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Scattering of Quantum Light by a Perfectly Conducting Cylinder

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Abstract

We consider transformation of quantum states of electromagnetic field by scattering from a perfectly conducting cylinder with a circular cross-section. We introduce a quantization technique for the scattered field, and demonstrate how quantumness (in particular, entanglement) of field states is transferred to the scattered field. The obtained results may be useful for designing quantum antennas and for quantum-enhanced far-field sensing, i.e., quantum lidars/radars.

1 Introduction

The recent progress in quantum optics [1] both for microwaves and visible light stimulated the development of quantum antennas and quantum radars/lidars [2,3]. As a result, it became important to study the states of EM-field with no classical analogs emitted by a quantum source and scattered by a target with the aim to control and to account for non-classicality (in particular, such non-classical correlations as entanglement) [4]. To that end, the techniques of classical electromagnetics should be adapted to the case of quantum light.

Multimodal (spectral) representations have always been an important tool for the analysis of waveguides as well as transmitting/receiving devices, and helping engineers to describe and use the subtle properties of the EM-fields. Such modal representations can also provide a natural platform for field quantization and quantum description of the field propagation and scattering. As a main conceptual problem, one needs to consider a physically correct quantization of the EM-field, which includes the introduction of multimode wave functions and field operators satisfying corresponding commutation relations. Among recent works in this area, one can note the analytical techniques based on the separation of variables [5] and a numerical approach based on the synthesis of the time-domain finite element analysis with the canonical quantization technique [6].

2 Field quantization

As an example of scattering by a lossless canonical object, in this paper, we will consider the problem of quantum plane-wave scattering by a perfectly conducting circular cylinder. The problem geometry is illustrated in the Fig. 1a. We assume $e^{-i\omega t}$ harmonic time dependence and consider the case of transverse magnetic (TM) polarization also referred as E-polarization (electric field directed along the axis of the cylinder). In general, our approach follows the conventional transformation of plane-wave basis to the radial cylindrical one, which allows satisfying the boundary condition at the surface of cylinder and radiation condition for every cylindrical harmonic separately [7]. We define such cylindrical modes satisfying the Helmholtz equation and the boundary condition as $f_{nk} = \sqrt{(k/2\pi)} B_n(k\rho) e^{in\varphi}$, where k is the wavenumber and the radial dependence is defined as

$$B_n(k\rho) = J_n(k\rho) + P_{nn} H_n^{(1)}(k\rho) = \\ = \frac{1}{2}\left(H_n^{(2)}(k\rho) + S_{nn} H_n^{(1)}(k\rho)\right) \tag{1}$$

Here ρ, φ are cylindrical coordinates, while $J_n(k\rho)$, $H_n^{(1)}(k\rho)$, and $H_n^{(2)}(k\rho)$ are the n-th order Bessel and Hankel functions of the first and second kinds, respectively. Also, $P_{nn} = -J_n(ka)/H_n^{(1)}(ka)$ and $S_{nn} = -H_n^{(2)}(ka)/H_n^{(1)}(ka)$ are the diagonal elements of the so-called perturbation and scattering matrices, respectively. These matrices in the chosen basis are diagonal, and the scattering matrix is unitary. For the quantization of the EM-field in the cylindrical basis, we first consider a suitable system with a discrete spectrum (2D-cylindrical cavity with the scatterer under consideration placed inside (Fig. 1b)). Thereafter, we make the transition $b \to \infty$ for its normal modes. Conventionally, quantum mechanical field operators are separated into positive- and negative-frequency components [1], which are denoted by superscripts $\pm$. The operators for vector potential and electric field are then given by

$$\hat{A}^{+}(\rho,\varphi;t) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{E_k}{\omega_k} \sum_{n=-\infty}^{\infty} \frac{1}{i^{-n}} B_n(k\rho) \cdot \\ \cdot e^{in\varphi} e^{-i\omega_k t} \hat{c}_{nk} \sqrt{k}\, dk \tag{2}$$

$$\hat{E}^{-}(\rho,\varphi;t)=\frac{i}{\sqrt{2\pi}}\int_{0}^{\infty}E_k\sum_{n=-\infty}^{\infty}i^{-n}B_n^{*}(k\rho)\cdot$$
$$\cdot e^{-in\varphi}e^{i\omega_k t}\hat{c}_{nk}^{+}\sqrt{k}dk \qquad (3)$$

where $E_k=(\hbar\omega_k/2\varepsilon)^{1/2}$ is a normalization factor, $\hbar$ is the Planck constant, ε is the permittivity of the surrounding space, $\omega_k=kc$, c is the velocity of light, and $\hat{c}_{nk},\hat{c}_{nk}^{+}$, are creation-annihilation operators in the new basis, which are defined in the conventional way [1] and satisfy the commutation relations $[\hat{c}_{nk},\hat{c}_{n'k'}^{+}]=\delta_{nn'}\delta(k-k')$.

