



## Fine Scale Partial Coherent Model Based on lidar Elevation Measurements for GNSS-R Applications

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### Abstract

In GNSS-R (Global Navigation Satellite System Reflectometry) land applications, bistatic scattering occurs in the vicinity of the specular direction, and the scattered waves can include both coherent and incoherent components. A description of land surfaces roughness is required to predict GNSS-R specular scattering. Here we consider roughness on three distinct length scales: a microwave roughness  $f_1(x, y)$  with correlation lengths of  $\sim 10$  centimeters, a coarse scale 30-meter planar topography  $f_3(x, y)$  based on digital elevation model (DEM) data, and a fine scale topography  $f_2(x, y)$  between these two length scales. The fine scale topography represents the length scales in which scattered waves can transition from coherence to partial coherence to incoherence. In this paper, we investigate  $f_2(x, y)$  using recent airborne lidar measurements of land surface heights. Using  $f = f_1 + f_2 + f_3$ , fine scale partial coherent FPCN and FPCP models are applied to predict bistatic scattering coefficients near the specular direction for 30 m surface area. Here “fine scale” means the fine scale topography of  $f_2(x, y)$  is included. The model uses complex electric field summation and Monte Carlo simulations within a large area. For non-overlapping large areas, we use intensity summations as in radiative transfer theory.

We consider a fine scale partial coherent numerical model (FPCN) that applies numerical integration to the Kirchhoff integral using 2 cm discretization. The fine scale partial coherent patch model (FPCP) uses planar patches of L size, where L is less than the correlation length of  $f_2$ . Numerical illustrations show that the results of the FPCN and FPCP are in good agreement with each other. Comparisons are also made with geometric optics model (GO) with and without the attenuation factor of microwave roughness.

### 1 Introduction

GNSS-R systems have been applied for land surface remote sensing in recent years. To retrieve the soil moisture from GNSS-R measurements, physical models for the observed land surface bistatic scattering have been proposed [1-4]. Because existing global DEM data is resolved at spatial resolutions no finer than 30 meters, local variations in elevations and slopes within the resolved 30

m by 30 m areas are missing. Physical models have introduced a fine scale  $f_2$  (intermediate scale) topography to describe such variations [2, 4]. In applying the geometric optics model (GO) to the total terrain height  $f_1 + f_2 + f_3$ , the contributions of  $f_2$  and  $f_3$  to the surface mean square slope should be taken into account [4, 5]. However, there are two versions of GO, one is with the attenuation factor of  $\exp(-4k^2 h_1^2 \cos^2 \theta_i)$  due to microwave roughness, and the other is without such attenuation factor. The conventional GO [5] is based on stationary phase approximation to the Kirchhoff integral and does not have the attenuation factor.

The fine scale partial coherent numerical model (FPCN) applies numerical integration at a spatial interval of 2 cm to the Kirchhoff integral [1]:

$$\bar{E}_s(\bar{r}) = \frac{ik}{4\pi} \sqrt{\frac{P_t G_t \eta_0}{2\pi}} \iint_s d\bar{r}' \frac{e^{ik(R_{pt} + R_{pr})}}{R_{pt} R_{pr}} \left( \bar{I} - \hat{k}_s \hat{k}_s \right) \cdot \bar{F}(\alpha, \beta) \quad (1a)$$

$$\bar{F}(\alpha, \beta) = \sqrt{1 + \alpha^2 + \beta^2} \left\{ R_v(\hat{e}_i \cdot \hat{p}_i) (\hat{n} \times \hat{q}_i) + R_h(\hat{e}_i \cdot \hat{q}_i) (\hat{n} \cdot \hat{k}_i) \hat{q}_i - R_v(\hat{e}_i \cdot \hat{p}_i) (\hat{n} \cdot \hat{k}_i) (\hat{k}_s \times \hat{q}_i) + R_h(\hat{e}_i \cdot \hat{q}_i) [\hat{k}_s \times (\hat{n} \times \hat{q}_i)] \right\} \quad (1b)$$

in which  $P_t$  is the emitted power of the transmitter, and  $R_{pt}$  and  $R_{pr}$  are the distances from the patches to the transmitter and receiver, respectively. The FPCN is used here as a benchmark because no approximation beyond the Kirchhoff tangent plane approximation is applied.

