

### Design and Simulation of Beam-Generating Shells with Near-Zero-Index Characteristics

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#### Abstract

We present design and simulation of three-dimensional shell structures with near-zero-index (NZI) characteristics that generate directive radiation patterns from isotropic sources. Both homogenized and actual models involving periodic arrangements of unit cells are investigated via accurate solvers based on surface integral equations. Geometric properties of the shells are studied for the purpose of high-quality beams and favorable radiation patterns, such as low side-lobe levels. Various strategies to enhance or suppress selected beams to form alternative beam configurations are also discussed to demonstrate extensive capabilities of NZI shells for diverse applications.

### 1 Introduction

Near-zero-index (NZI) structures have recently been topics of many studies, as they enable manipulation of electromagnetic waves that cannot be achieved via ordinary materials [1]–[7]. In addition to tunneling, coupling, and beam splitting, NZI objects can be used to generate directive beams or for focusing [7]. Specifically, when an isotropic source is enclosed in an NZI shell, alternative beam configurations can be obtained, depending on the geometric properties of the shell. Such a transformation can be useful in a plethora of applications, from radio to optical frequencies, considering that an NZI shell does not need any physical contact with the main source so that it can be rotated arbitrarily, leading to alternative operation capabilities. On the other hand, design of an efficient and effective NZI shell can be challenging, particularly due to the sensitive nature of NZI structures to nearby excitations.

In real life, NZI characteristics are realized in alternative ways, such as employing periodic arrangements of unit cells (like metamaterials). For example, when dielectric rods are arranged in array configurations [3], epsilon-and-mu-near-zero (EMNZ) properties can effectively be obtained, provided that material and geometric properties are adequately selected. On the other side, at the design level, homogeneous models of NZI objects (where near-zero permittivity and/or permeability values are inserted) can be useful to analyze them and to understand their behaviors in alternative scenarios [6],[7]. In both cases, computational solutions can be challenging, considering subwave-

length details in actual structures and near-zero parameters in homogeneous models.

In this study, our aim is to design NZI shells involving dielectric rods with various beam-generating characteristics. In addition to actual structures, homogenized versions are considered to facilitate design and analyses of alternative configurations. Simulations clearly demonstrate the sensitive nature of the designed shells, e.g., critical roles of cavities for optimal radiation properties. We show that, by properly modifying the outer surfaces of the shells, beams can effectively be enhanced or suppressed to obtain customized radiation patterns that can be useful in diverse applications.

### 2 Computational Analyses of NZI Shells

In this work, actual and homogenized versions of NZI shells are analyzed via different versions of the multilevel fast multipole algorithm (MLFMA) in the frequency domain. In both cases, surfaces of models are discretized by using planar triangles, on which the Rao-Wilton-Glisson functions are used to expand equivalent electric and magnetic currents. In the following examples, each problem



**Figure 1.** Results for homogenized models of  $5\lambda \times 5\lambda \times 5\lambda$  cubic shells with square cavities of different sizes.



Figure 2. Results for homogenized models of various shells with  $5\lambda \times 5\lambda$  cross sections and  $2\lambda \times 2\lambda$  square cavities.

is discretized with 250,000–350,000 unknowns. In actual models, where ordinary material properties are used, the electric-magnetic current combined-field integral equation [8] is used to formulate problems. In homogenized models, however, a recently developed mixed formulation [6] is employed for stable and accurate simulations. Iterative simulations are carried out by applying an innerouter scheme involving full and approximate forms of MLFMA [9]. In all results of this paper, the primary source is modeled as a Hertzian dipole. Once expansion coefficients are obtained for a problem, they are used to compute near-zone and far-zone electromagnetic fields to understand the behavior of the investigated structure.

### **3** Numerical Examples

First, we consider examples involving homogenized models, such as shown in Fig. 1, where  $5\lambda \times 5\lambda \times 5\lambda$  cubic shells with square cavities of different sizes are investigated ( $\lambda$  is the wavelength in the host medium, i.e., vacuum). Both the relative permittivity and the relative permeability are assumed to be  $5 \times 10^{-3}$ , leading to  $5 \times 10^{-3}$  refractive index for the material of the shells. In each case (and also for each result below), the primary source (Hertzian dipole) is located at the center of the cavity and oriented vertically (in the z direction). Fig. 1 depicts the power density distributions in  $100\lambda \times 100\lambda$  frames on the x-y plane, as well as the values of the far-zone electric field intensity with respect to the observation angle on the same plane. We observe that the size of the cavity is extremely critical to achieve highquality beams. Specifically, the cavities with  $1.9\lambda \times 1.9\lambda$ and  $2\lambda \times 2\lambda$  cross sections provide the desired four beams, while the beam-generation ability diminishes dramatically when the cavity is enlarged to  $2.1\lambda \times 2.1\lambda$ . We note that the  $2\lambda \times 2\lambda$  cavity leads to brighter beams, while it also results in more significant side lobes, in comparison to the  $1.9\lambda \times 1.9\lambda$  cavity. A close examination of the cavity region reveals that the resonating characteristics of the cavity is crucial to generate strong beams, while resonance conditions are not trivial to be derived without computational simulations.

