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Dynamic Metasurfaces for Energy Concentration

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Abstract

In this talk, we put forward the concept of metasurface energy concentrator which can fully reflect waves incident from multiple directions into one direction. The proposed structure is based on a spatiotemporally modulated impedance sheet mounted on a grounded substrate. We show what proper traveling-wave modulations of the sheet impedance provide this functionality. We introduce a general analysis and design method based on the generalized transmission-line model. Using mathematical optimizations, we demonstrate three-port and four-port designs of energy concentrators.

1 Introduction

Electromagnetic reciprocity imposes strong restrictions on wave propagation: the forward and backward transmission coefficients for waves propagating through reciprocal media are always equal. In the past years, studies of breaking reciprocity have been mostly focused on realizing one-way wave transmission. Most common nonreciprocal devices are isolators and circulators, which have essential applications in microwave and optical engineering. However, potential applications of nonreciprocal effects are not limited to unidirectional or unequal transmission between two ports. One of possible new applications is collection of energy carried by waves from different directions to a single spot. Obviously, such functionality cannot be achieved by any reciprocal device. For example, for a reciprocal convex lens [see Fig. 1(a)], plane waves from different illumination angles always focus at separated points on the focal plane. In other words, for a reciprocal lens, it is not possible to collect all the energy from different spatial directions to a single focusing point. Why? This can be explained considering the reciprocal scenario. If such an exotic effect takes place [see Fig. 1(b)], reciprocally, a radiator positioned at the focal point will transmit all the energy to every direction at the left side, meaning that each receiving antenna at the left side receives all the energy from the radiator, which is apparently impossible! Therefore, to realize energy collection from multiple incident routes, the system must be nonreciprocal, as illustrated in Fig. 1(b).

Electromagnetic reciprocity can be broken using external magnetic bias, nonlinear or time-varying materials, and other means [1, 2]. In recent years, time-varying structures for nonreciprocal applications have raised significant attention, because these systems are compact and compatible with modern fabrication technologies [3]. Within this framework, one method is to uniformly modulate material parameters in bianisotropic structures, realizing nonreciprocal wave amplification and attenuation [4]. Another method is to modulate at different points with synchronized differences of the modulation phase [5–8, 10]. This method is generally called space-time modulation. It has been demonstrated that space-time modulation, e.g, a travelling-wave modulation, allows unidirectional wave transmission and creation of such nonreciprocal devices as isolators [6], circulators [7], nonreciprocal phase-shifters and antennas [8, 9], and even a combination these functionalities in a reconfigurable system [10].

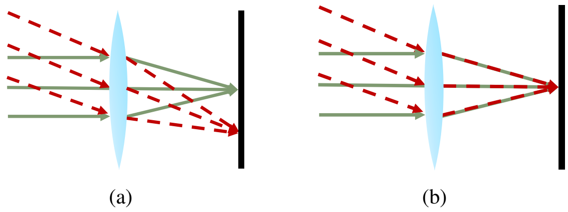

Figure 1. (a) Reciprocal and (b) nonreciprocal convex lens.

Here, we present a new nonreciprocal device which can focus energy from multiple ports to a single collection pool. Creation of such devices relies on space-time modulation of material parameters of thin sheets. We present a general theory for this functionality on a metasurface platform, and design three- and four-port energy collectors.

2 Theory

Let us consider a space-time varying impedance sheet positioned on a grounded dielectric substrate, as shown in Fig. 2(a). The impedance sheet is modelled as a series connection of spatiotemporally modulated sheet capacitance $C(z,t)$ and a fixed inductance L_0. The modulation of surface capacitance is in a travelling-wave form with spatial and temporal periods D and T, respectively. Due to the periodicity, the modulation function can be expanded in the

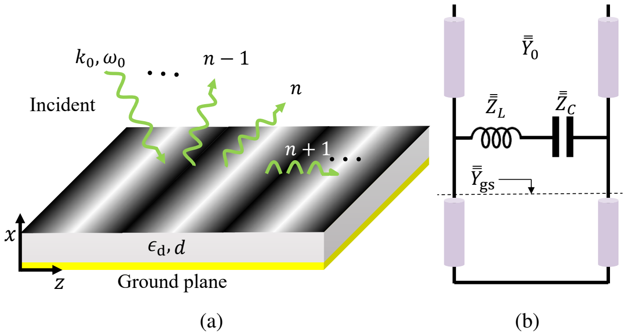

Figure 2. (a) Scattering scenario for a space-time modulated impedance sheet positioned on a grounded substrate. (b) The corresponding circuit model.

