Electromagnetic Scattering by Magnetic Biaxial Cylinders

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Abstract

In this work a volume integral equation method is proposed for the evaluation of the electromagnetic scattering by multilayered magnetic biaxial circular cylinders with infinite length. The key point of the proposed method is the expansion of the magnetic field and magnetic flux density in Dini-type cylindrical vector wave functions, which consitute an orthogonal vectorial basis in the transversal circular cross section of the cylinder. These expansions reduce the two-dimensional volumetric-type integrals to sets of algebraic equations. The cylinder is illuminated by a normally impinging, on cylinder's axis, plane wave. Both types of H- and E-wave polarization of the incident plane wave are considered. The formalism used in this method allows for an easy and direct incorporation of the multilayered magnetic biaxial permeability profiles through the constitutive relations. The method is validated by comparison of the normalized scattering cross section with the exact solution for the case of uniaxial anisotropy. Moreover, we employ HFSS commercial software to verify our results for homogeneous and inhomogeneous magnetic biaxial profiles.

1 Introduction

Electromagnetic scattering of a plane wave by magnetized cylinders has been studied in various works over the past years. Among others, such works include the scattering by homogeneous ferrite cylinders [1, 2, 3], finite periodic structures consisting of homogeneous ferrite cylinders [4], and conducting circular cylinders coated with homogeneous [5, 6] or nonuniform ferrite materials [7]. Also, the scattering by dielectric/ferrite cylinders in the presence of a half-plane has been investigated using equivalent electric and magnetic currents [8]. More recently, the scattering from an arbitrary convex cross sectional ferrite post has been examined with the use of the field matching method [9].

The purpose of this work is to examine the electromagnetic scattering by a multilayered magnetic biaxial circular cylinder whose tensorial permeability is referred to Cartesian coordinate system. The magnetic biaxial medium provides an extra degree of freedom for the manipulation of various electromagnetic properties, as compared to the respective uniaxial medium, due to its additional optic axis. The configuration of the problem consists of a multilayered magnetic biaxial circular cylindrical scatterer of infinite length, situated in free space, normally illuminated by a H- or E-polarized plane wave. Our solution is based on a volume integral equation method where the magnetic field and magnetic flux density are expanded in Dini-type orthogonal cylindrical vector wave functions. These vectorial functions constitute an orthogonal basis in the transversal circular cross section of the cylinder, by satisfying appropriate eigen-equations [10]. The main advantage of our formulation is that it allows for a straightforward incorporation of multilayered magnetic biaxial permeability profiles, by a simple integration of the constitutive relation than connects the magnetic field with the inverse permeability tensor and the magnetic flux density.

In order to examine the validity of the volume integral equation, under both H- and E-wave incidence polarization, an exact solution based on [11] is constructed for the special case of magnetic uniaxial permeability. For one-layered homogeneous and two-layered inhomogeneous magnetic biaxial permeability cases, we establish validity by employing the HFSS commercial software since no exact solution exists for a Cartesian magnetic biaxial permeability, and an excellent agreement is met.

2 Formulation of the Problem

The proposed method of solution is based on the magnetic field-volume integral equation given by

$$\mathbf{H}(\boldsymbol{\rho}) = \mathbf{H}^{\mathrm{inc}}(\boldsymbol{\rho}) + (k_0^2\mathbb{I} + \nabla\nabla\cdot) \int_{\boldsymbol{\rho}'\in S} g(\boldsymbol{\rho},\boldsymbol{\rho}')\mathbb{X}_{\mathrm{m}}(\rho)\mathbf{H}(\boldsymbol{\rho}')\mathrm{d}S', \quad \boldsymbol{\rho}\in\mathbb{R}^2 \qquad (1)$$

