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$\mathscr{H}$-matrix Accelerated Locally Corrected Nyström Solution of Electric Field and Magnetic Field Integral Equations

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In this work we demonstrate acceleration of the Locally Corrected Nystrom (LCN) solution with hierarchical matrices ($\mathscr{H}$-matrices) for the Electric Field Integral Equation (EFIE) and the Magnetic Field Integral Equation (MFIE). The new computational framework allows for fast direct solution of the pertinent $\mathscr{H}$-matrix equation via $\mathscr{H}$-LU decomposition and $\mathscr{H}$-back-substitution. While the higher-order (HO) LCN method ensures error-controlled discretization of the EFIE and MFIE, the error-controlled compression of the $\mathscr{H}$-matrix blocks via Adaptive Cross Approximation (ACA) and error-controlled $\mathscr{H}$-matrix arithmetics ensures error-controlled fast direct solution of the matrix equation. For smooth objects such as sphere and torus of electrically moderate sizes (up to approximately 100 wavelengths) it is demonstrated that the proposed approach allows to achieve the $O(h^p)$ HO error convergence in the solution (surface electric current density) with $O(N \log N)$ CPU time and memory use.

It is shown also that due to exactness of the far interactions in the LCN point-based discretization compression of the $\mathscr{H}$-matrix blocks is done more efficiently than compression of the far interactions in the Method of Moments (MoM) discretization as the latter feature numerical error introduced by the evaluation of the 4D reaction integrals.

Fast direct nature of the solution enables robust solution even when conditioning of the matrix deteriorates due to presence of the sharp edges in the scatterer, its oversampling, or presence of multi-scale features, provided tolerances in the LCN and $\mathscr{H}$-matrix arithmetics are set to resolve pertinent dynamic range of the interactions in the solution (e.g. dynamic range in the vector and scalar potential contributions of the EFIE).

Various numerical results demonstrating $O(h^p)$ error behaviour as well as CPU and memory complexity scaling will be presented at the conference.

References

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