Impact of Localization Error on Open-Loop Distributed Beamforming Arrays

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Abstract

We present an analysis on the required two-dimensional positioning accuracy of the location of nodes in open-loop distributed beamforming arrays. For distributed antenna arrays in dynamic motion to operate coherently, localization is required to adjust the relative phase shift between the nodes in the array to maintain coherent beamforming. We investigate the required localization accuracy in a hierarchical architecture, where the secondary nodes localize and synchronize themselves to a primary node. We discuss a model for distributed beamforming and present analysis of the required ranging and angle estimation accuracy as a function of the secondary node locations and the beamforming gain using wireless frequency synchronization.

1 Introduction

Beamforming from a set of wireless nodes in a coherent distributed array requires accurate synchronization of the relative electrical states of the nodes. Two of the most basic properties that must be synchronized to support beamforming are the frequencies of the oscillators on each node, and the relative phases of the signals transmitted by each node. Previous approaches relied on feedback from the destination, limiting the array to cooperative communications [1, 2, 3, 4]. The most flexible distributed array architectures operate open-loop, where the nodes in the array self-synchronize without any external input from the targeted location [5, 6]. Such arrays enable general wireless operations, including communications and sensing. Open-loop distributed beamforming arrays require accurate knowledge of the relative positions of the nodes in the array to adjust the relative phase shifts on the nodes, supporting beamforming. Prior works on relative ranging for distributed beamforming have shown that to obtain 90% of the ideal coherent gain level at a random beamsteering angle with a probability of 90%, the standard deviation of the ranging estimates needed to be at most $\lambda/15$, where λ is the wavelength of the carrier [5]. If wireless frequency synchronization is implemented, ranging accuracy no more than $\lambda/27$ is necessary [7]. While these metrics relate to ranging accuracy, they do not consider two-dimensional localization accuracy.

In this paper we investigate the impact of localization errors on distributed beamforming in dynamic distributed arrays.



Figure 1. Hierarchical open-loop distributed array with one primary and two secondary nodes. θ_0 is the beamforming angle, θ_1 and θ_2 are the angles of the primary node relative to the planes of the secondary nodes.

We explore limitations on array size (number of elements and physical two-dimensional extent) in terms of the impact on coherent beamforming gain in the presence of range and angle estimation error between nodes. We consider a hierarchical array format, where one node is designated as a primary node to which the secondary nodes synchronize their states (Fig. 1). Secondary nodes are assumed to use wireless frequency synchronization as in [6, 8]. The results provide bounds on the required estimation accuracy in range and angle between nodes to support high coherent gain with high probability.

2 Coherent Gain Modeling

Two phase offset terms are observed by each secondary node in a dynamic setting. A phase offset at the secondary node *n* is determined by the change in distance $\Delta d_1(n)$ between the transmitters of the primary and secondary nodes by

$$\Delta\phi_1(n) = -\frac{2\pi\Delta d_1(n)\sin(\theta_0 + \theta_n)}{\lambda} \tag{1}$$

A second phase offset term is due to wireless frequency synchronization, where the phase of the input signal at a secondary node n changes based on the relative distance



Figure 2. Radial error $\sqrt{\sigma_x^2 + \sigma_y^2}$ in terms of λ for maximum and minimum coordinates of ± 10 m for *x* and *y* in the case of $\sigma_R = 0.01\lambda$ and $\sigma_{\theta} = 1^{\circ}/m$.

change $\Delta d_2(n)$ between the frequency synchronization antennas on the two nodes, yielding [6]

$$\Delta\phi_2(n) = -\frac{2\pi\Delta d_2(n)}{\lambda}.$$
 (2)

