# Frequency Splitting and other Things that Filters and Wireless Power Transfer Systems have in Common

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## Abstract

In this paper, the phenomenon of frequency splitting in resonating Wireless Power Transfer Systems is considered. In particular, the property of impedance inverters of converting an impedance into an admittance is here exploited to give a simple and intuitive explanation of that phenomenon. Furthermore, some analogies between WPT systems and filters are here highlighted showing that the frequency splitting phenomenon is also present in Chebyshev filter responses.

## 1 Introduction

The basic principles of the Wireless Power Transfer (WPT) have been introduced by Nikola Tesla in the first years of the 20th century. After that, for a long time, not a lot effort was put in this kind of research. In the last years however, after the paper of the MIT group [1], a lot of contributions on this topic have been published and many different applications, ranging from biomedical devices to automotive, have been proposed.

One of the key points of the WPT is represented by the so-called frequency splitting phenomenon. It consists in the splitting of the resonant frequencies when the coupling between two coils of a WPT system became larger than a threshold value. Several papers were published to explain the frequency-splitting phenomenon. As an example, in [2]-[3] equivalent circuits are used, giving a quite rigorous explanation. In [4] instead, a full wave and more complex modelling has been used.

In this paper, an equivalent circuit like that used for filters and based on impedance inverters has been exploited to model an ideal WPT systems. Impedance inverters have the capability of transforming an impedance into an admittance (and vice versa). This property has been used here to explain in a very simple and intuitive way the splitting frequency phenomenon. Furthermore, some analogies between WPT systems and filters have been highlighted, showing that filter Chebyshev responses use the splitting frequency phenomenon (even though in filter theory is not used that name).

## 2 Equivalent Circuit

To better understand the phenomenon of the frequency splitting in Wireless Power Transfer (WPT), the model of the WPT system here considered is kept as simple as possible and losses are not considered.

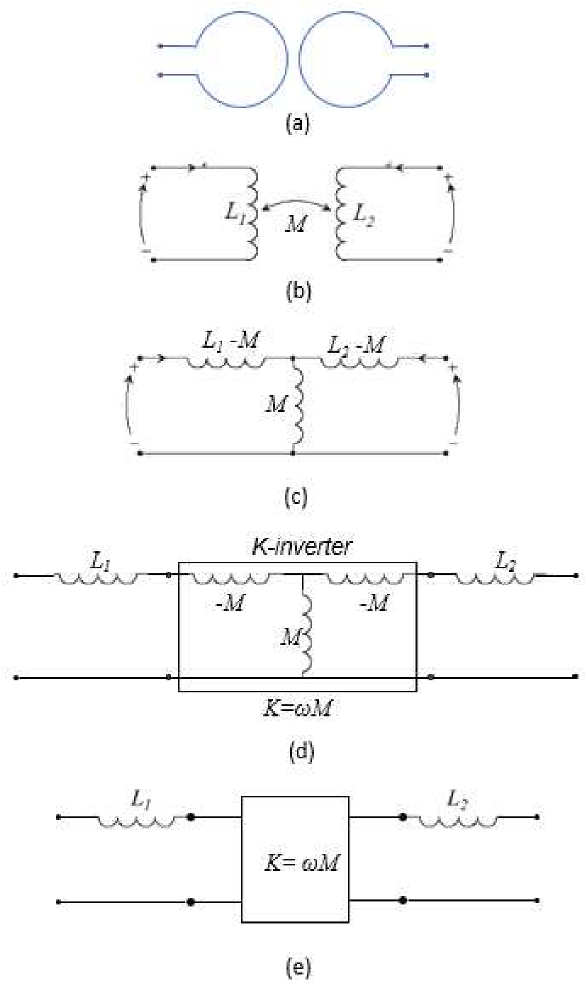


**Figure 1.** Coupled coils and their equivalent circuits. (a) Mutually coupled coils. (b) Electronic network symbol. (c) T-network equivalent circuit. (d) T-network equivalent circuit with separed induttance and mutual inductance contributions. (e) Impedance inverter based equivalent circuit.