The fine scale partial coherent patch model (FPCP) has been also proposed for GNSS-R land reflections [4]. The expression of FPCP is shown in Eq (2), in which  $G_t$  and  $G_r$  are the antenna gain of the transmitter and receiver, respectively. The model begins with the Kirchhoff integral, but divides the integration area into a set of tilted planar patches. The Kirchhoff approximation is then applied to carry out the integral over each planar patch to obtain its coherent scattering and diffuse scattering. Complex field summation is then applied:

$$\frac{P_r^{SWC}}{P_i} = \frac{G_t G_r}{64\pi^2 R_r^2 R_t^2} \left\langle \left| \sum_{n=1}^N L^2 \exp(ik(R_{nt} + R_{nr})) \exp(-2k^2 h_n^2 \cos^2 \theta_m) \right. \right. \\ \left. \left. \text{sinc} \left( \left( \frac{k_{dnc} + p_n}{k_{dnc}} \right) \frac{k_{dnc} L}{2} \right) \text{sinc} \left( \left( \frac{k_{dny} + q_n}{k_{dnc}} \right) \frac{k_{dnc} L}{2} \right) 2 \cos \theta_i R_{CP}(\theta_m) \right|^2 \right\rangle \quad (2a)$$

The diffuse scattering intensities are also added:

$$\frac{P_r^{SWCI}}{P_i} = \frac{G_t G_r}{64\pi^2 R_r^2 R_t^2} \sum_{n=1}^N 4 \cos^2 \theta_i |R_{CP}(\theta_m)|^2 \left[ \langle |I_p|^2 \rangle - \langle |I_p| \rangle^2 \right] \quad (2b)$$

Because of speckle effects, Monte Carlo simulations have been performed over realizations of a statistically described  $f_2$  as represented by average in Eq (2a). It is also noted that the geometrical optics result can be derived through an additional high frequency approximation of the Kirchhoff integral [5].

In this paper, we use airborne lidar measurements to extract the fine scale topography  $f_2$ , and the coarse topography  $f_3$  by numerical fit and average processing. We implement statistical methods to retrieve rms height and correlation length of  $f_1$  and  $f_2$ . Using a statistical description of  $f_2$  determined from the lidar data, we generate land surface profiles of  $f = f_1 + f_2 + f_3$  for the Monte Carlo simulation. Then we calculate bistatic scattering coefficients with the three models: FPCP, FPCN, and GO. The comparison of FPCP and FPCN shows good agreement, which implies the accuracy of FPCP method.

## 2 Methodology

### 2.1 Extraction of three scales of the land surface from lidar data

The profile of the land surface can be denoted as:

$$f(x, y) = f_1(x, y) + f_2(x, y) + f_3(x, y) \quad (3)$$

in which  $f_1$  is the microwave roughness with rms height of 2 cm or less,  $f_3$  is the coarse scale topography as given by DEM, which is modeled as a tilted planar surface, and  $f_2$  is a fine scale topography that is in between the microwave roughness and the coarse scale.

We process the lidar data with 30 cm resolution to separate the three scales of the land surface. To simplify our initial studies, we only consider 2D profiles  $f(x)$  by assuming the land surface is isotropic in all directions. The studied area is 30 meters long. The coarse scale is defined as a straight line. We do a least squares fit of  $f$  over 30 m to extract the straight line  $f_3$ . Next, we subtract  $f_3$  from  $f$  to obtain  $f_{12} = f_1 + f_2$ . With the assumption of the correlation length  $l_2$  of  $f_2$  to be much larger than the correlation length of  $f_1$ , we take the average of  $f_{12}$  over non-overlapping intervals of several correlation lengths of  $f_1$  (e.g. 1 m) to obtain  $f_2$ . Finally,  $f_1$  is extracted by subtracting  $f_2$  from  $f_{12}$ .

