Exact Scattering by Metal Structures With Strips and Right-Angle Wedges

Baker B. Al-Bahri and Piergiorgio L. E. Uslenghi

Abstract – The scattering of four plane waves by a two-dimensional structure consisting of any number of metal strips either parallel or perpendicular to one another is considered in the phasor domain. General conditions for the existence of exact geometrical optics solutions are derived and discussed. Several examples are provided.

1. Introduction

Electromagnetic scattering by metallic structures with sharp edges is important in a variety of practical applications. Although it is often desirable to blunt the scattering by edges, this objective cannot be achieved under incidence by a single plane wave but may be possible when several plane waves with the same frequency, each having an appropriate amplitude, phase, polarization, and direction of incidence, are impinging upon the metallic structure. In such instances, the linear, homogeneous, and isotropic space (e.g., air) surrounding the structure is filled with standing plane waves, the edges do not scatter, and geometrical optics (GO) provide the exact solution to the boundary-value problem. Such a solution has been obtained for a class of metallic wedges [1] and is valid for any frequency of the incident waves, because the wedges do not contain any characteristic length. Additional restrictions involving wavelength, characteristic dimensions, and angles of incidence may have to be imposed on the solution whenever characteristic lengths occur in the structure, as is the case in [2–5] and in this work.

The two-dimensional structures considered herein consist of simply or multiply connected bodies whose boundaries are any number of planar perfect electric conducting (PEC) strips that are parallel to one another and either parallel or perpendicular to the same plane. Under incidence by four appropriate plane waves, the edges of the structure do not scatter. The analysis is conducted in the phasor domain with a time dependence factor $\exp(+j\omega t)$ omitted throughout.

The PEC strips are of infinite length and are oriented in the z-direction of a rectangular coordinate system (x, y, z). Each strip occupies a portion of either the (x, z) plane or the (y, z) plane and is either isolated or in contact at right angle with a neighboring strip or two.

The first step in the analysis (Section 2) consists in considering four plane waves that create a uniform rectangular grid of standing waves. Each element of the grid is a rectangle. The total electric field due to the four waves is perpendicular to the sides of the rectangle and is zero at its vertices, meaning that one or more faces of the rectangle can be metallized without disturbing the field and that the metal edges do not scatter. This result is possible if the periods of the grid in the x and y directions, the wavelength, and the angles of incidence of the primary waves satisfy a couple of relations, which are discussed in detail. The analysis is performed for the two cases of electric field or magnetic field parallel to the z-axis.

If the penetrable space around the metal structure occupies only one, two, or three quadrants, the number of primary plane waves may be reduced to one, two, or three, respectively. The remaining waves needed to form the grid are supplied by the images of the incident waves into the PEC walls of the structure. Several examples are given in Sections 3–5, and some structures requiring four primary waves are discussed in Section 6. A general theorem establishing a sufficient condition for a GO solution to exist is stated in Section 7.

2. Grid Formation

The electric field amplitude of each of the incident plane waves is normalized to 1 V/m. For the case of E polarization (electric field parallel to the z-axis), the electric field of the four incident plane waves is selected as

$$E^i_{1z} = \exp[jk(x\cos\phi_0 + y\sin\phi_0)] \quad (1)$$

$$E^i_{2z} = -\exp[jk(x\cos\phi_0 - y\sin\phi_0)] \quad (2)$$

$$E^i_{3z} = -\exp[-jk(x\cos\phi_0 - y\sin\phi_0)] \quad (3)$$

$$E^i_{4z} = \exp[-jk(x\cos\phi_0 + y\sin\phi_0)] \quad (4)$$

where $k = 2\pi/\lambda$ is the wave number and $0 \le \phi_0 \le \pi/2$.

The related magnetic fields follow from Maxwell's equations. Consequently, the total incident field has components

$$E_z = -4\sin(kx\cos\phi_0)\sin(ky\sin\phi_0) \quad (5)$$

$$H_x = -4jY\sin\phi_0\sin(kx\cos\phi_0)\cos(ky\sin\phi_0) \quad (6)$$

$$H_y = 4jY\cos\phi_0\cos(kx\cos\phi_0)\sin(ky\sin\phi_0) \quad (7)$$

where Y is the intrinsic admittance of the medium surrounding the PEC structures. If the conditions

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B. B. Al-Bahri and P. L. E. Uslenghi are with the Department of Electrical and Computer Engineering, University of Illinois at Chicago, 851 South Morgan Street, Chicago, IL 60607, USA; e-mail: balbah2@uic.edu, uslenghi@uic.edu.

