Models for Resonance at Low Values of Adiabatic Invariant

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Cyclotron-resonant interactions of test particles with coherent, quasi-monochromatic waves in an inhomogeneous background magnetic field can be effectively analyzed with a Hamiltonian formalism. Typically, the Hamiltonian is Taylor-expanded to first order in the wave amplitude. The zeroth order term reflects the deviation from resonance and is in turn expanded to second order in the first adiabatic invariant *I* of the particle, which plays the role of generalized momentum. If the first order term is evaluated at the resonant value of *I* the well-known pendulum Hamiltonian is obtained. This can lead to reliable analytical approximations for the change in *I* due to nonlinear phase bunching and phase trapping, or (for low amplitude waves) quasi-linear, diffusive interactions.

However, this fails for particles with low initial values of *I*. The analytical expressions for phase bunching of electrons by chorus waves can predict decreases to unphysically negative values of *I*, while test particle simulations actually exhibit positive changes in *I* for both phase bunching and phased trapping, as reported in a recent study [1]. This behavior can be better treated by a "second fundamental model of resonance" [2], in which the first order term of the approximate Hamiltonian retains a dependence on the square root of the action:

$$H(I,\theta,t) = [I-I_{res}(t)]^2/2 + \varepsilon\,(2I)^{1/2}\sin\theta \tag{1}$$

rather than approximating the nonlinear perturbation as $\varepsilon\,(2I_{res})^{1/2}\sin\theta$. We investigate the nature of motion for the two forms of the Hamiltonian numerically, using symplectic integration algorithms which enforce the Hamiltonian area-preserving property of the equations of motion. Results and ranges of applicability of the two models will be presented and compared, and generalized, physically useful analytical descriptions will be developed.

References

[1] M. Kitahara and Y. Katoh, "Anomalous Trapping of Low Pitch Angle Electrons by Coherent Whistler Mode Waves," *Journal of Geophysical Research: Space Physics*, **124**, July 2019, pp. 5568-5583, doi:10.1029/2019JA026493.

[2] J. Henrard, and A. Lemaitre, "A Second Fundamental Model for Resonance," *Celestial Mechanics*, **30**, 1983, pp. 197-218, doi:10.1007/BF01234306.