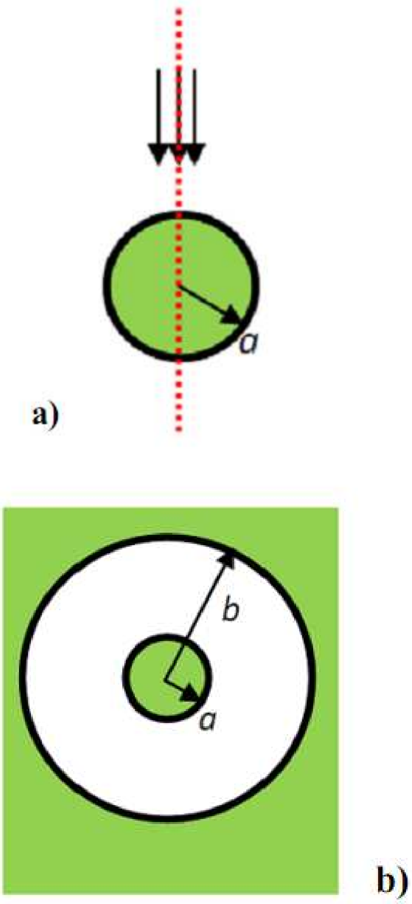

Figure 1. Configuration of the scattering problem: a) the cylinder illuminaited by a plane wave; b) the cylinder placed inside a cylindrical cavity.

3 State transformation by scattering

As a simplest example of a quantum state transformation, let us show how the single-photon state is transformed by scattering. It is easy to see from Eqs. (2,3) that the impinging photon is split between all the cylindrical modes. So, the scattering of the single photon in the incident plane wave leads to the following wave function for the total field

$$|\Psi\rangle=\sum_n S_{nn}\cdot$$
$$(\ldots\otimes|0:f_{0k}\rangle\otimes|0:f_{1k}\rangle\otimes\ldots\otimes|1:f_{nk}\rangle\otimes\ldots) \qquad (4)$$

where $|n_n:f_{nk}\rangle$ denotes the mode f_{nk} with n_n photons, and weights S_{nn} describe the photon distribution between the modes. So, even this simple example shows that scattering of quantum light can lead to actual creation of quantum correlations (and even entanglement). More involved quantum states (such as multi-photon Fock states, single and multi-mode squeezed states, etc.) are also being specifically transformed by the scattering, and quantum correlations can be produced. Moreover, one may aim for designing quantum states with the goal to produce scattered fields with specific quantum-correlation features (such as, for example, time and momentum-correlated photons flying in different directions).

4 Conclusion and Outlook

We have considered the scattering of quantum light by a perfectly conducting cylinder with a circular cross-section. As a convenient basis for the EM-field quantization, we have used the eigenmodes of the radial waveguide terminated by a perfectly reflecting surface of the circular cylinder, in detail studied in [8]. We demonstrate the feasibility of producing quantum correlations in the scattered field, and design these correlations by appropriate tailoring of the impinging quantum state. These effects are useful in designing devices for quantum-enhanced far-field sensing, in particular, for quantum lidars/radars, and for designing quantum antennas exploiting combination of classical and quantum interference effects for creating fields with required spatio-temporal configurations and correlation properties. Notice that the scattering process is defined by the properties of scattering matrix and therefore depends on the spectral shape of the impinging field. This opens the possibility to create entanglement between temporal and spatial degrees of freedom.

The developed approach may be applied to other problems of quantum light scattering, such as the H-polarization of the impinging field, the excitation of a cylinder by a current filament, as well as scattering by a wedge and half-plane as its limiting case. In this work, we considered the quantization in the radial representation (the partial mode propagates along the radial coordinate, while azimuth is considered as a transverse one). Such representation converges rapidly for relatively small values of the radius [8]. In classical electrodynamics, radial waveguides can be replaced with another choice of cylindrical modes. In this approach, the radial coordinate for every partial mode plays a role of transverse direction, while azimuth is considered as a direction of propagation [8]. This approach may be modified for the quantum light in a similar way and is expected to be more efficient for quasioptical regime when considering scattering by objects large compared to the wavelength.

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