Particle Swarm Optimization of Layered Media Cloaking Performance

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Abstract – Scattering problems are considered for the excitation of a layered spherical medium by an external dipole. The purpose of this work is to determine suitable parameters of the dielectric layers covering a perfectly conducting core so that the scattered far field is significantly reduced for a wide range of observation angles. A particle swarm optimization algorithm is developed and applied to the associated optimization problem. Numerical results exhibiting reduced values of the scattering cross section for realizable coating parameters are presented. The effect of the dipole's location on the cloaking performance is analyzed.

1. Introduction

Optimizations of the characteristic input parameters of devices, used in several applications of electromagnetics and photonics, are of primary importance for achieving required operational characteristics with respect to prescribed field variations. In this context, obtaining exact or semianalytic solutions for the involved electromagnetic scattering and radiation problems is significant for achieving fast and efficient optimization schemes. Such solutions can be exploited as suitable objective functions in optimization problems concerning the determination of the physical and geometrical parameters of the configuration under examination. Considering, in particular, the efficient design of layered media exhibiting desired far-field patterns, relevant optimizations involve the determination of the layers widths, as well as their permittivities and permeabilities. For achieving cloaking behavior of spherically layered media, aspects of related optimization problems have been investigated in [1–9] and mainly focus on plane incident waves. An overview of optimization techniques for the design of various metadevices is presented in [10].

In this work, we present numerical optimization results for reduction in the scattering cross section of a spherical medium containing a perfect electric conducting (PEC) core covered by a small number of dielectric layers. The primary excitation is due to an external magnetic dipole. When the dipole lies sufficiently far from the medium, approximations of the scattering performance due to a plane incident wave are obtained. The optimization variables are the radii, the permittivities, and the permeabilities of the spherical layers, while the core's radius is kept constant. An evolutionary algorithm on the basis of the particle swarm optimization (PSO) is developed and used for the determination of suitable values of the optimization variables yielding reduced scattered far-field contributions. The exact solution in the form of a Mie series of the considered scattering problem is crucial for the fast and efficient implementation of the PSO algorithm in the present setting. It is shown that the scattered far field can be reduced for a wide range of observation angles for coatings composed of two up to five dielectric layers occupied by ordinary materials. The obtained optimal designs exhibit efficient cloaking performance for different locations of the primary dipole.

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2. The Scattering Problem

A layered spherical medium V with radius a_1 is excited by an external magnetic dipole, with position vector $\mathbf{r}_0$ on the z-axis and dipole moment along the direction $\hat{\mathbf{y}}$. An arbitrary dipole can also be considered and treated by similar techniques with those presented in the following. The interior of V is divided by $P-1$ concentric spherical interfaces $r = a_p$ $(p = 2, \ldots, P)$ into $P-1$ homogeneous dielectric layers V_p $(p = 1, \ldots, P-1)$, consisting of materials with real relative dielectric permittivities ε_p and magnetic permeabilities μ_p, and surrounding a PEC core (layer V_P). The exterior V_0 of V has permittivity ε_0, permeability μ_0, and wavenumber k_0. A schematic of the scattering problem is shown in [11, fig. 1].

The exact solution of the scattering problem is determined analytically by applying a combined Sommerfeld T-matrix methodology, which is developed and analyzed in [11, 12]. More precisely, the electric fields in each region of the problem are decomposed into primary and secondary components, which are then expressed as series of the spherical vector wave functions [13, 14]. The unknown coefficients in the expansions of the secondary fields are determined analytically by imposing the transmission boundary conditions on the interfaces of the spherical shells and applying a T-matrix method. By this methodology, the following expressions are obtained for the bistatic (differential) scattering cross section and the total scattering cross section, respectively,

$$\sigma(\theta, \phi; \mathbf{r}_0) = \frac{4\pi}{k_0^2}\left[|S_\theta(\theta; \mathbf{r}_0)|^2\cos^2\phi + |S_\phi(\theta; \mathbf{r}_0)|^2\sin^2\phi\right],$$

$$\sigma^{\mathrm{t}}(\mathbf{r}_0) = \frac{1}{4\pi}\int_{S^2}\sigma(\theta,\phi;\mathbf{r}_0)\mathrm{d}s(\hat{\mathbf{r}}) = \frac{2\pi}{k_0^2}\sum_{n=1}^{\infty}(2n+1)\left[|\gamma_n|^2+|\delta_n|^2\right]$$