In (1), $\mathbf{H}(\boldsymbol{\rho})$ represents the total magnetic field, $\mathbf{H}^{\mathrm{inc}}(\boldsymbol{\rho})$ is the magnetic field of the incident plane wave, $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ is the free space wavenumber (with ε_0 and μ_0 the free space permittivity and permeability, respectively), $\mathbb{I}$ is the unity dyadic, $g(\boldsymbol{\rho},\boldsymbol{\rho}') = -i/4H_0^{(2)}(k_0|\boldsymbol{\rho}-\boldsymbol{\rho}'|)$ is the two-dimensional free space Green's function (with $H_0^{(2)}$ the zeroth order Hankel function of the second kind) [12], $\mathbb{X}_{\mathrm{m}}(\rho) = \boldsymbol{\mu}(\rho)/\mu_0 - \mathbb{I}$ is the normalized tensorial magnetic contrast function, and S is the circular domain of the cylinder having radius a. The assumed time dependence in (1) is $\exp(i\omega t)$. The multilayered magnetic biaxial permeability tensor $\boldsymbol{\mu}(\rho)$ appearing in the definition of $\mathbb{X}_{\mathrm{m}}(\rho)$, is given by

$$\boldsymbol{\mu}(\rho) = \mu_0 \begin{bmatrix} \mu_{1r}(\rho) & 0 & 0 \\ 0 & \mu_{2r}(\rho) & 0 \\ 0 & 0 & \mu_{3r}(\rho) \end{bmatrix}. \quad (2)$$

In (2), the matrix elements $\mu_{1r}(\rho)$, $\mu_{2r}(\rho)$, $\mu_{3r}(\rho)$ are relative quantities. The permittivity of the cylinder equals that of free space.

The first step for the solution of (1) is to introduce the magnetic flux density $\mathbf{B}(\boldsymbol{\rho})$ through the constitutive relation $\mathbf{B}(\boldsymbol{\rho}) = \boldsymbol{\mu}(\rho)\mathbf{H}(\boldsymbol{\rho})$ in the kernel of the integral. Thereafter, we expand $\mathbf{H}(\boldsymbol{\rho})$ and $\mathbf{B}(\boldsymbol{\rho})$ in cylindrical vector wave functions of Dini-type, i.e.,

$$\mathbf{H}(\boldsymbol{\rho}) = \sum_{m=-\infty}^{\infty} \sum_{l=1}^{\infty} \Big[\Sigma_{ml} \mathbf{M}_{ml}(k_{ml}^M, \boldsymbol{\rho}) + T_{ml} \mathbf{N}_{ml}(k_{ml}^N, \boldsymbol{\rho}) + \Pi_{ml} \mathbf{L}_{ml}(k_{ml}^L, \boldsymbol{\rho}) \Big], \quad (3)$$

and

$$\frac{1}{\mu_0}\mathbf{B}(\boldsymbol{\rho}) = \sum_{m=-\infty}^{\infty} \sum_{l=1}^{\infty} \Big[K_{ml} \mathbf{M}_{ml}(\gamma_{ml}^M, \boldsymbol{\rho}) + \Lambda_{ml} \mathbf{N}_{ml}(\gamma_{ml}^N, \boldsymbol{\rho}) \Big], \quad (4)$$