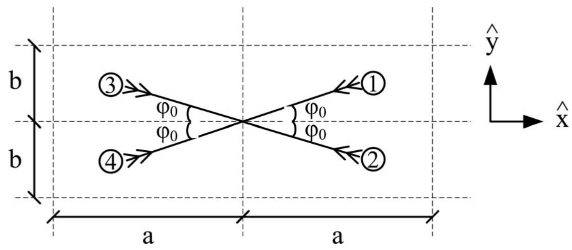

Figure 1. Grid and incident waves.

$$ka\cos\phi_0 = m\pi, \quad (m = 0, 1, 2, ...) \tag{8}$$

$$kb\sin\phi_0 = n\pi, \quad (n = 0, 1, 2, ...) \tag{9}$$

with m and n non-negative integers are imposed, it follows that

$$E_z|_{x=ha} = E_z|_{y=hb} = 0 \tag{10}$$

where $h = 0, \pm 1, \pm 2, \ldots$ is any integer. The entire space is filled with standing waves in the x and y directions, forming a grid of equal rectangles of dimension a in the x direction and b in the y direction. The electric field is perpendicular to the boundaries of all rectangles and is zero at the vertices. Consequently, PEC strips can be inserted at any of the grid boundaries without disturbing the field distribution, and GO is the exact solution to the boundary value problem.

A similar analysis may be performed for H polarization (magnetic field parallel to the z-axis). The magnetic field of the four incident plane waves is

$$H^i_{1z} = Y\exp[jk(x\cos\phi_0 + y\sin\phi_0)] \tag{11}$$

$$H^i_{2z} = Y\exp[jk(x\cos\phi_0 - y\sin\phi_0)] \tag{12}$$

$$H^i_{3z} = Y\exp[-jk(x\cos\phi_0 - y\sin\phi_0)] \tag{13}$$

$$H^i_{4z} = Y\exp[-jk(x\cos\phi_0 + y\sin\phi_0)] \tag{14}$$

and therefore the total incident field has components

$$H_z = 4Y\cos(kx\cos\phi_0)\cos(ky\sin\phi_0) \tag{15}$$

$$E_x = 4j\sin\phi_0\cos(kx\cos\phi_0)\sin(ky\sin\phi_0) \tag{16}$$

$$E_y = -4j\cos\phi_0\sin(kx\cos\phi_0)\cos(ky\sin\phi_0) \tag{17}$$

Under (8) and (9), we have that

$$E_x|_{y=hb} = E_y|_{x=ha} = 0 \tag{18}$$

where $h = 0, \pm 1, \pm 2, ...$ is any integer. As for the case of E polarization, the electric field is perpendicular to the boundaries of all rectangles in the grid; hence, any boundary can be metallized without disturbing the field. A sketch of the grid and incident waves in any plane $z =$ constant is shown in Figure 1.

It follows from the fundamental conditions in (8) and (9) that for given a and b, the incidence angle ϕ_0 depends on the ratio n/m:

$$\tan\phi_0 = \frac{na}{mb} \tag{19}$$

whereas the wavelength depends on both m and n:

$$\lambda = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} \tag{20}$$

Note that (8) and (9) require that

$$\lambda \le 2a, \quad \lambda \le 2b \tag{21}$$

respectively, or equivalently

$$m_{\max} \le \frac{2a}{\lambda}, \quad n_{\max} \le \frac{2b}{\lambda} \tag{22}$$

Hence, for given a and b, an exact GO solution is possible only at sufficiently high frequencies. Furthermore, observe that ϕ_0 and λ are unchanged if a is replaced by a multiple integer of a and b is replaced by a multiple integer of b, provided that m and n are multiplied by those same multiple integers, respectively.

All widths of the metal strips in the x direction must be multiples of a basic length a, and all widths of metal strips in the y direction must be multiples of a basic length b, as conditions for the construction of the grid and the existence of GO solutions. Thus, the choice of a and b is not unique; however, it is desirable to select a and b as large as possible to lower the minimum value of frequency for which GO solutions are possible.

Conditions (8) and (9) are sufficient but may not be necessary, depending on the geometry of the metallic structure. For example, consider the case of any number of metallic strips of arbitrary width and spacing but contained in the same plane $y =$ constant. Only two incident plane waves (either 1 and 2 or 3 and 4 in Figure 1) that are the image of each other with respect to the plane containing the strips are needed to satisfy the boundary conditions; (8) and (9) are not needed, and no restrictions need to be imposed on ϕ_0 and λ for a GO solution to exist. There are innumerable structures that can be analyzed via the grid developed herein; some of them are shown as examples in the following sections.

3. One-Quadrant Structures

If the metallic structure, with or without ridges and trenches, occupies the entire half planes $(x \ge 0, y = 0)$ and $(x = 0, y \ge 0)$ without gaps, the penetrable medium in contact with the structure occupies only the first quadrant $(x \ge 0, y \ge 0)$ of space, plus eventual trenches of finite depth. We refer to such structures as *one-quadrant* structures for which only the primary wave 1 of Figure 1 is needed, with the other three waves in Figure 1 being reflections of wave 1 on the metal walls.