where S^2 denotes the unit sphere in $\boldsymbol{R}^3$, and

$$S_\theta(\theta;\mathbf{r}_0) = \sum_{n=1}^{\infty}\frac{(-1)^n(2n+1)}{\sqrt{n(n+1)}} \times\left[\delta_n\frac{P_n^1(\cos\theta)}{\sin\theta}-\gamma_n\frac{\partial P_n^1(\cos\theta)}{\partial\theta}\right],$$

$$S_\phi(\theta;\mathbf{r}_0) = \sum_{n=1}^{\infty}\frac{(-1)^n(2n+1)}{\sqrt{n(n+1)}} \times\left[\gamma_n\frac{P_n^1(\cos\theta)}{\sin\theta}-\delta_n\frac{\partial P_n^1(\cos\theta)}{\partial\theta}\right]$$

with P_n^1 the first-order Legendre function of degree n, and

$$\gamma_n = \frac{h_n(k_0r_0)}{h_0(k_0r_0)}\mathrm{i}^n\alpha_n, \quad \delta_n = \frac{\hat{h}'_n(k_0r_0)}{\hat{h}_0(k_0r_0)}\mathrm{i}^{n-1}\beta_n,$$

where h_n is the spherical Hankel function of order n, $\hat{h}_n(z) = zh_n(z)$, while α_n and β_n are defined in [11].

3. The PSO Algorithm

The PSO is an evolutionary optimization algorithm on the basis of biological behaviors that can be observed in real life. Such biological mechanisms usually have to do with how a population of a specific species (e.g., a swarm, herd, or a pride) acts in unison, changes in time, and solves problems. The related algorithms are required to handle and organize the mentioned populations, using metaheuristic methods (i.e., self-training methods) to simulate a specific biological evolutionary mechanism.

In PSO, the population evolving in time is a group of particles (points with no mass), which move in $\boldsymbol{R}^n$ (n being the number of variables of the problem's objective function) searching for solutions representing a maximum (or minimum). Biologically, this is interpreted as particles seeking the greatest path toward "food". Each particle represents a possible set of variables that serve as a potential solution to the optimization problem.

The two main characteristics of each particle are its position $\mathbf{p}$ and its velocity $\mathbf{u}$. These describe the current state and the short-term evolution of the particle. The n-dimensional vector $\mathbf{p}$ includes the variables of the objective function, which are allowed to have continuous variations in specific chosen intervals. The components of the n-dimensional vector $\mathbf{u}$ describe the movement of the particle. Each particle holds a memory "spot" for its best attained position $\mathbf{p}_{\text{best}}$ alongside with the obtained value $\mathbf{g}_{\text{best}}$ at this position. Additionally, the swarm, as a whole, keeps track of the best position $\mathbf{P}_{\text{best}}$ per all iterations with its corresponding value $\mathbf{G}_{\text{best}}$. These two types of memory allude to swarm particles having both *cognitive* and *social* learning taking place as the algorithm runs. Note that the swarm has three fundamental characteristics: 1) cohesion (particles move as a swarm and not completely independently); 2) separation (particles do not merge with other particles or hinder them); and 3) alignment (particles have common cause and judgment):

Algorithm 1: PSO
Input: N, l, u, c_1, c_2, $i_{\max}$
Output: A swarm S of size N with each current position(s)
Initialize S with random values for the position of each particle with regard to the domain of the objective function;
Initialize all velocities $\mathbf{u}$ to zero;
Initialize best positions (and respective values) both for individual particles and S;
Choose randomly $r_1, r_2 \in [0, 1]$;
Iteration $i = 0$;
Initialize $\theta_{\min}, \theta_{\max}$;
While $i < i_{\max}$ do
Calculate inertia: $\theta = \theta_{\max} - \frac{\theta_{\max}-\theta_{\min}}{i_{\max}} i$;
For each particle in S:
  Update velocity: $\mathbf{u}(i) = \theta\,\mathbf{u}(i-1) + c_1 r_1 [\mathbf{p}_{\text{best}} - \mathbf{p}(i-1)] + c_2 r_2 [\mathbf{P}_{\text{best}} - \mathbf{p}(i-1)]$;
  Update position: $\mathbf{p}(i) = \mathbf{p}(i-1) + \mathbf{u}(i)$;
  Check/update: $\mathbf{p}_{\text{best}}$, $\mathbf{g}_{\text{best}}$;
Upgrade $\mathbf{P}_{\text{best}}$ and $\mathbf{G}_{\text{best}}$;
Upgrade iteration: $i = i + 1$;
(Optional) check for convergence;
end
Return S;