with Σ_{ml}, T_{ml}, Π_{ml}, K_{ml} and Λ_{ml} unknown expansion coefficients. In addition, the cylindrical vector wave functions $\mathbf{M}_{ml}$, $\mathbf{N}_{ml}$ and $\mathbf{L}_{ml}$ are given by $\mathbf{M}_{ml}(k_{ml}^M, \boldsymbol{\rho}) = e^{-im\varphi}[-im/\rho J_m(k_{ml}^M \rho)\hat{\rho} - \mathrm{d}J_m(k_{ml}^M \rho)/\mathrm{d}\rho\,\hat{\varphi}]$, $\mathbf{N}_{ml}(k_{ml}^N, \boldsymbol{\rho}) = e^{-im\varphi} k_{ml}^N J_m(k_{m\ell}^N \rho)\hat{z}$ and $\mathbf{L}_{ml}(k_{ml}^L, \boldsymbol{\rho}) = e^{-im\varphi}[\mathrm{d}J_m(k_{ml}^L \rho)/\mathrm{d}\rho\,\hat{\rho} - im/\rho J_m(k_{ml}^L \rho)\hat{\varphi}]$, with J_m the Bessel function. It should be stressed that in these expressions, $\partial/\partial z = 0$ since the cylinder is infinitely extended along z direction. The special arguments k_{ml}^M, k_{ml}^N in the cylindrical vector wave functions $\mathbf{M}_{ml}$, $\mathbf{N}_{ml}$ are used to establish orthogonality of $\mathbf{M}_{ml}$ and $\mathbf{N}_{ml}$ vectors within domain S, while the argument k_{ml}^L in $\mathbf{L}_{ml}$ is used to decouple $\mathbf{M}_{ml}$ and $\mathbf{L}_{ml}$ vectors [10].

Substitution of the expansions (3), (4) into (1), allows us to carry out analytically the two-dimensional integrals over the circular domain S, if we take into account the aforesaid orthogonality and decoupling properties of Dini-type $\mathbf{M}_{ml}$, $\mathbf{N}_{ml}$, $\mathbf{L}_{ml}$ vectors. This leads to two algebraic sets of equations involving Σ_{ml} with K_{ml} and T_{ml} with Λ_{ml}. Another two equations are obtained by Galerkin technique on the constitutive relation $\mathbf{H}(\boldsymbol{\rho}) = \boldsymbol{\mu}^{-1}(\rho)\mathbf{B}(\boldsymbol{\rho})$, where $\boldsymbol{\mu}^{-1}(\rho)$ is the inverse tensor of (2). In particular, these two additional equations relate Σ_{ml} with K_{mp}, $K_{m\pm2,p}$ and T_{ml} with Λ_{mp} via the relations

$$\Sigma_{ml} = \sum_{p=1}^{\infty} \Big(K_{mp} O_{mlp} + K_{m+2,p} R_{mlp} + K_{m-2,p} S_{mlp} \Big),$$
$$T_{ml} = \sum_{p=1}^{\infty} \Lambda_{mp} \Phi_{mlp}, \quad (5)$$

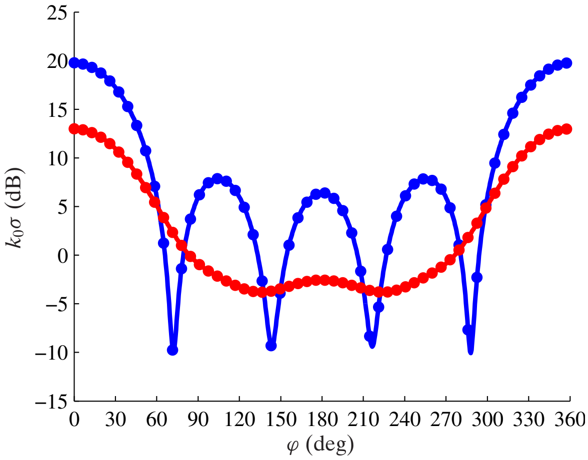

Figure 1. Normalized scattering cross section $k_0\sigma$ versus observation angle φ for a homogeneous magnetic uniaxial cylinder. Values of parameters: $k_0 a = 0.6\pi$, $\mu_{1r} = 2$, $\mu_{2r} = 2$, $\mu_{3r} = 4$. Blue curve/dots: H-wave polarization; curve: this method; dots: exact solution. Red curve/dots: E-wave polarization; curve: this method; dots: exact solution.

