URSI GASS 2021, Rome, Italy, 28 August - 4 September 2021

Far-Field Changes Induced by Rounding Corners: Dependence on Corner Angle

Paul D. Smith*(1) and Audrey J. Markowskei(1)
(1) Department of Mathematics and Statistics, Macquarie University, Sydney, 2109, http://www.mq.edu.au

Diffraction of electromagnetic waves by an appropriate canonical structure can be very illuminating in extracting the dominant features of a scattering scenario. Scattering by sharp edges and corners is informed by, for example, the diffraction from the half-plane and the wedge (of infinite extent). The nature of the singularities in the field and its derivatives is described in J. Van Bladel's well-known book [1]. There has been much work to develop analytical and numerical methods to account for these singularities and enable accurate modelling; however, these methods can be time consuming to implement and at times become very specialised. When numerical methods are employed, a common approach used when dealing with domains with corners is to round the corners, producing a smooth surface. This eliminates the singularities introduced by the corners and allows for standard numerical quadratures to be used, though leaving the researcher with no clear estimate of the error or difference thus induced. Although there is an extensive literature on scattering and diffraction from sharp cornered objects and those with smooth boundaries, there does not seem to be a systematic treatment of the transition from one to the other, as the radius of curvature of the rounded corner points tends to zero. The purpose of this paper is to elucidate the changes in the far-field pattern in that transition for two-dimensional scatterers illuminated by an E-polarised time-harmonic electromagnetic wave. A time variation of $e^{-i\omega t}$ is assumed, with associated wavenumber k.

A prototypical object ∂D_0 possessing one corner of angle 2Ω is considered with a parameterised family of rounded counterparts ∂D_ε. The lemniscate-like structure ∂D_0 and the curves ∂D_ε are parametrised by, respectively,

$$\begin{aligned} \mathbf{x}_0(t) &= a\,(2\sin(t/2), -\tan\Omega\sin t)\,, \\ \mathbf{x}_\varepsilon(t) &= a\left(2\sqrt{\varepsilon^2+(1-\varepsilon^2)\sin^2(t/2)}, -\tan\Omega\sin t\right), \quad t\in[0,2\pi]. \end{aligned} \tag{1}$$

where the geometrical parameter a is set to unit value, and the parameter ε lies between 0 and 1. At the rounded corner, the radius of curvature ρ equals $2\varepsilon\tan^2\Omega + O(\varepsilon^3)$, as $\varepsilon \to 0$.

The non-dimensionalized maximum difference (L^∞ norm) $\sqrt{k}\left\|u_0^\infty - u_\rho^\infty\right\|_\infty$ between the scattered far-field u_0^∞ of the obstacle, of corner angle 2Ω, and the corresponding quantity u_ρ^∞ of the rounded object, with radius of curvature ρ at the corner, was numerically examined in [2] and [3]. Setting $\nu = (2\pi - 2\Omega)/\pi$, it was shown that as $k\rho \to 0$,

$$\sqrt{k}\left\|u_0^\infty - u_\rho^\infty\right\|_\infty \approx C\,(\theta_0)\,(k\rho)^{2/\nu}\,, \tag{2}$$

for some constant C dependent on the direction of the plane wave, θ_0. Similar results were also obtained for multi-cornered scatterers. An analytic study [3] established the result for the special case of the right-angled lemniscate.

In this paper we undertake the analysis of the scattered field difference for the family (1) with the angle Ω arbitrary and satisfying $0 < \Omega < \pi/2$. An integral equation is obtained for the difference in the surface distributions on each obstacle; its approximate solution is shown to be $O((k\rho)^{1/\nu})$, as $k\rho \to 0$. It can then be deduced that the non-dimensionalised far-field difference is $O((k\rho)^{2/\nu})$, as $k\rho \to 0$. Thus these results are consistent with the numerical results (2) previously obtained, and provide a rule of thumb for interpreting results of general purpose codes when smoothed surfaces are used to approximate structures with sharp corners.

References

[1] J. Van Bladel, *Singular Electromagnetic Fields and Sources*. Oxford University Press, 1991.

[2] A. J. Markowskei and P. D. Smith, "Measuring the effect of rounding the corners of scattering structures," *Radio Science*, 2017, 2017RS006276. [Online]. Available: http://dx.doi.org/10.1002/2017RS006276

[3] A. J. Markowskei, "Two-dimensional diffraction by structures with corners," Ph.D. dissertation, Macquarie University, Sydney, Australia, April 2019.