The 2D Hybrid Spectral Element Spectral Integral Method for Electromagnetic Scattering from Multiregion Inhomogeneous Objects Embedded in Cylindrically Layered Media

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Abstract

A novel hybrid spectral element-spectral integral (SESI) method is proposed to simulate electromagnetic scattering by objects in multiregion cylindrically layered media. The basic idea is to solve the interior inhomogeneous regions with multiple scatterers by the spectral element method (SEM), while the spectral integral method (SIM) is used as an exact boundary condition for the exterior regions with cylindrically layered media. A final system with recursive SIM matrix element is obtained and the fast Fourier transform (FFT) is used for the Toeplitz matrix inversion. Numerical tests show that the hybrid method is efficient and accurate by comparing its results with analytical solutions and the conventional SEM, and with the finite element method (FEM) results.

1 Introduction

Multiple regions of scatterers may exist in a cylindrically layered medium for electromagnetic scattering problems. In such problems, the objects are embedded in multiple regions of homogeneous medium; each such region is surrounded by an arbitrary number of cylindrical layers (see Figure 1). Electromagnetic wave propagation and scattering in such complex media with cylindrical layers have potential applications in photonics, electronics and geophysics, but their solutions can be usually time-consuming due to the extremely fine mesh or large scale by using the conventional numerical methods, such as the FEM or the SEM [1]. However, in principle such problems can be solved efficiently with the hybrid finite element-boundary integral (FEBI) method as the exact radiation boundary condition can be provided by a boundary integral method, while the inhomogeneous regions with the scatterers are solved by the FEM [2, 3]. Nevertheless, so far the FEBI method has been only reported for a homogeneous background medium; no cylindrical layers have been considered.

The innovation of this paper lies in two points: first, multiple cylindrical layers have been first considered by the hybrid method and the number of layers can be arbitrary; second, the inhomogeneous scatterers can be multiple and located in multiple regions.

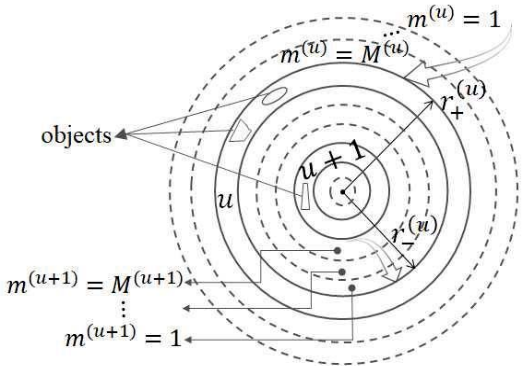

Figure 1. The 2D cylindrically layered model for electromagnetic scattering. The scatterers are embedded in multiple regions (U regions); adjacent regions are separated by arbitrary cylindrical layers. For example, the u-th region with inhomogeneous objects has $M^{(u)}$ outer cylindrical layers within $r_-^{(u-1)} \le r \le r_+^{(u)}$, and $M^{(u+1)}$ inner cylindrical layers within $r_-^{(u)} \le r \le r_+^{(u+1)}$.

2 Hybrid Formulation

2.1 Interior Multiregion Inhomogeneous Problem

As for the interior problem for the u-th inhomogeneous region, it is solved by the SEM originating from the 2D TMz Helmholtz equation

$$-\nabla_t \times (\mu_r^{-1} \nabla_t \times \hat{z} E_z) + k_0^2 \epsilon_r \hat{z} E_z = \hat{z} f \tag{1}$$

Taking the product of (1) with the test function and integrating by parts yields the weak formulation

$$\begin{aligned} &\boldsymbol{Z}_u^{ii} \boldsymbol{e}_u^{(i)} + \boldsymbol{Z}_u^{ib_+} \boldsymbol{e}_u^{(b_+)} + \boldsymbol{Z}_u^{ib_-} \boldsymbol{e}_u^{(b_-)} = \boldsymbol{f}_u \\ &\boldsymbol{Z}_u^{b_+ i} \boldsymbol{e}_u^{(i)} + \boldsymbol{Z}_u^{b_+ b_+} \boldsymbol{e}_u^{(b_+)} + \boldsymbol{Z}_u^{b_+ b_-} \boldsymbol{e}_u^{(b_-)} + \boldsymbol{Z}_{u,s}^{+} \tilde{\boldsymbol{j}}_u^{(b_+)} = 0 \\ &\boldsymbol{Z}_u^{b_- i} \boldsymbol{e}_u^{(i)} + \boldsymbol{Z}_u^{b_- b_+} \boldsymbol{e}_u^{(b_+)} + \boldsymbol{Z}_u^{b_- b_-} \boldsymbol{e}_u^{(b_-)} + \boldsymbol{Z}_{u,s}^{-} \tilde{\boldsymbol{j}}_u^{(b_-)} = 0 \end{aligned} \tag{2}$$

where i is the index of discrete nodal points in the interior region, while the b_+ and b_- are the index of nodal points on the boundaries $r_+^{(u)}$ and $r_-^{(u)}$ respectively.

