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A Circuital Approach to Control the Response of Conformal Metasurfaces

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Abstract

In this paper we introduce an analytical method to control the response of conformal metasurfaces with respect to an impinging plane wave as excitation source. We first develop and present the analytical framework in order to achieve a full control over the current distribution within the metasurface. Then, in order to prove the theoretical model, we design a test-case through a numerical solver, consisting in a curved magnetic metasurface excited by a plane wave. The obtained numerical results are in excellent agreement with the theory, confirming the validity of the proposed approach. In this way, an accurate and effective control over conformal magnetic metasurfaces can be accomplished, extending the possible technological applications with respect to the traditional planar configuration.

1 Introduction

Metamaterials and metasurfaces have received an incredible attention in the last decades [1]. This interest is motivated by the extraordinary electromagnetic properties that can be achieved by similar structures. Indeed, it has been proved that negative permittivity or permeability, and even negative refractive index (i.e., the "superlens" concept) can be achieved [2]-[3].

Among this wide topic, a particularly interesting class is constituted by magnetic metasurfaces [1]. Metasurfaces retains a fundamental advantage with respect to their 3D counterpart (i.e., metamaterials). As a matter of fact, metasurfaces are commonly designed as thin films that present a desired surface impedance and, because of their compactness, can be easily integrated within modern technological applications. Magnetic metasurfaces are typically employed in magnetic resonance imaging, resonant inductive wireless power transfer and also for electromagnetic absorbers [4]-[6].

Typically, the traditional approach to design metasurfaces is by adopting some fundamental hypothesis. Indeed, the unit-cell must have subwavelength dimensions, in order to apply the homogenization criterion [7]. Further, the array constituting the metasurface is considered infinite in extent; in this sense, periodic boundary conditions are commonly employed to design the unit-cell. Finally, an impinging plane-wave is adopted as excitation source. Nonetheless, these hypotheses are far too ideal, especially when the operating frequency is relatively low. Indeed, in such cases, the array constituting the surface is not infinite but constituted by a limited number of unit-cells. Hence, strong truncation effects may arise and the actual metasurface response is significantly different from the ideal case. Moreover, this typical approach does not consider conformal shapes. Although different works in the literature faced this latter problem [8], nevertheless a method to practically control the metasurface behavior is still lacking.

Thus, in order to overcome these limitations, this paper propose an analytical method to model and control magnetic metasurfaces, starting from a circuital approach. We demonstrate through preliminary full-wave simulations that it is possibile to shape the response of a conformal metasurface and to eliminate the truncation effects. The availability of a simple and effective model can be significantly helpful to facilitate the design and to avoid computationally expensive simulations for the optimization of the system.

The paper is organized as follows. Section 2 is devoted to introducing the analytical framework to model and control the response of a generic magnetic metasurface; in Section 3, we present the numerical test-case and the obtained results are discussed. Finally, Conclusions follow.

2 Methods

Supposing to have a spherical shape, externally covered by a magnetic metasurface, and excited by an impinging plane wave (Fig. 1). Further, we hypothesize that the metasurface is composed by N magnetic resonant inclusions (like spiral or split rings resonators, for instance) and that the operating frequency is relatively low to consider the magnetic induction as the prevalent exciting mechanism for the unit-cells. At this point, it is possible to derive the circuital equations for the N unit-cells composing the metasurface:

$$\begin{cases} j\omega H cos(\theta_1)A + Z_{11}I_1 + Z_{12}I_2 \cdots + Z_{1N}I_N = 0 \\ \vdots \\ j\omega H cos(\theta_N)A + Z_{N1}I_1 + Z_{N2}I_2 \cdots + Z_{NN}I_N = 0 \end{cases} \quad (1)$$

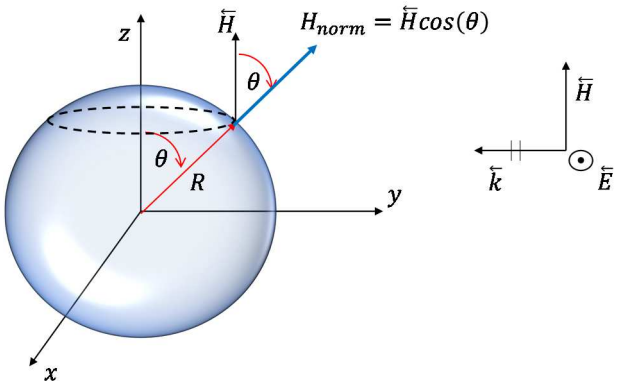


Figure 1. Schematic draw depicting the considered case: a plane wave impinging on a spherical shape where a magnetic metasurface can be laid.

where θ_i is the angle between the incident H-field and the normal direction to the i-th unit-cell surface, in turn represented by the quantity A.
By expressing the current flowing in each unit-cell I_i as a linear combination of a reference current I_x ($I_i = c_i I_x$), then the loading condition for the i-th cell to obtain the desired current distribution becomes:

$$Z_{ii} = cos(\theta_i)\left[Z_{IN}^{i} - \frac{\sum_{\substack{j=1 \\ j\neq i}}^{N} c_j Z_{ij}}{c_i cos(\theta_i)}\right] \quad (2)$$

where $Z_{IN}^{i} = \frac{-j\omega HA}{c_i I_x}$ represents the input impedance of the i-th unit-cell.