The simplest structure of this type is the dihedral reflector that has no characteristic lengths and therefore

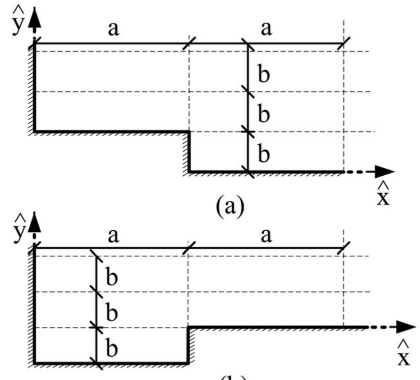

Figure 2. (a) Step and (b) trench in a dihedral reflector.

no restrictions on ϕ_0 and λ for a GO solution to exist. If either a rectangular step or trench is inserted at the reflector's corner, then (8) and (9) apply; a suitable choice of the lengths a and b is shown in Figure 2, with the grid boundaries shown as broken lines.

If the parameter a tends to zero or to infinity or the parameter b tends to zero, then both step and trench in Figure 2 vanish and one reverts to the simple dihedral reflector. If b tends to infinity, the step belongs to a two-quadrant structure (Section 4), but the trench becomes a parallel plate waveguide of semi-infinite length, and the field reflected at the bottom of the trench becomes a mode propagating in the $+\hat{y}$ direction inside the guide. This is obviously a limiting case between existence and nonexistence of gaps in the PEC structure. For both step and trench, only (8) is needed when b goes to infinity.

As another example, consider a dihedral reflector whose faces consist of uniform arrays of square ridges and trenches of equal width and depth, as shown in Figure 3. The grid consists of squares of size a, the incidence angle $\phi_0 = \arctan(n/m)$, and the wavelength $\lambda = 2a/\sqrt{m^2 + n^2}$.

4. Two-Quadrant Structures

If the PEC structure, with ridges and trenches of finite height, occupies the entire plane $y = 0$, only the primary waves 1 and 3 of Figure 1 are needed for the possible existence of exact GO solutions, with the other two waves 2 and 4 being reflections of 1 and 3 into the metal plane. The simplest such two-quadrant structures are the step in a metal plane and the strip perpendicular to a metal plane, both of height b, as shown in Figure 4. For these structures, only condition (9) is needed. The scattering of a simple plane wave incident at an arbitrary angle on the strip of Figure 4 has been solved exactly in terms of an infinite series of elliptic-cylinder wave functions [6]. By combining two such solutions and imposing (9), it can be verified that an exact GO solution is obtained.

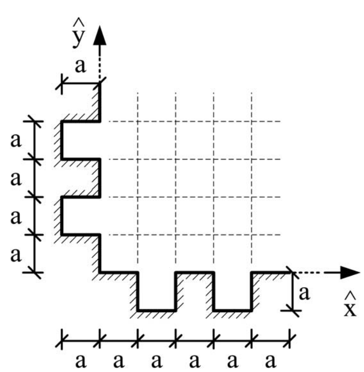

Figure 3. Corrugated dihedral reflector.

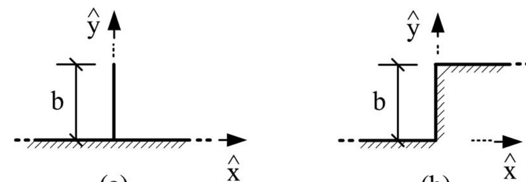

Figure 4. (a) Strip and (b) step on a metal plane.

Some other simple two-quadrant structures that require (8) and (9) are shown in Figure 5. If the depth b of the trench tends to infinity, the remarks made for Figure 2 apply.

A peculiar structure is a finite or infinite array of equally spaced half planes $(y \leq 0)$ parallel to the y-axis with edges in the plane $y = 0$. The two incident waves 1 and 3 of Figure 1 and (8) assure the existence of an exact GO solution. The two waves combine into a mode propagating in the $-\hat{y}$ direction between any two half planes, and waves 2 and 4 of Figure 1 are not needed. If the half planes were thick instead of infinitesimally thin and truncated, so as to have two right-angle wedges each, then a mode propagating in the $+\hat{y}$ direction would also have to be present to have waves 2 and 4 incident on the right-angle wedges of the thick half planes. This structure can be analyzed exactly under a single incident plane wave via the Wiener–Hopf method (see, e.g., [7]). By combining two such solutions, the existence of an exact GO solution can be verified. Strictly speaking, an infinite array of half planes belongs to two-quadrant structures, a semi-infinite array to three-quadrant structures, and a finite array to four-quadrant structures.