The two coupled coils (Figure 1a) have a very well-known electronic symbol (Figure 1b) which behavior is described by the equation:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = j\omega \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}, \tag{1}$$

Where $L_i$ represents the self-inductance of the i-th coil (i=1,2) whereas M represent the mutual coupling [5]. It is very simple to demonstrate that the same response can be obtained by the T-network of Figure 1c. By splitting the contribution of the horizontal branches into two contributions the circuit of Figure 1d is obtained. The T-network consisting of an inductance M in the vertical branch and -M in the horizontal branches has the following impedance matrix.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = j\omega \begin{bmatrix} 0 & M \\ M & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}. \tag{2}$$

That impedance matrix also represents the impedance matrix of an impedance inverter $K = \omega M$ and this leads to the well-known circuit of Figure 1d where two coupled coils are represented by the self-inductances of each coil connected through an impedance inverter.

Considering that we are dealing with resonating wireless power systems, according to Figure 2., two capacitances are added to the system to make it resonant. Obviously, in the physical system the added capacitances can model both lumped capacitances added to the coil and parasitic capacitance of the coil itself.

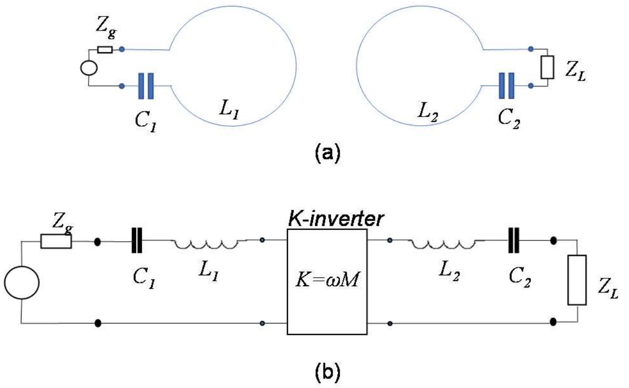


**Figure 2.** Resonant synchronous coupled coils. (a) Physical structure. (b) Equivalent circuit.

The circuit of Figure 2b is a well-known circuit used for both WPT systems and filters. Actually, that circuit, especially for filters, is used in the version with two additional impedance inverters between the input (output) impedance and first (last) series resonator, but for our scope it is convenient to use the one in Figure 2. The fact that filters and coupled coils shares the same equivalent circuit means that coupled resonant coils can be seen as a two-pole filter for proper values of circuit components.

## 3 Frequency splitting

In this paragraph, a very simple method for the explanation of frequency splitting behavior based on a property of impedance inverters is shown.

As suggested by its name, one of the most important property of an impedance inverter is its capability of inverting the load impedance. This can be easily demonstrated by using eq. (2) and it is illustrated in Figure 3: an impedance connected to the port 2 of the impedance inverter is seen from port 1 as the inverse of the impedance multiplied by the square of K, where K is the value of the impedance inverter.

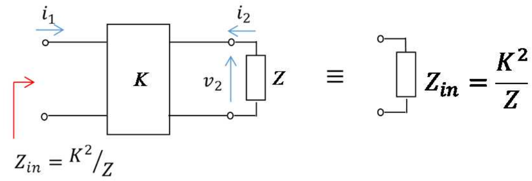


**Figure 3.** Impedance inverter and its capability of inverting load impedence.

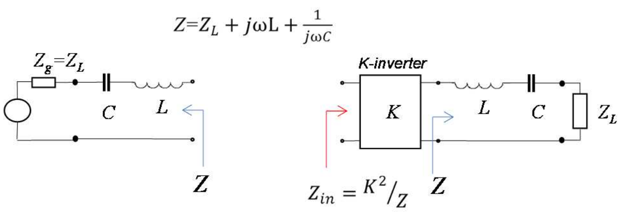


**Figure 4.** Equivalent network of the synchronous resonant coupled coils separated in two branches.

This property is exploited to analyze the resonances of the circuit in Figure 2. According to Figure 4, the circuit is divided into two branches. The impedance inverter is left in the right branch. The left branch can be seen as a generator with an internal impedance Z where Z is:

