Metasurface design constraints in Metasurface-based Virtual absorbers

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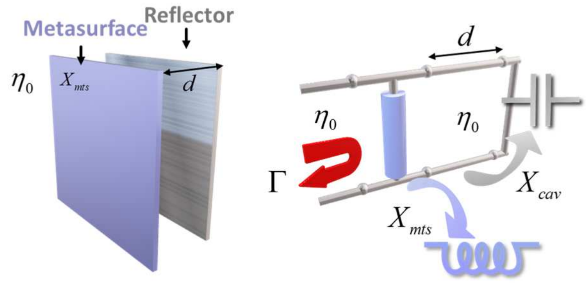
Abstract

In this contribution, we will present our investigation on design limits of the metasurfaces to be used for enabling virtual absorbing in metasurface-based virtual absorbers. To reach this goal, we analytically identify the conditions under which a properly modulated monochromatic plane wave can be virtually absorbed by the system, deriving the required metasurface values for a given cavity. The analysis is developed by modelling the system components, *i.e.,* the metasurface reactance and the cavity input impedance, through an equivalent transmission line model, exploring the scattering zeros positions in the complex frequency plane and determining whether the metasurface-based system can support the virtual absorption or not by evaluating the time-constant from its equivalent circuit.



(a) (b)

Figure 1. Schematic representation of (a) a virtual metasurface absorber and (b) its transmission line model, where the equivalent reactances are modelled reactive lumped elements as capacitance and inductance.

1 Introduction

Besides several studies of anomalous interaction between light and artificial meta-structures [1–10], *virtual absorption* theory [11–15], can be considered a competitive technique for storing and releasing electromagnetic energy, due to lossless and passive structures that can support the desired energy accumulation. Firstly, virtual absorption has been achieved by impinging with a plane wave a lossless dielectric slab of finite thickness with two perfectly synchronized signals at its opposite sides, showing the possibility to achieve zero total scattering and energy accumulation when the illuminating signals have a proper complex temporal frequency $\omega = \omega_r + j\omega_i$ [11]. This special temporal excitation is fundamental for engaging the complex zero scattering eigenmodes of the system and enabling its anomalous electromagnetic behaviour of zero scattering and energy storing.

Lately, we proposed a metasurface-based virtual absorber [16,17], able to store energy from an impinging plane wave under arbitrary illumination condition, and without the use of dielectric materials, to outdo the strictly constraints of pure lossless dielectrics and perfect coherent illumination of the structure. A metasurface virtual absorber as shown in Fig. 1a, is composed of a one-dimensional cavity filled by vacuum and bounded by a reflector and a metasurface.

In this contribution, exploiting the transmission line equivalence as shown in Fig. 1b, we present our investigation on design limits of the metasurface to be used for enabling virtual absorbing in metasurface-based virtual absorbers. We analytically derive the required complex frequency of the illuminating plane wave as a function of the surface impedance X_{mts} of the metasurface, the electrical size of the cavity, polarization and incidence angle of the illuminating wave, suggesting that the virtual absorption phenomenon does not occur for any pair of metasurface and cavity impedances. To reach this goal, we reduce the analysis to the circuital response of the transmission line equivalent in Fig.1b. The metasurface reactance X_{mts} and input reactance X_{cav} of the short-circuited stub modelling the metal-backed cavity can assume independently capacitive and inductive reactive values, according to the metasurface design and electrical dimension of the cavity [18]. Such a representation significantly simplifies the analysis by reducing the problem of virtual absorption to a very well-established concept in microwave circuit, that is the *perfect impedance matching* [19,20].

Hereafter, we focus our attention on the response from the load in terms of reflection coefficient when an excitation signal with complex frequency is propagating along the feeding transmission line. Two possible scenarios are investigated: firstly, the metasurface reactance X_{mts} and input reactance X_{cav} modelling the metal-backed cavity show the same electrical response, *i.e.* $X_{mts} \cdot X_{cav} > 0$; at last, the two same reactances show opposite electrical behaviour, *i.e.* $X_{mts} \cdot X_{cav} < 0$.

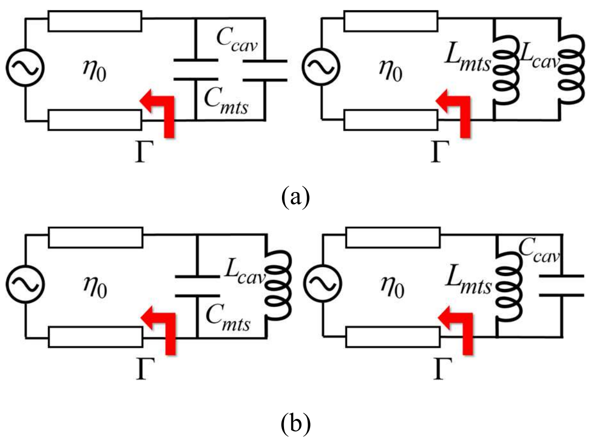


Figure 2. Transmission line model of a metasurface virtual absorber where the equivalent reactances are modelled showing the (a) same electrical response and (b) opposite electrical response.