### 2.2 Retrieving rms height and correlation length of profiles using stochastic methods

The rms height of the profile can be calculated using:

$$h = \frac{1}{N} \left[ \sum_{i=1}^N |f(x_i)|^2 \right]^{\frac{1}{2}} \quad (4)$$

The correlation function can be computed as:

$$C(\rho) = \langle f(x)f(x+\rho) \rangle \quad (5)$$

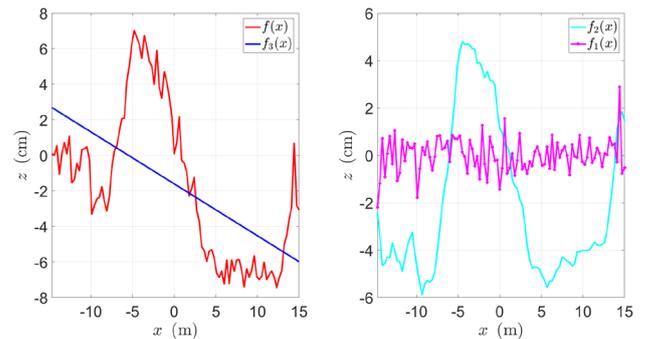
The correlation length  $l$  corresponds to a value  $\rho$  at which  $C(\rho) = e^{-1}$ . In this study, the rms height and correlation length of the extracted  $f_1(x)$  and  $f_2(x)$  are retrieved by these methods.

### 2.3 Comparison of results from FPCN, FPCP, and GO

With the statistical parameters retrieved from the 2D profile, we can generate a virtual 3D land surface with the assumption that the land surface is isotropic in all directions. Next, the bistatic scattering coefficients of the reflected fields are calculated with the three models. Specifically, the partial coherent field, diffuse incoherent intensity, and the total reflected fields are considered.

## 3 Results and Discussions

In this section, we show the extracted profile of the land surface, and simulation results of models. The lidar data has a resolution of 30 cm. We process a 30-meter 2D profile to extract the three scales. The extracted profiles are shown in Figure 1. With the statistical analysis, the rms height  $h_1$  of  $f_1$  is about 0.72 cm, while  $h_2$  of  $f_2$  is around 3.3 cm, which is much larger than  $h_1$ . A 3.3 m correlation length  $l_2$  for  $f_2$  is also retrieved. It is noted that this analysis was performed only for a single profile, and that these statistics are expected to vary with terrain location. The location considered here is an unusually “flat” location.



**Figure 1.** Extracted three scales of the land surface profile.  $f_1$  is the microwave roughness with  $h_1 = 0.72$  cm.  $f_3$  is the DEM topography with slope  $p_3 = -0.17$  deg.  $f_2$  is the

fine scale topography in between  $f_1$  and  $f_3$ , with  $h_2 = 3.3$  cm, and  $l_2 = 3.3$  m.

With the retrieved parameters of the profiles, we apply FPCN, FPCP, and GO to calculate the bistatic scattering coefficients of the profile. Here we use  $h_2$ , and  $l_2$  from the extraction. However, the retrieved  $h_1$  and  $l_1$  from lidar measurements are small and below the resolution of the lidar. Thus, we use in-situ ground measurements of  $h_1$  and  $l_1$ . The simulation results are shown in Table 1. As shown, the difference between the results of FPCP and FPCN is less than 0.2 dB, which means the two models are in good agreement with each other. The GO models are less accurate for this "very flat" single 30 m area. The GO model without the attenuation factor is more accurate than GO model with the attenuation factor.

**Table 2.** Simulation results of the bistatic scattering coefficients (dimensionless). The simulation is implemented with  $h_1 = 1.5$  cm,  $h_2 = 3.3$  cm,  $l_1 = 15$  cm,  $l_2 = 3.3$  m, and large-scale topography slope  $p_3 = -0.17$  deg.

Model	$\gamma_{coh}$	$\gamma_{incoh}$	$\gamma_{tot}$
FPCN	25.75 dB	25.16 dB	28.48 dB
FPCP	25.93 dB	25.27 dB	28.62 dB
GO (no attenuation)	-	-	26.56 dB
GO (with attenuation)	-	-	23.29 dB

## 4 References

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