

# Nonlinear Wave Growth Analysis of Chorus Emissions modulated by field line resonance and mirror-mode ULF waves

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#### Abstract

Previous studies have found that chorus waves can be generated in the troughs of the compressional ULF waves. Here, we report for the first time the periodic excitation of chorus waves near ULF wave crests, which is attributed to different modes of the observed ULF waves. We demonstrate that latitudinal profile of the ULF waves can play important roles in excitation of chorus waves on the basis of nonlinear generation theory of chorus emissions. Field line resonance (FLR) mode results in chorus wave excitation near ULF wave troughs, while the mirror mode causes chorus near the wave crest. Chorus wave occurrence near the ULF wave crests is attributed to the antisymmetric field profiles of mirror-mode ULF waves, which periodically modulate the threshold amplitude by modifying the second-order derivative of the background dipole field. FLR ULF wave with the symmetric profile of magnetic field with respect to the equator fosters chorus wave excitation near ULF wave trough. The good agreement between the theory and the observations highlights the effects of ULF wave field configuration in modulating chorus waves.

#### 1 Introduction

ULF waves in the mHz frequency range play an important role in accelerating and transporting charged particles that contribute to the formation of Van Allen radiation belts [1,2]. Whistler-mode chorus waves are known to facilitate the pitch-angle scattering of electrons over a broad energy range, which results in their atmospheric precipitation and consequently, auroral activities [3,4,5].

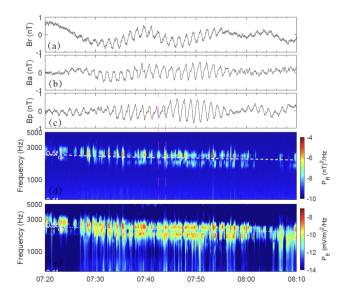
Chorus waves often coexist with, and are modulated by ULF waves [6,7]. Previous studies have found that chorus waves can be generated in the troughs of the ULF waves. Previous work [8] demonstrated that this phenomenon can be alternatively explained by the periodic variations of the magnetic field configurations. The nonlinear theory is used to investigate the role of the compressional ULF waves on chorus wave generation, in which the wave growth enters the nonlinear phase if its amplitude becomes greater than the threshold amplitude  $\Omega_{th}$  and stops near the optimum wave amplitude  $\Omega_{op}$  [9]. The ULF waves can regulate the

second-order spatial derivative of the magnetic field, which in turn leads to the  $\Omega_{th}$  variations.

This work directly utilizes the nonlinear theory of chorus wave excitation [8] to explain and compare the observed modulation of the chorus waves by FLR and mirror mode ULF waves. Moreover, different ULF wave structures lead to different first-order spatial derivatives of the compressional magnetic field, which then affect the inhomogeneity factor S for nonlinear growth of chorus waves by adjusting the background magnetic field gradients.

### 2 Observations

We first consider a case where chorus emissions are observed near the FLR ULF wave troughs by RBSP-A on 11 June 2016. Figures 1 summarizes these observations, presenting (a) the radial  $(B_r)$ , (b) azimuthal  $(B_a)$ , (c) parallel  $(B_{\mu})$  magnetic fields in the MFA coordinates [10], which oscillate at the period of ~120 s, and (d-e) the power spectra of the magnetic field and electric field oscillations in the frequency range between 200 and 5000 Hz, respectively, which show periodic occurrence of the chorus waves for almost 1 hour.

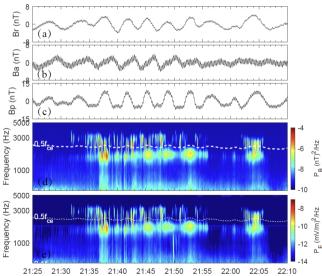


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**Figure 1.** Whistler-mode chorus waves modulated by field line resonance ultralow frequency (ULF) waves observed by Van Allen Probe A on 11 June 2016. (a) The radial magnetic field  $B_r$ . (b) The  $B_a$  oscillations. (c) The compressional magnetic field  $B_{\mu}$ . (d-e) Power spectrum density of chorus wave field.

Rising tones can be observed in high-resolution burst mode data (not shown), which provides strong observational support for nonlinear chorus wave generation mechanism. A more careful examination of the wave modulation indicates that the chorus waves are mostly confined to the troughs of the  $B_{\mu}$  pulsations (e.g., see the vertical dashed line). The resonant behavior of energetic electrons and the nature of standing waves are consistent with the theory of FLR [11,12].

We next report in Figure 2 another event on 01 January 2016, which appears to be very different from Figure 1. The magnetic fields in the MFA coordinates shown in Figures 2a-2c oscillate at the period of  $\sim 200$  s. Figures 2d and 2e present the power spectra of the higher-frequency magnetic field oscillations with the dashed lines denoting  $0.5f_{ce}$  (electron cyclotron frequency). Obviously, both the upper-band and lower-band chorus waves turned on and off periodically with a period identical to the ULF wave period. A closer examination shows that the upper band chorus waves occur mostly at the troughs of the compressional ULF waves, and the amplitude of the lower band chorus waves observed in this case to the drift mirror modes since the drift mirror mode instability condition is well satisfied.