Fourier series

$$C(z,t) = \sum_{m=-\infty}^{+\infty} c_m e^{-jm(\beta_M z - \omega_M t)}, \tag{1}$$

where $\beta_M = 2\pi/D$ and $\omega_M = 2\pi/T$ are the spatial and temporal modulation frequencies, respectively. We consider a TE-polarized wave incident from $\theta = +\theta_i$ at the frequency of ω_0. The wavenumber in the surrounding space is denoted as k_0. Due to the spatial modulation, the incident wave is scattered into infinitely many spatial harmonics with tangential wavenumbers $k_{zn} = k_0 \sin\theta_i + n\beta_M$ including free-space propagating and evanescent modes, and due to the time modulation, the n-th spatial channel supports waves at $\omega_n = \omega_0 + n\omega_M$ [10]. Therefore, we can write the total tangential electric and magnetic fields on the surface as

$$\begin{aligned} E^t_{tot} &= E^t_i + E^t_s = \sum_{n=-\infty}^{+\infty} E^t_n e^{-j(k_{zn}z - \omega_n t)} \\ H^t_{tot} &= H^t_i + H^t_s = \sum_{n=-\infty}^{+\infty} H^t_n e^{-j(k_{zn}z - \omega_n t)}. \end{aligned} \tag{2}$$

To find the complex amplitudes of all the harmonics, we use the boundary conditions at each material interface of the metasurface structure. Alternatively, similarly to classical time-invariant systems, such spatiotemporal varying structures can be analyzed using the transmission-line model, where the current and voltage in the circuit are analogized to electric and magnetic fields in the structure. Since the electric/magnetic field is composed of an infinite number of harmonics, the voltage/current in the equivalent circuit is also a combination of all these harmonics. If we only consider harmonic orders from $-N$ to $+N$, we can consider the voltage/current variables as arrays composed of $2N+1$ scalar values that represent the complex amplitudes of all current/voltage harmonics. The surface impedance and characteristic impedance of the transmission line should be written as $(2N+1)$ dimensional square matrices to relate the current and voltage arrays. Reference [10] presents a detailed method for deriving the impedance/admittance matrices of all layers of space-time varying planar multilayer structures. Here, without elaborating the derivation process, we denote the admittance matrix of free space and

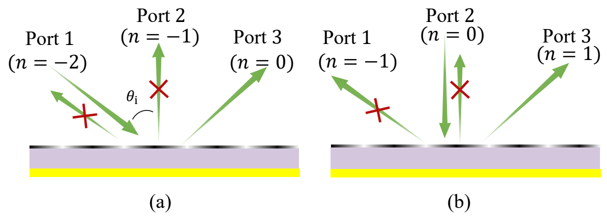

Figure 3. A three-port metasurface energy concentrator for incidences from (a) Port 1 and (b) Port 2.

the grounded substrate as $\bar{\bar{Y}}_0$ and $\bar{\bar{Y}}_{gs}$, the impedance matrix of the fixed inductance and time-varying capacitance as $\bar{\bar{Z}}_L$, and $\bar{\bar{Z}}_C$. Knowing the impedance/admittance matrices of all the circuit components, we can find the reflection matrix of the structure:

$$\bar{\bar{R}} = (\bar{\bar{Y}}_0 + \bar{\bar{Y}}_{in})^{-1}(\bar{\bar{Y}}_0 - \bar{\bar{Y}}_{in}), \tag{3}$$

where $\bar{\bar{Y}}_{in} = (\bar{\bar{Z}}_L + \bar{\bar{Z}}_C)^{-1} + \bar{\bar{Y}}_{gs}$ is the input admittance matrix of the whole structure. The reflection matrix relates the amplitudes of the incident and scattering harmonics: $\vec{v}_s = \bar{\bar{R}} \cdot \vec{v}_i$. Knowing the array of the incident-wave harmonics and the reflection matrix (determined by the metasurface properties), we can calculate the array of scattered harmonics, which includes both propagating modes in free space and evanescent modes, bound to the metasurface.

In practical metasurface engineering, it is usually desired that one or several scattering modes can be excited under a specific illumination. This is actually an optimization problem, in which, by optimizing the material parameters (c_m, ω_M, L_0, ε_d, and d), the elements in the scattering array are engineered. Here, the optimization is done by MATLAB Optimization Toolbox using the algorithm of '*fmincon*'. In the optimization process, MATLAB imposes a set of material parameters to construct the impedance/admittance matrix, and calculates the scattering coefficients. This process is repeated until the desired harmonics appear in the calculated scattered array of voltage coefficients $\vec{v}_s$.

3 Multi-channel energy concentration

In this section, based on the developed theory, we optimize the material parameters to design multichannel metasurfaces for unidirectional energy concentration. Let us first consider a three-channel metasurface. The spatial periodicity of modulation is chosen as $D = \lambda_0/\sin\theta_i$ ($\theta_i = 45°$), such that three plane-wave propagation channels $\theta_1 = \theta_i$ (Port 1), $\theta_2 = 0$ (Port 2), and $\theta_3 = -\theta_i$ (Port 3) are open in free space. In general, waves incident from any port (Port 1, 2, or 3) can be retro-reflected or reflected to the other two channels. We aim to determine the modulation function and the other material parameters that can realize full energy transmission to Port 3 for incidences from both Port 1 and Port 2.