Moreover, there is an inertia mechanism added to the particles movement [15, 16]. Inertia may improve the results by slowing down particles that move too fast and, hence, might miss an optimum. The developed algorithm is presented in a pseudocode form, where l and u are the lower and upper limits of the positions domain, $\theta_{\min} = 0.4$ and $\theta_{\max} = 0.9$, and c_1 and c_2 are the cognitive and social learning rates (both set to 2 here).

4. Implementation of PSO to the Scattering Problem and Numerical Results

The objective function we consider in the optimization schemes is the *normalized total scattering cross section* $\sigma^{\mathrm{t}}(\mathbf{r}_0)/(\pi a_{\text{PEC}}^2)$ (instead of the backscattering cross section, as in a previous work [17]), where a_{PEC} is the radius of the PEC sphere to be cloaked when covered by suitable coating layers.

The previously described PSO algorithm was implemented in MATLAB R2018a. The swarm was a MATLAB struct for which we followed the steps in

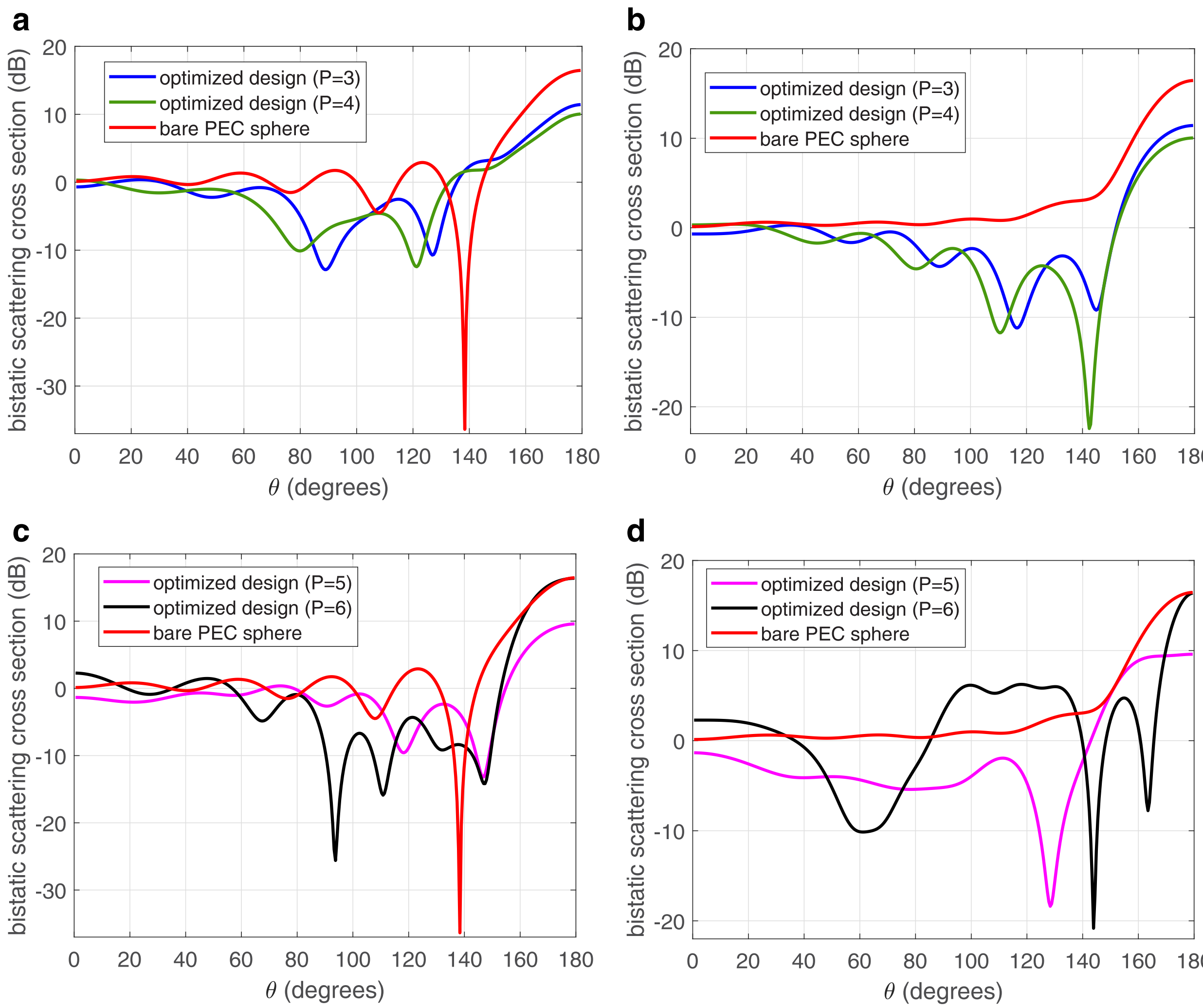

Figure 1. Normalized bistatic cross sections versus the angle θ for (a) and (b) $P=3$, 4 and (c) and (d) for $P=5$, 6 optimized layers with parameters computed by the PSO algorithm; (a) and (c) and (b) and (d) refer to the xOz and the yOz planes, respectively. The dipole lies at $r_0=10$ a_{PEC}; thus, the cross sections approximate those due to plane wave incidence.

Algorithm 1. The chosen N for the experiments was 20, and the iterations were 800. The components of the position vector consisted of the optimization variables a_p of the radii, ε_p of the permittivities, and μ_p of the permeabilities of the first $P-1$ dielectric layers. The radius a_P of the PEC core