Basic Differential Equations of the Theoretical Physics After the Concept of Generalized Functions in the Sense of Distribution and the Double Current Sheet Problem of Van Bladel

To the memory of Jean Van Bladel

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Almost all differential equations of the theoretical physics had been established within the three centuries preceding the second quarter of the 20 th century. They were admitted to be valid for differentiable (or piece-wise differentiable) functions in the 4-dimensional space. In just the middle of that century, L. Schwartz had published his theory of *distributions*[1], which introduced into the mathematical literature a new class of entities that were coined later on as the *generalized functions* (in the sense of distribution!) . An appealing property of these functions was that without any restriction they had derivatives of all orders (in the sense of distribution !), which yielded naturally the question [2, 4]:

Are the already known differential equations of the theoretical physics valid in the 4-dimensional space in the sense of distribution ?.

It goes without saying that the answer to this question is not *naturally* and *imperatively* affirmative because the generalization made in the sense of the distribution was in a rather particular direction chosen by L. Schwartz. For example, the multiplication of this kind of two generalized functions is not defined, which excludes the non-linear equations such as the Navier-Stokes equations of the fluid mechanics. Therefore, an affirmative answer to the aforementioned question is a new *postulat*e put on the equations in question. Furthermore, adoption of this postulate enforces also the acceptance of the existence of some singular terms in the expressions of some field components, which can not be expressed in terms of ordinary functions. For example, in the case of electromagnetic field, when the field may have jump type discontinuities on a certain surface S, the electric field may have an expression of the form

$$\boldsymbol{E} = \{\boldsymbol{E}\} + \boldsymbol{E}_0\, \delta(S) + \boldsymbol{E}_1\, \delta'(S) + \ldots + \boldsymbol{E}_m\, \delta^{(m)}(S).$$

Here the first term on the right hand side stands for the expression valid beyond the discontinuity surface *S* while the additional terms are the singular part concentrated on the surface *S*. This kind of expressions are also valid for all other components and permit ones to discover new properties of the field.

The aim of the present work is to show the application of this postulate to the double current sheet problem considered three decades ago by Van Bladel without recourse to this postulate [5], and compare the results. The sheet considered in the present work is assumed to be in motion, which makes it more general as compared to the Van Bladel's case where it is at rest.

The approach used by Van Bladel is based on his power of intuition in addition to his large and deep knowledge of mathematics and physics. His results obtained by *intuition* are completely in accord with those obtained by the direct application of the postulate in question. But, in order to elucidate his intuition and to legitimate some of his mathematical operations, one has to make some small changes in his presentation.

References

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