To do that, we define two objective functions in the optimization. The first one is $|\vec{v}_s(n=-1)| + |\vec{v}_s(n=-2)| = 0$

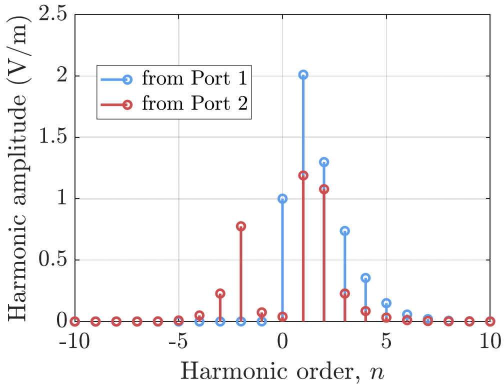

Figure 4. (a) Tangential amplitudes of scattering harmonics for incidences from Port 1 and Port 2. The amplitude of the tangential electric field component of the incident wave is 1 V/m. The optimized material parameters are $\varepsilon_{\rm d} = 4$, $d = 5$ mm, $\omega_{\rm M} = 0.0561\omega_0$ ($\omega_0 = 2\pi \times 10$ GHz), and $C(z,t) = 9.83\,[1 - 0.1741\cos(\beta_{\rm M}z - \omega_{\rm M}t)]$ with the unit of fF.

for incidence from Port 1. This objective ensures zero retro-reflection and zero transmission to Port 2, meaning that all the energy is delivered to Port 3. The second objective is $|\vec{v}_{\rm s}(n=-1)| + |\vec{v}_{\rm s}(n=+1)| = 0$ for incidence from Port 2. This objective ensures that all the energy from Port 2 is delivered to Port 3. The caption of Fig. 4 lists one set of the optimized material parameters (note that the solution is not unique). Figure 4 shows the calculated harmonic amplitudes for the optimized solution. It is easy to figure out the device scattering matrix (only free-space propagating modes are considered):

$$|\bar{\bar{S}}| = \begin{pmatrix} 0.0002 & 0.0624 & 0.7417 \\ 0.0003 & 0.0388 & 0.3157 \\ 1 & 1 & 0.6018 \end{pmatrix}. \qquad (4)$$

As we can see from Eq. (4), the scattering matrix is asymmetric, meaning that the system is nonreciprocal.

The above results show unidirectional energy flowing in a three-port system. In principle, the method can be generalized to design energy collectors with an arbitrary number of ports. However, as the number of ports increases, the control of port responses in the optimization will be more difficult. In this case, one must introduce more and more Fourier coefficients of the modulation function to increase the possibilities of finding a solution. Here, as an example, we consider a four-port metasurface (as shown in Fig. 5), with the spatial periodicity $D = 3\lambda_0/(2\sin\theta_{\rm i})$, where $\theta_1 = \theta_{\rm i}$ is the incident angle of Port 1. The spatial orientations of Port 2 and 3 are defined as $\theta_{2,3} = \pm\arcsin[1/(3\sin\theta_{\rm i})]$. Similarly to the design procedures of the three-port device, we set up three objective functions in optimization for incidences from Port 1, 2, and 3, and each of them ensures full transmission to Port 4. The caption of Fig. 5 lists one set of optimized material parameters. In this case, we apply two modulation harmonics,

$$\begin{aligned} C(z,t) = 9.81[1 &+ 0.0076\cos(\beta_{\rm M}z - \omega_{\rm M}t) \\ &- 0.1748\cos(2\beta_{\rm M}z - 2\omega_{\rm M}t + 0.39\pi)] \end{aligned} \qquad (5)$$

with the unit of fF. We can see that the total modulation function becomes non-harmonic.

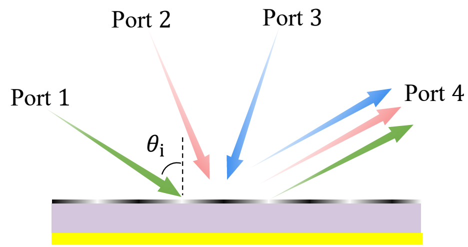

Figure 5. Schematic of four-channel operation ($\theta_{\rm i} = 45°$). The optimized material values are $\varepsilon_{\rm d} = 4$, $d = 9.47$ mm, $\omega_{\rm M} = 0.0343\omega_0$ ($\omega_0 = 2\pi \times 10$ GHz).

Following the mode matching method introduced in Sec. 2, the scattering harmonics for incidence from each port can be calculated and therefore the scattering matrix of this device can be obtained:

$$|\bar{\bar{S}}| = \begin{pmatrix} 0.0028 & 0.0745 & 0.0775 & 0.5360 \\ 0.0032 & 0.0302 & 0.0067 & 0.0529 \\ 0.0231 & 0.0238 & 0.0424 & 0.8657 \\ 0.9997 & 0.9976 & 1.0000 & 0.0798 \end{pmatrix}. \qquad (6)$$

4 Conclusions

This paper presents an idea of unidirectional energy concentration using space-time varying metasurfaces. It is shown that such functionality must break reciprocity. The idea has been verified on an example of a spatiotemporally modulated metasurface. We have demonstrated three- and four-channel energy concentrators. We expect that similar nonreciprocal structures can find applications in telecommunications, offering a possibility to effectively receive signals coming to the receiver from different directions. Another possibility is to use frequency conversion of waves coming from different directions to realize arrival directions sensors.

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