The estimated change in the distances $\Delta d_{1,e}(n)$ and $\Delta d_{2,e}(n)$ must be determined by the secondary node, after which the estimated phase shifts $\Delta \phi_{1,e}(n)$ and $\Delta \phi_{2,e}(n)$ are calculated. The beamformed signals from *N* nodes at the far-field is then

$$s_r(t) = \sum_{n=1}^{N} C_n e^{j2\pi ft} e^{j\left[\Delta\phi_1(n) + \Delta\phi_2(n) - \Delta\phi_{1,e}(n) - \Delta\phi_{2,e}(n)\right]}$$
(3)

where C_n represents the amplitude scaling from propagation and f is the carrier frequency. The ideal summation of the signals is

$$s_i(t) = \sum_{n=1}^{N} C_n e^{j2\pi ft}$$
(4)

where $\Delta d_{1,e}(n)$ and $\Delta d_{2,e}(n)$, and the estimated angles $\theta_{n,e}$ are equal to the actual $\Delta d_1(n)$ and $\Delta d_2(n)$, and θ_n . The coherent beamforming gain is calculated from $G_c = |s_i(t)s_i^*(t)| / |s_r(t)s_r^*(t)|$.

3 Error Analysis

The impact of range and angle estimation errors on coherent gain G_c was investigated by selecting a standard deviation on the estimated range σ_R and angle σ_{θ} for all the secondary nodes, with σ_{θ} expressed as angle per wavelength λ . The positional accuracy of the primary node is dependent on its relative location to the secondary node, since angle errors translate to larger positional errors at longer distances. The accuracy of estimating the position of the primary node is illustrated in Fig. 2, where $\sigma_R = 0.01\lambda$, $\sigma_{\theta} = 1 \ ^{\circ}/m$, and the radial error $\sqrt{\sigma_x^2 + \sigma_y^2}$ was expressed in terms of λ , with σ_x^2 and σ_y^2 representing the variance of estimating the relative coordinates *x* and *y*. The probability



Figure 3. Probability of the coherent gain exceeding 0.9 versus the maximum and minimum allowed coordinates *x* and *y* for the distribution of secondary nodes. $\sigma_R = 0.01\lambda$ and $\sigma_{\theta} = 0.1^{\circ}/m$.



Figure 4. The required σ_{θ} to maintain a coherent gain of 90% with a 90% probability for 10 nodes with multiple values of σ_R .

of the coherent gain exceeding 90% versus the the maximum allowed area of secondary nodes distribution was analyzed in Fig. 3 for 2, 4, 10, and 100 nodes. A minimum of 0.5 m x and y separations between the primary and secondary nodes were imposed. 10,000 Monte Carlo simulations were generated where $\Delta d_1 = \Delta d_2$, θ_0 and the locations of the secondary nodes were uniformly distributed, and it was assumed that the same antenna was used for frequency synchronization and beamforming. The other parameters were selected as follows: $C_n = 1$, $\delta \phi = 0$, $\sigma_R = 0.01\lambda$, and $\sigma_{\theta} = 0.1^{\circ}/m$, to allow for the interpretation of the results for any desired frequency or wavelength. Fig. 4 was generated in order to evaluate the required upper bound for σ_R and σ_{θ} to maintain a coherent gain of 90% with at least a 90% probability for 10 nodes. It can be seen that with $\sigma_R \leq 0.01\lambda$, the requirements for σ_{θ} are similar, since the bottleneck in this case is σ_{θ} . For $\sigma_R \ge 0.045\lambda$ it was not

possible to maintain a coherent gain of 90% with a 90% probability for any value of σ_{θ} .

4 Conclusion

An analysis on the requirements for two-dimensional positioning accuracy for open-loop distributed beamforming arrays was presented. The accuracy in estimating the range and angle is expressed in terms of wavelength and can thus be applied to any frequency of interest. For arrays with internode ranging accuracy of 0.044λ or better, localization accuracy can be obtained with sufficient angle estimation accuracy to support 90% coherent beamforming gain. For ranging accuracies of 0.01λ or better, angle estimation accuracy becomes the limiting factor on beamforming performance.

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