3 Numerical Test-case

3.1 Adopted CAD Model

In order to verify our proposed analytical method, we designed a numerical test-case, exploiting an electromagnetic solver based on the Method of Moments (Feko Suite, Altair, Troy, MI, USA).

We realized a 7×7 array of spiral resonators. Each unit-cell consists of a 8-turns planar spiral with a 4 cm diameter, realized with a 1.4 mm diameter lossy copper wire. We loaded each cell with an opportune capacitor in order to obtain the desired response at the chosen working frequency (6 MHz). At this point, we projected the realized planar structure onto the surface of a sphere with radius 40 cm in order to realize an example of conformal surface.

Then, we used as excitation source a plane wave directed along the x-axis with respect to Fig. 2. In this way, we recreated the same geometrical configuration described in Fig. 1 between the H-field vector and the unit-cells' surface. The overall CAD model developed in Feko is reported in Fig. 2.

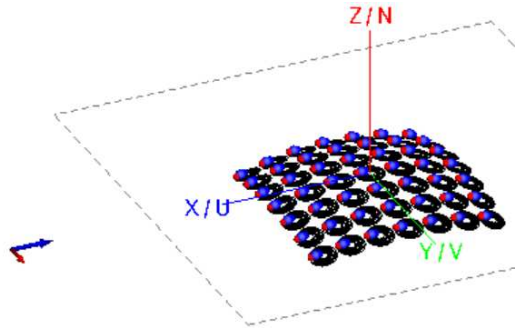
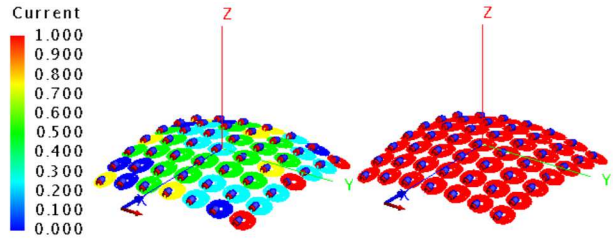


Figure 2. CAD model of the proposed test-case: the 7×7 metasurface is projected onto the surface of a sphere with radius 40 cm.



(a) (b)

Figure 3. Obtained numerical results: the "standard" (a) and the "homogenized" (b) configurations are compared in terms of current distribution. As evident, the "homogenized" case is able to fully consider the finite size and the curved shape of the array.

The proposed test-case was conceived to demonstrate that it is possible to eliminate truncation effects and inhomogeneities in the unit-cells' response when they are placed in a finite-size and conformal array. In particular, we compare the case when the unit-cells are all loaded with the same capacitor (i.e., as if the ideal hypotheses about metasurface design are matched) against the case when the loads are corrected following (2). From a theoretical point of view, this is equivalent to impose to zero the unit-cells' input impedance Z_{IN}^{i} and to choose the current coefficient c_i all equal to 1.

3.2 Results

We performed full-wave simulations for the two configurations described before. In the "standard" configuration, the unit-cells have been all loaded with the same capacitor making them resonate singularly at 6 MHz; in the "homogenized" configuration, the capacitive loads were selected based on (2), thus fully considering the finite array dimension and its conformal shape.

Fig. 3 reports the obtained results in terms of normalized currents, in the "standard" (a) and "homogenized" (b) configurations. As evident from the figure, the "homogenized" case is able to perfectly smooth the current distribution inside each unit-cell, thus eliminating the truncation effect. Moreover, our proposed theoretical

approach takes into account also the conformal shape of the structure through the angle θ_i in (2). These numerical results demonstrate the validity of our approach; thus, the proposed analytical model can be exploited in order to achieve a more effective and practical design for conformal metasurfaces.

4 Conclusions

In this paper we introduced an analytical approach to describe and control the response of conformal magnetic metasurfaces. Differently from the classical approach in the literature, our proposed method fully considers the finite extent of the slab along with the eventual curved profile.

In order to demonstrate the validity of the proposed procedure, we designed a numerical test-case, consisting of a curved magnetic metasurface excited by an impinging plane wave. We obtained an excellent agreement with the theoretical considerations, proving that it is possible to homogenize the response of all the unit-cell within the conformal surface, avoiding undesired truncation effects.

The methodology herein presented can pave the way to a more effective way to design metasurfaces when conformal shape must be considered. Further development will be directed to analyze more complex current distributions and to explore possible exotic properties of conformal metasurfaces.

5 References

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