was chosen constant at $k_0 a_P = k_0 a_{\mathrm{PEC}} = 2\pi$ (one free space wavelength). Thus, for a medium with P layers, the number of optimization variables for the particles position is $3(P-1)$. We focused on experiments with small values of P for potentially easier realization of the coating. The differences $k_0(a_{p+1} - a_p)$ between two consecutive layers radii were considered in two different ranges: $\left[\frac{\pi}{10}, \pi\right]$, $\left[\frac{\pi}{10}, \frac{\pi}{2}\right]$, while the values of ε_p and μ_p in [0.5,10], [0.5,3]. For the numerical solution of the scattering problem, we used the code developed in [11], which is valid for an arbitrary number P of layers.

First, the distance of the dipole from the scatterer was taken $r_0 = 10\ a_{\mathrm{PEC}}$, for which case the obtained far field is close to the one due to plane wave incidence; see [11]. We applied the developed PSO algorithm to minimize the normalized total cross section for a spherical medium with $P=3$, 4, 5, and 6 total number of layers, respectively. The actual reduction in the far field with respect to the angles of observation is demonstrated in Figure 1, depicting the normalized bistatic scattering cross sections $\sigma(\theta, \phi; \mathbf{r}_0)/(\pi a_{\mathrm{PEC}}^2)$ versus the angle θ in the xOz and yOz planes. Reduced far-field contributions with respect to the bare PEC sphere are observed for large ranges of the observation angles; in fact, $P=3$, 4 offer reduced values for a wider angular range than $P=5$, 6.

Then, we considered optimizations for dipoles lying close to the scatterer. Particularly, we fixed the dipole at $r_0 = 1.2\ a_1$ and applied the PSO algorithm to compute the coating's parameters.