2.2 Exterior Cylindrically Multilayered Homogeneous Problem

As for the exterior problem for u ($u = 1, \cdots, U$), which is bounded by inner radius $r_-^{(u)}$ and outer radius $r_+^{(u)}$, the relation between the unknown current density $\tilde{\boldsymbol{j}}_u^{(b_\pm)}$ and electric field $\boldsymbol{e}_u^{(b_\pm)}$ along the boundaries in equation (2) can be obtained through the linear system by the SIM [4]

$$\begin{aligned} \boldsymbol{Y}_u^{j+}\tilde{\boldsymbol{j}}_u^{(b_+)} + \boldsymbol{Y}_u^{e+}\boldsymbol{e}_u^{(b_+)} &= \boldsymbol{v}_u^{(b_+)} \\ \boldsymbol{Y}_u^{j-}\tilde{\boldsymbol{j}}_u^{(b_-)} + \boldsymbol{Y}_u^{e-}\boldsymbol{e}_u^{(b_-)} &= \boldsymbol{v}_u^{(b_-)} \end{aligned} \tag{3}$$

$\boldsymbol{Y}_u^{j\pm}$ and $\boldsymbol{Y}_u^{e\pm}$ are the SIM matrices obtained by 2D cylindrically layered medium Green's function in [4]. Note that these matrices are evaluated using the Green's functions for the cylindrically layered media surrounding region u. $\boldsymbol{v}_u^{(b_\pm)}$ is the incident wave at $r_\pm^{(u)}$ of the u-th region for the sources outside region u

2.3 Hybrid System for Multiple Scatterers Embedded in Cylindrically Layered Media

The interior and exterior problems are coupled by eliminating the coexisting current density $\tilde{\boldsymbol{j}}_u^{(b_\pm)}$ along the boundaries of inhomogeneous u ($u = 1, \cdots, U$), then the electric field for each layer can be solved. Letting the SIM relation system (3) into the SEM linear system (2) provides an inversion matrix $\left(\boldsymbol{Y}_u^{j\pm}\right)^{-1}$ as the matrix element. The matrix inversion of $\boldsymbol{Y}_u^{j\pm}$ possessing the Toeplitz properties is handled by the FFT algorithm with $O(N \log N)$ complexity [5, 6]. We denote $\boldsymbol{W}_u^{j+} = \boldsymbol{Z}_{u,s}^{+}\left(\boldsymbol{Y}_u^{j+}\right)^{-1}$ and $\boldsymbol{W}_u^{j-} = \boldsymbol{Z}_{u,s}^{-}\left(\boldsymbol{Y}_u^{j-}\right)^{-1}$, the hybrid system for multiregion U can be obtained by

$$\begin{bmatrix} \boldsymbol{Z}_U^{ii} & \boldsymbol{Z}_U^{ib_+} & \boldsymbol{Z}_U^{ib_-} \\ \boldsymbol{Z}_U^{b_+i} & \boldsymbol{Z}_U^{b_+b_+} - \boldsymbol{W}_U^{j+}\boldsymbol{Y}_U^{e+} & \boldsymbol{Z}_U^{b_+b_-} \\ \boldsymbol{Z}_U^{b_-i} & \boldsymbol{Z}_U^{b_-b_+} & \boldsymbol{Z}_U^{b_-b_-} - \boldsymbol{W}_U^{j-}\boldsymbol{Y}_U^{e-} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_U^{(i)} \\ \boldsymbol{e}_U^{(b_+)} \\ \boldsymbol{e}_U^{(b_-)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_U \\ -\boldsymbol{W}_U^{j+}\boldsymbol{v}_U^{(b_+)} \\ -\boldsymbol{W}_U^{j-}\boldsymbol{v}_U^{(b_-)} \end{bmatrix}$$

3 Numerical Results

3.1 Ten Circular Layers with Two Homogeneous/Inhomogeneous Regions

In order to verify the accuracy and efficiency of the proposed SESI method, a ten-layer cylinder with two homogeneous or inhomogeneous regions is considered. As shown in Fig.2, there are ten cylindrical layers with radius r_i=10-i m (i=1,2,⋯,9) from outside to inside, the medium parameters in each region are different but stratified. L_5 and L_9 are considered as the interior regions which need to be meshed by using the SESI method, while the other layers are the background cylindrically homogeneous layers and solved by integral equations. The comparison between the SESI and SEM is listed in Table I. The analytical solution solved by [7] is the reference solution. The SEM solver is from [8].

Then we simulate a model with the same size and the same media except for the region L_5 and L_9. An elliptic object lies in the region L_5 arising in inhomogeneous media in L_5, and two square objects lie in L_9. The CPU time is 16.56 s for SESI and 93.06 s for SEM respectively, the two methods match well and the relative error is 1.23%.