where O_{mlp}, R_{mlp}, S_{mlp} and Φ_{mlp} are radial integrals, with integration limits from $\rho = 0$ to $\rho = a$, involving the tensorial relative permeability elements $\mu_{1r}(\rho)$, $\mu_{2r}(\rho)$ and $\mu_{3r}(\rho)$. The new terms $K_{m+2,l} R_{mlp}$ and $K_{m-2,l} S_{mlp}$ that contribute in (5) were absent in the formulation of [10], where the problem of a gyrotropic cylinder was addressed, but not the case of a magnetic biaxial permeability tensor. These additional terms stem from the more complicated transformation of the permeability tensor (2) from Cartesian to cylindrical coordinates (as compared to the transformation used in [10]), required for the application of the constitute relation $\mathbf{H}(\boldsymbol{\rho}) = \boldsymbol{\mu}^{-1}(\rho)\mathbf{B}(\boldsymbol{\rho})$. Overall, relations (5) yield an additional complexity to the present solution, as compared to the respective solution of the gyrotropic case [10].

Once the expansion coefficients Σ_{ml}, T_{ml}, K_{ml} and Λ_{ml} are computed, the scattered magnetic field is evaluated by the expansion

$$\mathbf{H}^{\mathrm{sc}}(\boldsymbol{\rho}) = \sum_{m=-\infty}^{\infty} \sum_{l=1}^{\infty} \Big[C_{ml}^M \mathbf{M}_m^{(4)}(k_0, \boldsymbol{\rho}) + C_{ml}^N \mathbf{N}_m^{(4)}(k_0, \boldsymbol{\rho}) \Big], \quad (6)$$

where C_{ml}^M depends on Σ_{ml} and K_{ml} and C_{ml}^N on T_{ml} and Λ_{ml}. Moreover, $\mathbf{M}_m^{(4)}$, $\mathbf{N}_m^{(4)}$ are again given by the above formulas for $\mathbf{M}_{ml}$, $\mathbf{N}_{ml}$ with J_m replaced by $H_m^{(2)}$ and $k_{ml}^M = k_{ml}^N \equiv k_0$. Once $\mathbf{H}^{\mathrm{sc}}(\boldsymbol{\rho})$ is known, the normalized scattering cross section $k_0\sigma$, where $\sigma = \lim_{\rho\to\infty} 2\pi\rho |\mathbf{H}^{\mathrm{sc}}(\boldsymbol{\rho})|^2 / |\mathbf{H}^{\mathrm{inc}}(\boldsymbol{\rho})|^2$, can be readily computed.

3 Numerical Results

Herein we validate our method for various scenarios. We assume plane wave excitation where the incident wave impinges from the negative to positive x axis, having a zero degree angle with respect to the Ox semi-axis. Both incidence

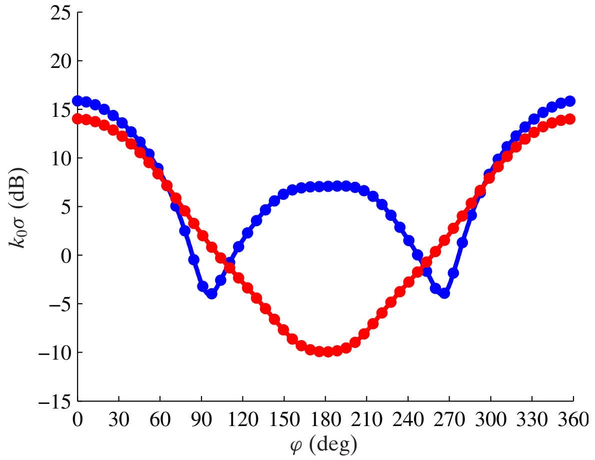

Figure 2. Normalized scattering cross section $k_0\sigma$ versus observation angle φ for a homogeneous magnetic biaxial cylinder. Values of parameters: $k_0a = 0.6\pi$, $\mu_{1r} = 2$, $\mu_{2r} = 1.5$, $\mu_{3r} = 3$. Blue curve/dots: H-wave polarization; curve: this method; dots: HFSS. Red curve/dots: E-wave polarization; curve: this method; dots: HFSS.