Figure 2 depicts the normalized bistatic scattering cross sections versus θ in the xOz and yOz planes for the

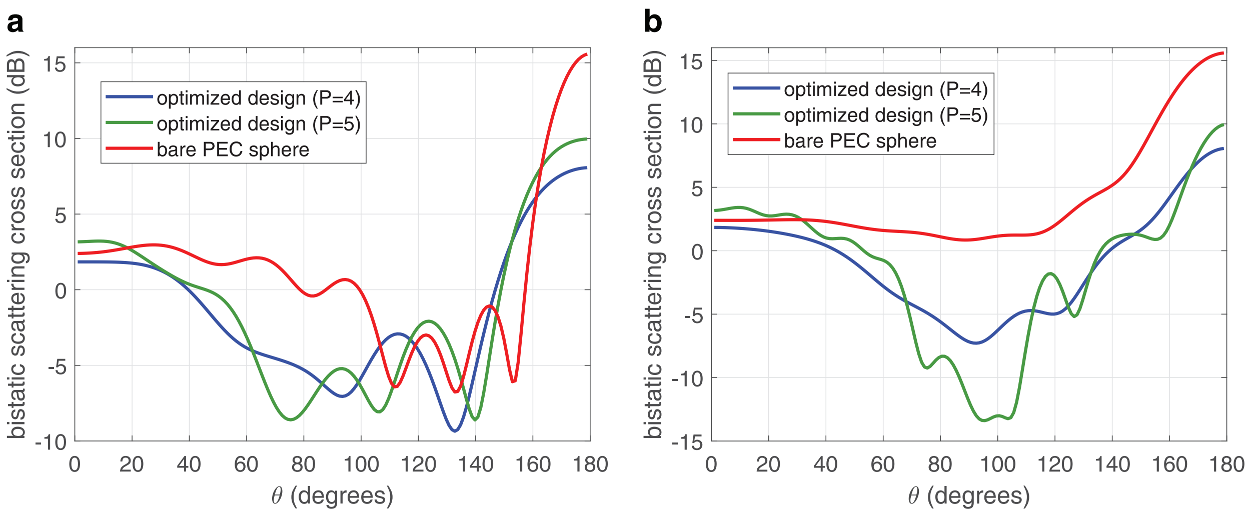

Figure 2. As in Figure 1, except the dipole at $r_0 = 1.2a_1$.

two previously described spheres. As in the plane wave case of Figure 1, also here, notable reduction in the bistatic cross section is evident for a wide range of observation angles (approximately 80% and 90% of the angles in the xOz and yOz planes, respectively).

Next, we examine the effectiveness of the cloaking performance for the determined set of coating parameters as we change the dipole's location. To this end, we retain the same coating parameters for which cross-section reduction was obtained for $r_0 = 1.2a_1$ (cf. Figure 2) but change the dipole's distance from the center of the sphere by 10%, namely, consider that $r_0 = 1.1a_1$ and $r_0 = 1.3a_1$. The respective normalized bistatic cross sections versus θ are depicted in Figure 3. It is concluded that the parameters offering significant cloaking for $r_0 = 1.2a_1$ work very well for both $r_0 = 1.1a_1$ and $r_0 = 1.3a_1$.

Finally, we test the sensitivity of the results with respect to inevitable fabrication imperfections. In Figure 4, we show the variations of the cross sections for the optimal design with $P = 4$ of Figure 1, when the radius k_0a_2 of the second layer is perturbed by $\pm$ 2% from the optimal value. The cloaking performs equally well in the yOz plane and exhibits a small deterioration in the xOz plane.

5. Conclusions and Prospects

A PSO algorithm was developed and applied for optimizing the permittivities, permeabilities, and thicknesses of the spherical layers surrounding a PEC core so that the scattering cross section is significantly reduced. The primary field was due to a magnetic dipole. Numerical results showed that the bistatic cross section was notably reduced for around 80% of the observation angles and for various dipole's locations. The optimal