2 Metasurface virtual absorber transmission line equivalent operative bounds

The transmission line equivalent of a metasurface-based virtual absorber are presented in Fig.2a and Fig. 2b. A lossless transmission line with characteristic impedance $\eta_0 \cong 377\Omega$ is loaded by a parallel connection between the reactance $X_{mts}(\omega)$ and $X_{cav}(\omega)$.

When metasurface and cavity exhibit the same electrical behaviour, the parallel load simply consists of two capacitances or two inductances, as shown in Figure 2a. The reflection coefficient can be expressed by:

$$\Gamma^L(\omega) = \frac{j\omega\tau - 1}{j\omega\tau + 1}, \quad \Gamma^C(\omega) = -\Gamma^L(\omega), \qquad (1)$$

where τ assumes the values $\tau^L = L/\eta_0$ and $\tau^C = \eta_0 C$ in case of inductive and capacitive load, respectively. Assuming a complex frequency excitation $\omega = \omega_r + j\omega_i$, *i.e.*, a harmonic signal of frequency ω_r, whose amplitude is modulated over time as defined by ω_i. The reflection coefficient in (1) vanishes for:

$$\omega_0^{eq} = \omega_r + j\omega_i = 0 - j\tau^{-1}, \quad \text{with} \quad \tau = \tau^{L,C} \qquad (2)$$

It is clear that in (2) the zero-reflection frequencies are purely imaginary. The impinging signal should be thus a growing exponential, whose steepness in time is given by the exponential factor $\exp(-\omega_i t)$. A metasurface virtual absorber cannot be composed of a metasurface and cavity reactance showing the same electrical behaviour, since only a static field growing in amplitude may lead to a virtual energy absorption within the cavity. Conversely, when metasurface and cavity exhibit an opposite electrical response, the equivalent circuit is a parallel connection between an inductance and a capacitance. Thus, the reflection coefficient of such circuit takes the following form:

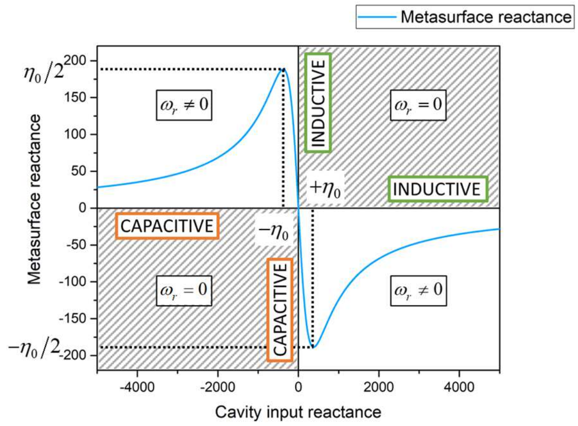


Figure 3. Metasurface reactance and cavity input reactance relation. The curve exists only when the two reactances show values of opposite sign.

$$\Gamma(\omega,\tau) = -\frac{(\omega^2 LC - 1) + j\omega\tau}{(\omega^2 LC - 1) - j\omega\tau}, \qquad (3)$$

where $\tau = L/\eta_0$ is the circuit time constant defined according to circuit theory. In this case the reflection coefficient in (3) vanishes for:

$$\omega_0^{opp} = \frac{1}{j}\left(\frac{\tau \pm \sqrt{\tau^2 - 4LC}}{2LC}\right) \qquad (4)$$

The sign of the square root argument in (4) is defined by the value of τ: for $\tau^2 < 4LC$, it is negative and the frequencies defined by (4) are two complex values, whereas for $\tau^2 > 4LC$, it is positive and the frequencies are still purely imaginary, as in the previous case. Therefore, the value of the equivalent capacitance or inductance for the metasurface for virtual absorbers must satisfy two fundamental constraints:

i) the electrical behaviour must be always opposite to the electrical behaviour of the cavity.
ii) the surface reactance must be chosen in order to have an equivalent lumped circuital element that satisfy the relation $\tau^2 < 4LC$.

For further verifying this link between metasurface and cavity reactance, in Fig. 3 we report the curve describing the relationship between the two reactance: as expected, the curve exists only when the two quantities show opposite sign values, *i.e.*, $X_{mts} \cdot X_{cav} < 0$. It is worth noting that the metasurface reactance shows a limit value, that is $\pm\eta_0/2$ when the cavity reactance equals η_0, that is, the input impedance is $Z_{cav} = \pm j\eta_0$.

3 Conclusions

In this contribution, we have presented the design limits of a metasurface-based virtual absorber. A monochromatic signal adjusted with a proper modulation must imping the structure, and only some required metasurface reactance values are allowable to enable virtual absorption. We have pursued the analysis by modelling both the metasurface reactance and the cavity input impedance through an equivalent transmission line model, detecting the corresponding complex scattering zeros in the complex frequency plane, thus establishing the underlying constraints of the impedance matching concept based on time-modulated excitation signals, in terms of load configurations. The feasibility of the virtual absorber can be extended at optical frequencies employing metasurfaces [21,22] or bidimensional arrays of scatterers [23,24].

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