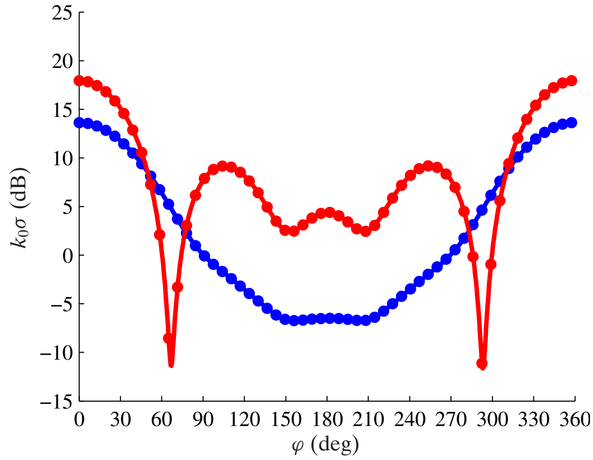

Figure 3. Normalized scattering cross section $k_0\sigma$ versus observation angle φ for an inhomogeneous two-layered magnetic biaxial cylinder. Values of parameters: $k_0a = 0.6\pi$, $b = 0.6a$. Core $(0 \le \rho \le b)$: $\mu_{1r} = 1.5$, $\mu_{2r} = 3$, $\mu_{3r} = 2$. Shell $(b < \rho \le a)$: $\mu_{1r} = 2.5$, $\mu_{2r} = 2$, $\mu_{3r} = 4$. Blue curve/dots: H-wave polarization; curve: this method; dots: HFSS. Red curve/dots: E-wave polarization; curve: this method; dots: HFSS.

polarizations are considered. In Fig. 1 the normalized scattering cross section $k_0\sigma$ in dB versus the observation angle φ is plotted for the case of a homogeneous magnetic uniaxial cylinder where $\mu_{1r} = \mu_{2r} = 2 \neq \mu_{3r} = 4$ and $k_0a = 0.6\pi$. It should be noted that in all examples the permittivity is equal to ε_0. The values of the parameters are also given in Figure's caption. It is evident that our method coincides with the results from the exact solution [11] for both H- and E-wave polarization.

Next, we consider a homogeneous magnetic biaxial cylinder with relative permeability values $\mu_{1r} = 2$, $\mu_{2r} = 1.5$, $\mu_{3r} = 3$. This case cannot be handled by an exact solution, so we employ the HFSS commercial software to evaluate our method. As Fig. 2 depicts, there is a very good agreement between the proposed method and HFSS, for both polarizations.

As a final scenario, we consider a two-layered cylinder consisting of two concentric magnetic biaxial layers. The inner radius is $b = 0.6a$ while the relative permeability elements of each layer are given in the caption of Fig. 3. In Fig. 3, in particular, we illustrate $k_0\sigma$ as computed by our method and HFSS. As it is evident, validity is established for the volume integral equation for this inhomogeneous magnetic biaxial permeability profile.

4 Conclusion and Future Work

A volume integral equation method for the electromagnetic scattering of a normally incident plane wave by a multilayered magnetic biaxial circular cylinder was developed in this work. Key element of the proposed solution is the expansion of the magnetic field and magnetic flux density in Dini-type cylindrical vector wave functions which constitute an orthogonal vectorial basis in the transversal circular cross section of the cylinder. The validation of our method was established by comparisons with the exact solution for the case of a magnetic uniaxial anisotropy, and with HFSS commercial software for one-layered homogeneous and two-layered inhomogeneous magnetic biaxial permeabilities. In the future we intend to extend the present formulation and develop a method capable of supporting a fully populated tensorial permeability tensor.

5 Acknowledgements

This work was supported by Greece and the European Union (European Social Fund-ESF) through the Operational Programme "Human Resources Development, Education and Lifelong Learning" in the context of the project "Strengthening Human Resources Research Potential via Doctorate Research" (MIS-5000432), implemented by the State Scholarships Foundation (IKY).

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