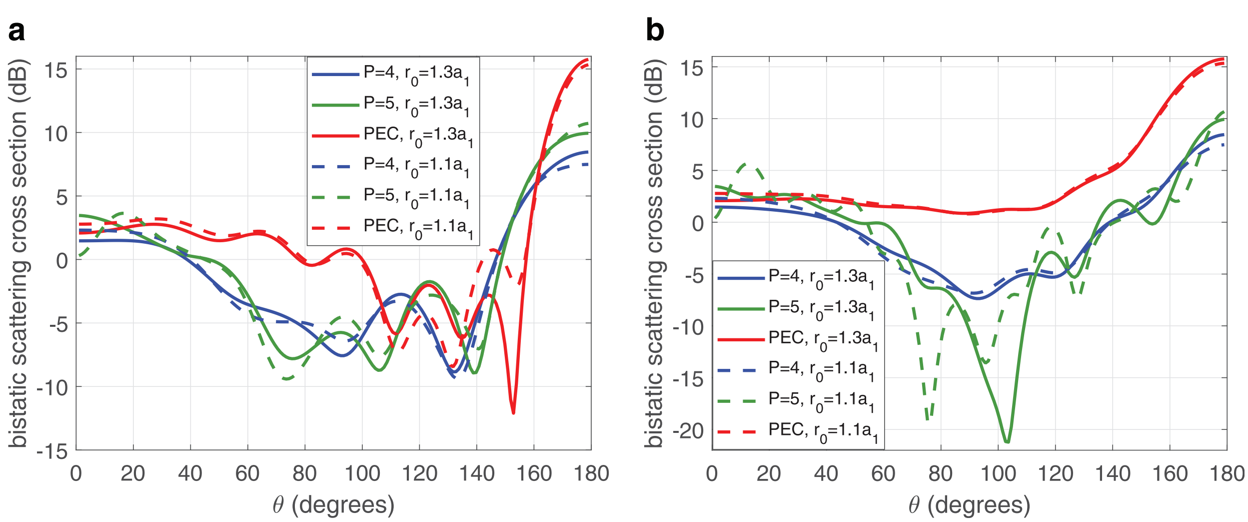

Figure 3. As in Figure 2, except $r_0 = 1.1a_1$ and $r_0 = 1.3a_1$.

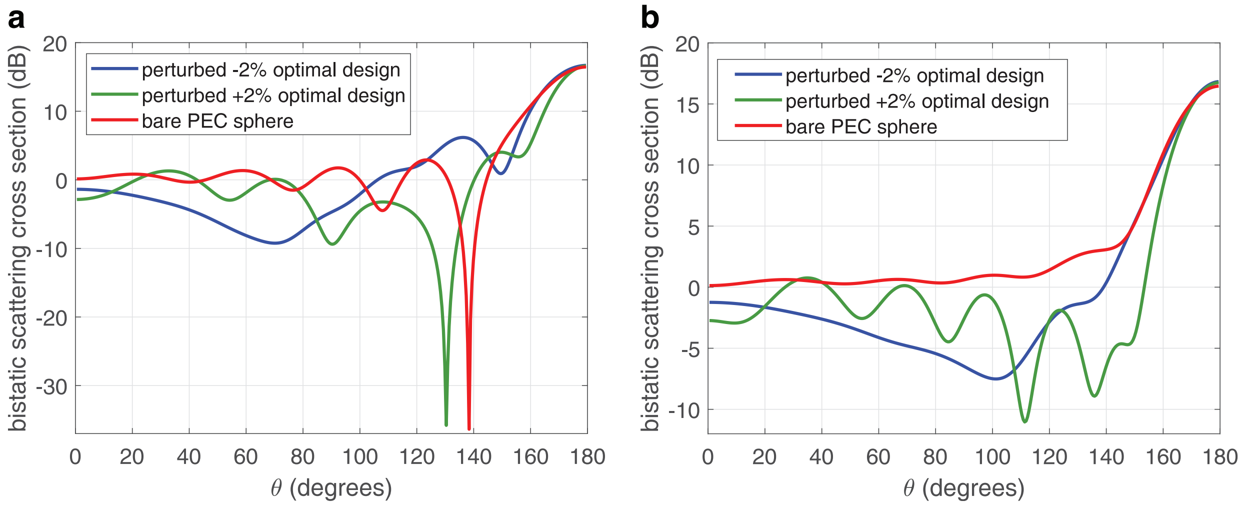

Figure 4. Cloaking performance of the design of Figure 2 for $P = 4$ after perturbing the inner radius k_0a_2 by $\pm 2\%$.

coating parameters can be realized by ordinary materials without resulting to epsilon-, mu-near-zero, single-, double-negative or plasmonic (meta)materials. The total number of layers was chosen to be small for easier fabrication. Deterioration of the cloaking due to material losses needs also to be tested. Extensions to spherical antennas [18] and inhomogeneous media [19] are also expected.