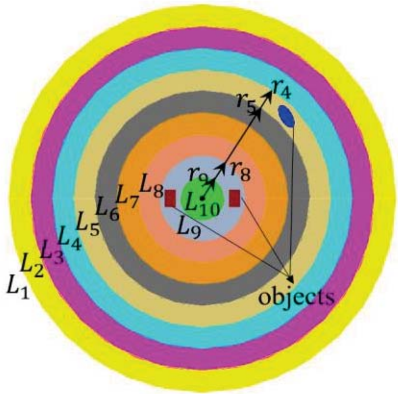

Figure 2. Model cross-sections with ten cylindrically layers. Region L_5 and L_9 are regarded as interior regions.

Table 1. Comparison between the SESI and the SEM.

N	SESI method Time (s)	SESI method Err of $\lvert E_z \rvert$	SEM Time (s)	SEM Err of $\lvert E_z \rvert$
100	9.78	1.58 %	23.19	4.13 %
150	13.86	0.69 %	58.98	1.48 %
200	22.13	0.64 %	96.77	1.22 %
250	29.77	0.62 %	133.72	0.86 %
300	43.84	0.60 %	379.11	0.80 %
350	66.67	0.59 %	515.28	0.72 %
400	69.75	0.58 %	607.77	0.71 %

3.2 Double-Layer Arc-shaped Frequency Selective Surface (FSS)

A double-layer conformal arc-shaped FSS which has been used for the design of ultrawideband electromagnetic shielding application is simulated as shown in Fig. 4. A metal-dielectric ring structure with the radii r_1=1.2 m and r_2=1.0 m is placed in the background air. The staggered laminated type PEC strips are uniformly distributed in the dielectric FR-4 substrate with (ϵ_r, μ_r)=(4.4-j0.01,1). The double-layer PEC strips are located on the circumferences of r_3=1.13 m and r_4=1.10 m. There are 40 periods in the circle, each period occupying $\alpha = \pi/20$ angle. For each period, there are two alternating PEC strips staggered up and down. The width of a PEC strip is $d = 11\pi/400$. COMSOL is also employed to simulate the double FSS model besides the SESI and SEM.

The monostatic RCS is used to consider the reflectivity of the scatterer to the plane wave and the best frequency of penetration is 300 MHz. At this frequency, the real and imaginary parts of E_z on r=1.0 m are shown in Fig. 4. The relative error of E_z is 0.36% between SESI and SEM, and 3.27% between SESI and COMSOL. The CPU time is 15.37 s, 50.57 s and 209.00 s for the SESI, SEM and COMSOL, respectively. We can also observe that for this complex model, the hybrid SESI method is also faster than the SEM and COMSOL.

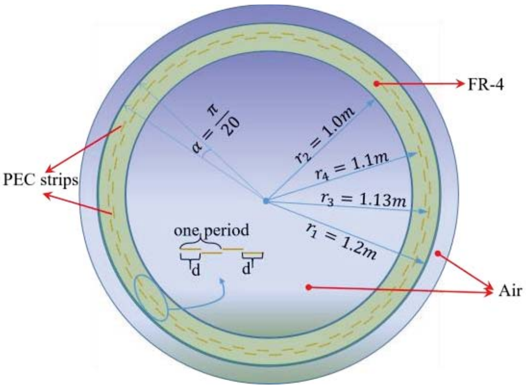

Figure 3. A double-layer FSS structure with the staggered laminated PEC strips.

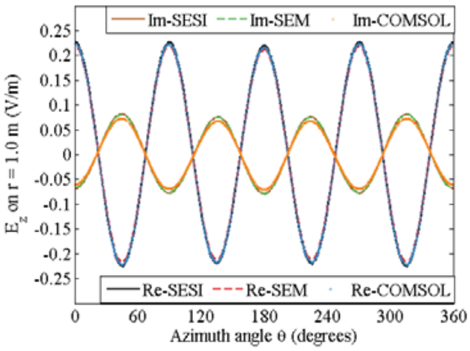

Figure 4. The real and imaginary parts of the electric field $\boldsymbol{E_z}$ on $\boldsymbol{r}$ = 1.0 m by the SESI, the SEM and the COMSOL at 300 MHz.

4 Conclusion

The 2D hybrid spectral element spectral integral method is developed for electromagnetic scattering by multiregion inhomogeneous objects embedded in cylindrically layered media. The interior problem is meshed and solved by the SEM while the exterior problem is handled by the SIM with cylindrically stratified homogeneous layered media. The SESI method only needs to discretize the regions with inhomogeneous objects instead of all layers. Therefore, compared with traditional numerical methods such as SEM and FEM, it can reduce lots of degrees of freedom and thus save computation costs. On the other hand, the fields at the boundaries of the inhomogeneous regions are obtained by the SIM as the exact boundary conditions, thus making it more accurate than using a pure absorbing boundary condition for the SEM and FEM.

5 References

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