$$Z = Z_L + j\omega L + \frac{1}{j\omega C} \tag{3}$$

Considering the right branch, because of the symmetry of the circuit, the impedance connected to right port of the K-inverter is the same of (3). According to the property of the K-inverter, the input impedance $Z_{in}$ seen at the left port of the K-inverter is then the inverse of Z multiplied by the square of K:

$$Z_{in} = \frac{K^2}{Z}. \tag{4}$$

According to the maximum power transfer theorem, when $Z_{in} = Z^*$ all incoming power from the left branch (behaving as a generator with internal impedance Z) is delivered to the right branch (load). In that condition all the power arrives to the load $Z_L$, the scattering parameter $S_{11}$ has a reflection zero and the two coils resonate. This is illustrated in the graphs from Figure 5 to Figure 8. For all those graphs the following values have been considered: $Z_L = Z_g = 0.1\,\Omega$, $L = C = \frac{1}{\omega_0}$ and $\omega_0 = 1$, where $\omega_0$

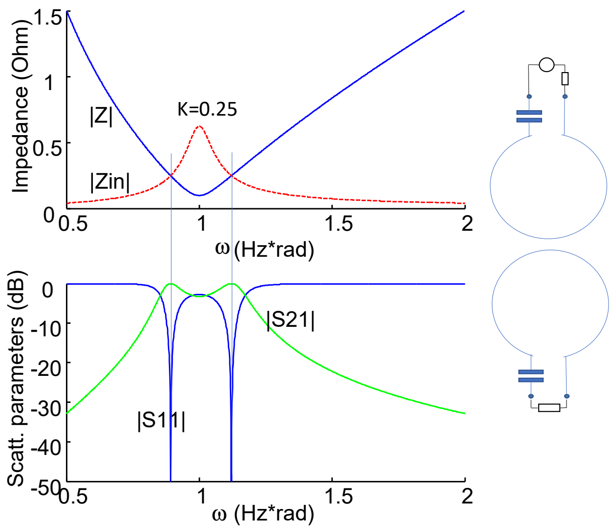


**Figure 5.** Coupled coils: branch input impedances |Z| and |Zin| and scattering parameters for K=0.25.

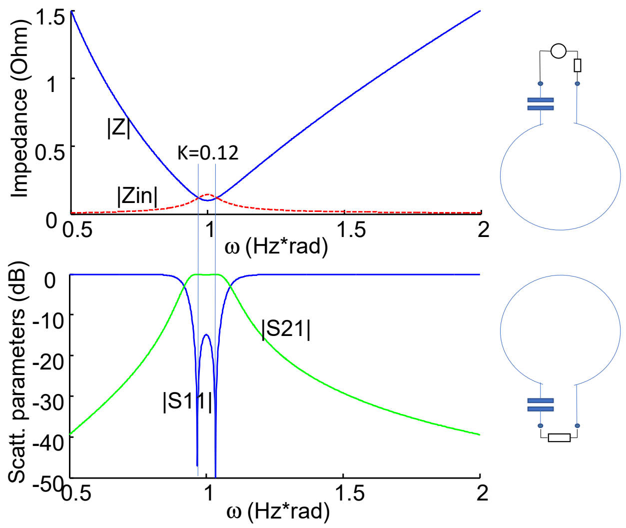


**Figure 6.** Coupled coils: branch input impedances |Z|, |Zin| and scattering parameters for K=0.12. This configuration corresponds to the Chebyshev filter response.

represents the resonant frequency of the isolated coils (more precisely: the isolated coil plus its capacitance, according to Figure 2b).

In Figure 5, the case of two closed coils which mutual coupling corresponds to an impedance inverter K = 0.25 is shown. In the upper graph the amplitude of the impedances Z and $Z_{in} = K^2/Z$ are plotted. In practice the graph of $Z_{in}$ corresponds to the graph of the inverse of Z multiplied by a constant ($K^2$). That constant is the only value that changes when the distance between the coils changes. There are two frequency points where the two curves intersect. At those frequencies coils resonate, and all the power generated by the source arrives to the load.

