



## Modal Expansions using Symmetric and Non-Symmetric Characteristic Mode Formulations for Dielectric Objects

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The theory of characteristic modes (CM) based on the electric field integral equation for conducting objects [1] is well established and popular in antenna design [2]. For a fixed frequency, the CM analysis yields a set of characteristic currents and corresponding eigenvalues, where the fields radiated by the currents are orthogonal and the eigenvalues are related to power ratios measuring how well each mode radiates. Moreover, any field scattered or radiated by the structure can be expressed as a modal expansion using a linear combination of the characteristic currents.

CM analysis of dielectric objects has been a somewhat open problem, but we are now confident that there are several surface integral based CM formulations that give physically sound characteristic modes [3–6]. Previous work on CM formulations for dielectric objects has mostly focused on verifying that the eigenvalues are correct. In this presentation, we continue the work by considering the modal expansions in more details.

For the symmetric PMCHWT-based formulation [4,5], the modal expansion coefficients are calculated in a similar way as in [1]. For formulations with non-symmetric operators, we also need the modes of the transpose (or adjoint) operators to calculate the modal expansion coefficients. The needed theory is briefly explained for conducting objects in [7]. In this presentation, we show how this theory is applied for dielectric objects.

In conclusion, symmetric and non-symmetric CM formulations for dielectric objects appear equally good for calculating the characteristic eigenvalues and both can be used for modal expansions. However, for a modal expansion, it is easier and computationally cheaper to use a symmetric formulation.

### References

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