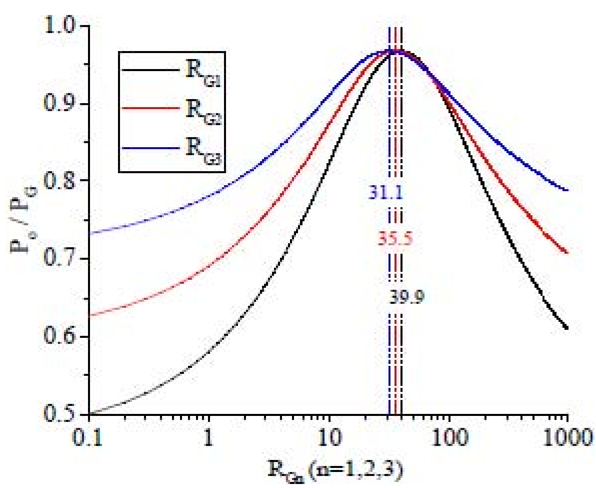

Figure 3. Results obtained by using power sources. The parameters of the link and the values assumed for the powers delivered by the generators are those reported in Tables 1 and 2, respectively. The generators have in series the resistances $R_{\mathrm{G}n}$ and the capacitors $C_{\mathrm{G}n}$ (see Table 2). Results obtained for the output power by varying a generator resistance $R_{\mathrm{G}n}$ at a time, while the others are set to their optimum value.

4. Conclusion

An inductive resonant WPT link with TXs and a single receiver has been analyzed. By modeling the link as an $(N+1)$-port network, the maximum efficiency solution is derived from a generalized eigenvalue problem; the analytical expression of the maximum achievable efficiency and those of the optimal load and generators have been presented. Different solutions for the generators have been discussed (current, voltage, and power sources), and the optimal amplitudes have been derived. It has been demonstrated that for the general case of any couplings between all the resonators of the link, it is necessary to act on both the load and the generators (i.e., input currents/voltages) for achieving the best efficiency.

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