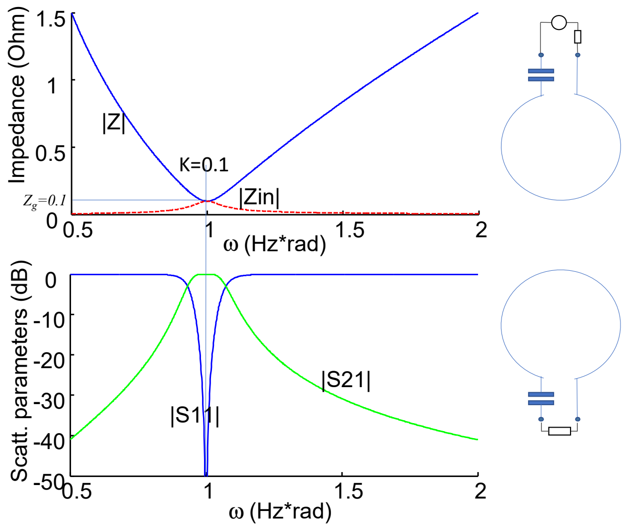


**Figure 7.** Coupled coils: branch input impedances |Z|, |Zin| and scattering parameters for K=0.1. This configuration corresponds to the maximally flat filter response.

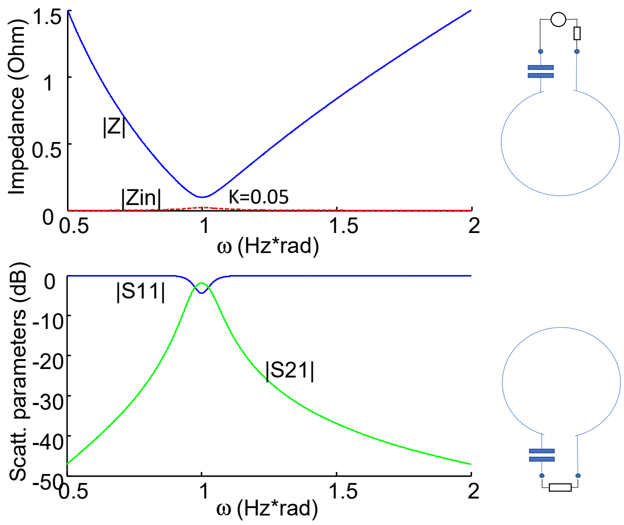


**Figure 8.** Coupled coils: branch input impedances |Z|, |Zin| and scattering parameters for K=0.12.

This can be also seen in the scattering parameter graph shown below the impedance graph, where $S_{11} = 0$ and $S_{12} = 1$ at the frequency where Z and $Z_{in}$ intersect. The fact that the two curves intersect, means that $|Z| = |Z_{in}|$ as curves represent the amplitude only. However, in order to obtain the matching, additional conditions are necessary: real parts shall have the same amplitude and sign and imaginary parts shall have same amplitude and opposite sign. Even though for the sake of conciseness the graph of real and imaginary parts is not shown here, this condition is satisfied in all cases when symmetry is preserved in the circuit and this is true for all graph here presented.

Increasing the coil distance, the value of the impedance inverter decreases. In Figure 6, the case with K = 0.12 is shown. The impedance Z remains the same of that in Figure 5, while the inverse of Z used to obtain $Z_{in}$ is multiplied by a smaller constant. Consequently, the two intersection frequencies become closer. The system is now working as a filter with Chebyshev response, as can be clearly seen from the graph of the scattering parameters. By keeping increasing the coil distance (i.e. decreasing K), at a certain point the two intersections collapse into one. In our example this happens when K=0.1, as shown in Figure 7 where Z and Zin have the same value in just one frequency point, corresponding to the resonance frequency of the isolated coils at $\omega_0$. As shown in the scattering parameter graph, this results in a response with a single resonance. This corresponds to a filter having a maximally flat response.

Increasing the coil distance, K became lower than 0.1 and, as shown in Figure 8, Z and $Z_{in}$ no longer intersect. This means that the matching condition $Z_{in} = Z^*$ is no longer satisfied and part of the power is back-scattered, as can be seen from the scattering parameters graph. In any case, because Z has a minimum point in $\omega_0$ (in our example $\omega_0 = 1$), that point represent the angular frequency where the curves are closer, and this explains why $\omega_0$ is still the angular frequency at which there is the higher amount of power that is dissipated on the load.

# 7 References


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