**Figure 2.** The same format as Figure 1 except that whistlermode chorus waves are modulated by drift mirror ULF waves.

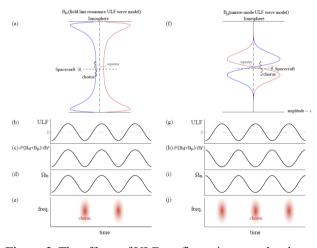
#### 3 Modeling and Analysis

We next utilize the nonlinear theory of chorus wave excitation [8] to explain the different observational features in two typical ULF wave modes. Here, the threshold wave amplitude  $\Omega_{th}$ , after considering the contribution of the ULF wave compressional wave field  $B_{\mu}$ , can be expressed as

$$\Omega_{th} = f(\omega, \omega_{pe}/\omega_{ce}, n_h, V_{t\parallel}, V_{t\perp}) \left(\frac{\partial^2 (B_d + B_\mu)}{\partial h^2}\right)^2.$$
(1)

where the term  $f(\omega, \omega_{pe}/\omega_{ce}, n_h, V_{t\parallel}, V_{t\perp})$  denotes a function varying with chorus wave frequency  $\omega$ , the ratio of electron plasma frequency to cyclotron frequency  $\omega_{pe}/\omega_{ce}$ , hot electron density  $n_h$ , and the parallel and perpendicular components  $V_{t\parallel}$  and  $V_{t\perp}$  of the electron thermal velocity (see [9] for the form of the *f* function). The term in brackets on the right-hand side of Equation (1) is the second-order derivatives of the background dipole magnetic field  $B_d$  and compressional wave magnetic field  $B_{\mu}$  with respect to the distance *h* along the magnetic field line.

Figure 3a shows the ULF wave magnetic field as a function of latitude in the near-equatorial region in the FLR model [12], which shows the symmetrical wave field with respect to the equator. Here, the relevant parameters in the model are determined based on the observations in Figure 1. Substituting the second-order derivatives of  $B_d$  and  $B_{\mu}$ from the modeled ULF wave field, we obtain the threshold amplitude in Figure 3d. It is the in-phase correlation that reduces the threshold amplitude and favors the nonlinear growth of the chorus emissions (see Figure 3e) near the troughs of the ULF waves, which explains the spacecraft observations in Figure 1.



**Figure 3.** The effects of ULF configuration over the chorus wave generation. (a) The compressional magnetic field components in modeled ULF waves with field line resonance structure [12]. The colored lines indicate the temporal evolution of ULF waves within one complete wave cycle (i.e., ULF wave crests and troughs). The variation of (b)  $B_{\mu}$ , (c) the second spatial derivative of  $B_{\mu}$  and (d) threshold amplitude for nonlinear growth of the chorus waves. (e) The power spectrum of chorus wave magnetic fields. (f-j) The same as (a-e) but the modeled drift mirror mode ULF waves with the antisymmetric profile with respect to the equator [13] (refer to [14]).

The chorus wave excitation in drift mirror ULF waves could be very different from the above scenario, since the associated compressional field is antisymmetric with respect to the equator (see Figure 3f, using a drift mirror ULF wave model [13]) resulting in antiphase relationship between  $B_{\mu}$  and second-order derivatives (compare Figures 3g and 3h), which in turn leads to antiphase relationship between  $B_{\mu}$  and threshold amplitude, which means that the chorus wave power (Figure 3j) peak in the crest of  $B_{\mu}$  signals. These calculations show good agreement with Van Allen Probes observations in Figure 2.

## 4 Conclusions

Modulation of one plasma wave mode by another has been an important topic in studies of radiation belt physics over the past years. A typical example is the periodic generation of whistler-mode chorus waves modulated by ULF waves.

In this paper, we report observations of periodic chorus wave emissions modulated by two typical ULF wave (i.e., field line resonance and mirror mode). Chorus waves are more likely to be excited at the trough of the FLR ULF waves and are suppressed at the crest, on the contrary, chorus waves are more likely to be excited at the crest of the drift mirror ULF waves and are suppressed at the trough.

Different ULF wave structures lead to different secondorder spatial derivatives of the compressional magnetic field. To quantitatively evaluate the ULF wave field configuration and its role in the modulation process, we utilize a field line resonance model to demonstrate that ULF waves with a symmetric compressional magnetic field with respect to the equator decrease the threshold amplitude for chorus wave excitation at the crest of the ULF wave, thereby determining the chorus wave growth and damping. Drift mirror ULF waves with an antisymmetric compressional magnetic field with respect to the equator increase the threshold amplitude for chorus wave excitation at the troughs of the ULF wave. At the ULF wave crests, the threshold amplitude is reduced to foster the nonlinear growth of chorus waves. These features of chorus wave growth are caused by the anticorrelation between the ULF wave field and its second-order spatial derivatives, which agrees well with the observations. This work provides the first evidence for the modulation of chorus wave growth by different types of ULF waves through perturbing the background field configurations.

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