Fig. 2 presents the results for homogenized shells (EMNZ with  $5 \times 10^{-3}$  refractive index) with  $2\lambda \times 2\lambda$  square cavities. In these comparisons, the overall cross section is fixed to  $5\lambda \times 5\lambda$  for all shells, while the height changes from  $\lambda$ 



**Figure 3.** Results for homogenized models of  $5\lambda \times 5\lambda \times 5\lambda$  cubic shells with circular cavities.



**Figure 4.** Results for various shell structures made of dielectric rods with different cavity formations.

to  $7\lambda$ . Considering the power density distributions, again on the *x*-*y* plane, we observe that neither very short nor very long shells provide the desired radiation characteristics (with four clear beams), while even  $2\lambda$  height is sufficient to generate directive beams. Main beams are generally observed whenever a shell has a sufficient length, but side lobes become significant as the height increases.



**Figure 5.** Far-zone radiation results for various shells made of dielectric rods  $(3\lambda)$  with different cavity formations.



**Figure 6.** Results for various shell structures made of dielectric rods. Two sides of the regular grids (with  $2\lambda$  and  $3\lambda$  rods) are modified by extracting rods.

Fig. 3 illustrates a type of useful modification on outer surfaces of NZI shells to enhance selected beams. In these results,  $5\lambda \times 5\lambda \times 5\lambda$  cubic shells with circular cavities (having  $2\lambda$  and  $2.1\lambda$  diameters) are considered. Similar to the results above, the shells are assumed be made of an EMNZ material with  $5 \times 10^{-3}$  refractive index. In each case (cavity dimension), by curving one of the surfaces, the effective aperture of the surface is increased. The geometry of a curvature must be designed carefully since it also increases the beam width (sometimes leading to splitting), in addition to increased radiation in the direction of the surface. Modifications like curving (in cylindrical forms) are favorable as they provide desired changes in radiation characteristics while being applicable in actual structures (involving rods).



Figure 7. Results for various shell structures with modified sides to suppress beams.

Next, we consider actual structures involving periodic arrangements of dielectric rods. Based on the study [3], we select cylindrical rods with 8.8 relative permittivity and  $0.13\lambda$  radius as unit cells, while the center-to-center distance is set to 0.58 $\lambda$ . Fig. 4 presents the results for  $9 \times 9$ grids, where three different cavities (circular, elliptical, and square) are formed by removing center rods. Considering near-zone power density distributions obtained for different lengths of the rods, we observe successful generations of four beams for the circular and square cavities. Considering these structures, intensity of the side lobes depends on both cavity type and the length of the rods. Three-dimensional far-zone radiation patterns shown for the structures with  $3\lambda$ rods clearly demonstrate the generated beams, as well as undesired (but limited) upward and downward radiations due to finite sizes of the rods. Fig. 5 further depicts the far-zone electric field intensity on the x-y plane (when the length of the rods is  $3\lambda$ ) to demonstrate the quality of the beams for the circular and square cavities.

Fig. 6 presents the results of computational experiments when two sides of  $9 \times 9$  grids with square cavities are modified. These modifications, which involve removal of rods, correspond to introducing curvatures (as in Fig. 3) on shell surfaces. Despite that the resulting practical structures do not possess ideal curvatures, we observe that the corresponding beams can effectively be enhanced, leading to bidirectional radiation characteristics with two (wider) main beams. This is particularly visible in the three-dimensional patterns depicted for the structure with  $2\lambda$  rods.

An effective way to suppress beams is applying welldesigned corrugations on shell surfaces. In homogeneous



Figure 8. Results for various shell structures involving dielectric rods of length  $\lambda$ .

models, two-dimensional pyramidal textures can be used to mimic conventional absorbers, while actual models are restricted to removal or movement of rods. Fig. 7 presents several examples involving  $9 \times 9$  grids of  $3\lambda$  rods. Both square and circular cavities are considered, whereas corrugations (removal of selected rods) are applied on one, two, and three sides of the structures. We observe that, when a single side is modified, the corresponding beam is effectively suppressed, leaving three main beams (in addition to some side lobes). Corrugations on the second and third surfaces further lead to radiations with two and single main beams, respectively, while side lobes become significant.

Finally, Fig. 8 presents the possibility of more compact (flat) shells involving  $\lambda$ -length rods. Alternative configurations and the corresponding results (near-zone power density distributions and far-zone radiation patterns) are depicted to demonstrate the diversity of radiation characteristics that can be obtained via relatively simple structures.

# 4 Concluding Remarks

A computational study of beam-generating shells with NZI characteristics is presented. Simulations involving both homogenized and actual models of shells involving dielectric rods demonstrate both sensitive and useful properties of these structures in terms of beam generation from isotropic sources. As shown with examples, practical modifications in rod arrangements can enhance or suppress selected beams to obtain customized radiation patterns that can be useful in diverse applications.

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