

R* Differential Forms for Calculating the Scattering Area in PSCS

Michael Walter^{* (1)}, Martin Schmidhammer⁽¹⁾, and Dmitriy Shutin⁽¹⁾ (1) German Aerospace Center (DLR), 82234 Wessling, Germany, e-mail: m.walter@dlr.de; martin.schmidhammer@dlr.de; dmitriy.shutin@dlr.de

In this contribution our goal is to discuss the concept of differential forms in order to calculate the scattering area in a non-Cartesian coordinate system. Specifically, we consider a prolate spheroidal coordinate system (PSCS), which is suitable for so called two-center problems. Models involving two-center problems appear in modeling non-stationary mobile-to-mobile propagation channels, e.g., between a moving transmitter and a moving receiver. In contrast to a channel description in classical Cartesian coordinates, the prolate spheroidal coordinates are curvilinear coordinates (see Figure 1 for an example of constant coordinate surfaces) which significantly simplify analytical tractability of the models, yet require special formalism.

The channel scattering function characterizes the probability or power density distribution of the received signal. Our intention is to determine the area of an arbitrarily oriented scattering plane. For scatterers located in a scattering plane, the area of the scattering plane can be easily determined in Cartesian coordinates. If we, however, condition the scattering on a specific delay, the scattering area becomes characterized by an intersection of a plane and an ellipsoid characterizing the delay. The determination of the corresponding intersection boundary becomes cumbersome. Thereby, a transformation of the problem into a PSCS simplifies the calculations and allows closed form solutions.

Thus, differential forms are needed to calculate the area in the PSCS. In electromagnetic field theory, for instance, differential forms have been used to simplify the Maxwell equations (see e.g., [1]). A differential form can be simply seen as an integrand see [2], e.g., the integrand for the scattering area. It naturally extends the concepts of gradient, divergence and rotation to higher dimensions. The most important property is the antisymmetry of the wedge product, see (1). Only with the help of the wedge product the differential area in the PSCS can be determined.

$$d\xi \wedge d\eta = -d\eta \wedge d\xi. \tag{1}$$

We present why differential forms are necessary to change the variables from Cartesian coordinates and differentials to prolate spheroidal coordinates and differentials.



Figure 1. A prolate spheroidal coordinate system is shown with its constant coordinate surfaces, i.e., ellipsoid, hyperboloid, and half-plane. Furthermore, the relationship to a local Cartesian coordinate system is indicated.

The presented approach is useful to calculate the area of an arbitrarily oriented planes bounded by certain geometric objects. For example, a plane surface bounded by a sphere, where a spherical coordinate system would be used to describe the sphere and the resulting area of the intersection of sphere and the plane would be an arbitrarily oriented circle.

References

- [1] K. F. Warnick and P. H. Russer, "Differential Forms and Electromagnetic Field Theory (Invited Paper)," Progress in Electromagnetics Research, Vol. 148, 83-112, 2014, doi:10.2528/PIER14063009.
- [2] D. Bachmann, A Geometric Approach to Differential Forms. Birkhäuser Boston, October 2012, doi:10.1007/978-0-8176-8304-7.