5. Three-Quadrant Structures

In all configurations where the quadrant $(x < 0, y < 0)$ is excluded, the three primary waves 1, 2, and 3 of

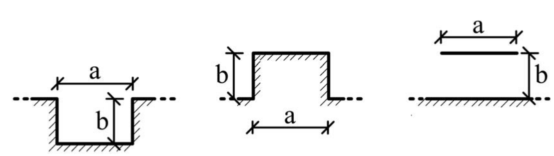

Figure 5. (a) Rectangular trench, (b) rectangular ridge, and (c) strip on a metal plane.

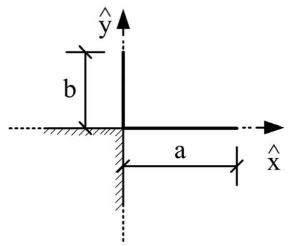

Figure 6. Right-angle wedge with two metal baffles.

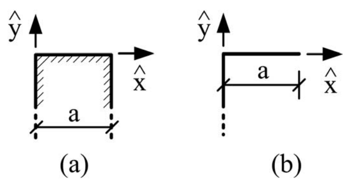

Figure 7. (a) Thick half plane and (b) bent half plane.

Figure 1 are needed to ensure the existence of a GO solution, with the image wave 4 being the reflection of waves 2 and 3 on the surfaces ($x = 0, y < 0$) and ($x < 0, y = 0$), respectively. The simplest three-quadrant structure is the right-angle wedge, previously discussed in [1] and for which no restrictions on ϕ_0 and λ are needed; if one or two metal baffles are present (Figure 6), then (8) or (9) or both are required. A variety of more complicated three-quadrant structures may be created by inserting rectangular ridges and trenches, as well as metal strips, on the faces of a right-angle wedge.

6. Four-Quadrant Structures

Aside from the half plane, the simplest four-quadrant structures are the thick half plane and the bent half plane, shown in Figure 7. These structures only require (8) to yield exact GO solutions under incidence of all four waves in Figure 1. Also, they are amenable to an exact solution under a single incident plane wave, via the Wiener–Hopf function theoretic method (see, e.g., [7]), meaning that repeated application of the Wiener–Hopf method to all four primary waves and summation of the four solutions leads to the GO solution.

Simple structures requiring both (8) and (9) are the bent strip and the rectangular prism, shown in Figure 8. Innumerable other four-quadrant structures, with simply or multiply connected cross-sectional shapes in any plane $z =$ constant, can yield exact GO scattering solutions under excitation by four waves,

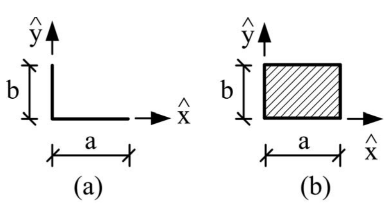

Figure 8. (a) Bent strip and (b) rectangular prism.

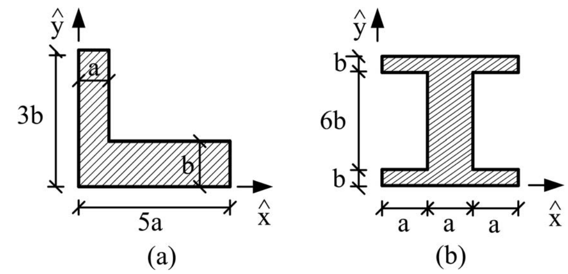

Figure 9. (a) L and (b) I beams.

provided that all linear dimensions in the same direction are commensurable to one another (i.e., the ratio of any two dimensions equals the ratio of two integer numbers). Examples of such structures are the L and I beams of Figure 9.

7. Conclusion

A general procedure has been developed for obtaining exact GO scattering solutions under incidence by four primary or imaged plane waves impinging upon a two-dimensional structure consisting of simply or multiply connected PEC strips either parallel or perpendicular to one another. From the analysis of the previous sections, the following theorem can be stated:

A sufficient condition for the existence of an exact GO solution to the scattering of four primary or imaged plane waves of sufficiently high frequency by a two-dimensional structure consisting of plane PEC strips either parallel or perpendicular to one another is that the ratio of any two characteristic dimensions in the same direction be equal to the ratio of two integer numbers.

The two-dimensional results obtained in this work may be extended to oblique incidence of the primary waves with respect to the z-axis and truncation of the structure with a PEC plane perpendicular to the z-axis, by following the general procedure described in [8].

This work is part of the doctoral dissertation of the first author conducted under the guidance of the second author [9].

This work is important for two reasons: it provides novel canonical solutions to scattering problems, and it may be helpful in validating complicated analytical solutions and commercially available computer solvers.

8. References

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