6. References

1. A. Alù and N. Engheta, "Achieving Transparency with Plasmonic and Metamaterial Coatings," *Physical Review E*, **72**, 1, July 2005, 016623.
2. A. Alù and N. Engheta, "Plasmonic Materials in Transparency and Cloaking Problems: Mechanism, Robustness, and Physical Insights," *Optics Express*, **15**, 6, March 2007, pp. 3318-3332.
3. A. Alù and N. Engheta, "Multifrequency Optical Invisibility Cloak with Layered Plasmonic Shells," *Physics Review Letters*, **100**, 11, March 2008, 113901.
4. C.-W. Qiu, L. Hu, B. Zhang, B.-I. Wu, S. G. Johnson, and J. D. Joannopoulos, "Spherical Cloaking Using Nonlinear Transformations for Improved Segmentation Into Concentric Isotropic Coatings," *Optics Express*, **17**, 16, August 2009, pp. 13467-13478.
5. G. Castaldi, I. Gallina, V. Galdi, A. Alù, and N. Engheta, "Analytical Study of Spherical Cloak/Anti-Cloak Interactions," *Wave Motion*, **48**, 6, September 2011, pp. 455-467.
6. T. C. Martins and V. Dmitriev, "Spherical Invisibility Cloak with Minimum Number of Layers of Isotropic Materials," *Microwave and Optical Technology Letters*, **54**, 9, June 2012, pp. 2217-2220.
7. X. Wang, F. Chen, and E. Semouchkina, "Spherical Cloaking Using Multilayer Shells of Ordinary Dielectrics," *AIP Advances*, **3**, 11, November 2013, 112111.
8. K. Ladutenko, O. Peña-Rodriguez, I. Melchakova, I. Yagupov, and P. Belov, "Reduction of Scattering Using Thin All-Dielectric Shells Designed by Stochastic Optimizer," *Journal of Applied Physics*, **116**, 18, November 2014, 184508.
9. A. Sheverdin and C. Valagiannopoulos, "Core-Shell Nanospheres Under Visible Light: Optimal Absorption, Scattering, and Cloaking," *Physical Review B*, **99**, 7, February 2019, 075305.
10. S. D. Campbell, D. Sell, R. P. Jenkins, E. B. Whiting, J. A. Fan, and D. H. Werner, "Review of Numerical Optimization Techniques for Meta-Device Design," *Optical Materials Express*, **9**, 4, April 2019, pp. 1842-1863.
11. N. L. Tsitsas and C. Athanasiadis, "On the Scattering of Spherical Electromagnetic Waves by a Layered Sphere," *Quarterly Journal of Mechanics and Applied Mathematics*, **59**, 1, February 2006, pp. 55-74.
12. P. Prokopiou and N. L. Tsitsas, "Electromagnetic Excitation of a Spherical Medium by an Arbitrary Dipole and Related Inverse Problems," *Studies in Applied Mathematics*, **140**, 4, March 2018, pp. 438-464.
13. J. J. Bowman, T. B. Senior, and P. L. Uslenghi, *Electromagnetic and Acoustic Scattering by Simple Shapes*, Amsterdam, North-Holland, 1969.
14. C.-T. Tai, *Dyadic Green Functions in Electromagnetic Theory*, Piscataway, IEEE Press, 1994.
15. S. S. Rao, *Engineering Optimization: Theory and Practice*, Hoboken, Wiley, 2009.
16. Y. Shi and R. C. Eberhart, "Parameter Selection in Particle Swarm Optimization," Proceedings of the Seventh Annual Conference on Evolutionary Programming, EP98, San Diego, California, USA, March 25–27, 1998, pp. 591-600.
17. Z. Tsitsoglou, P. Prokopiou, and N. L. Tsitsas, "Dipole-Scattering by Spherical Media and Related Optimization Problems," presented in the 2nd URSI Atlantic Radio Science Meeting, Meloneras, Spain, 28 May–1 June 2018.
18. C. A. Valagiannopoulos and N. L. Tsitsas, "On the Resonance and Radiation Characteristics of Multi-Layered Spherical Microstrip Antennas," *Electromagnetics*, **28**, 4, May 2008, pp. 243-264.
19. C. A. Valagiannopoulos and N. L. Tsitsas, "Linearization of the T-Matrix Solution for Quasi-Homogeneous Scatterers," *Journal of the Optical Society of America A*, **26**, 4, April 2009, pp. 870-881.