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Efficient Computation of Fock Integrals

L. Wilson Pearson[(1)], Reyhan Baktur*[(2)]
(1) Professor Emeritus, The Holcombe Department of Electrical and Computer Engineering, Clemson University, Clemson, SC 29634
(2) Departmental of Electrical and Computer Engineering, Utah State University, Logan, UT 84322

The Russian mathematical physicist. V. A. Fock was highly prolific through the middle part of the twentieth century. His work encompassed quantum mechanics and electromagnetics, In this presentation, we review an efficient method for computing the so-called Fock-type integrals as shown below, where expression (1) is associated with the far-field diffracted by a locally cylindrical impedance surface and (2) with the field observed on a locally cylindrical impedance surface.

$$G(X,q) = \frac{e^{-j(\pi/4)}}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{V'(t) - qV(t)}{W_2'(t) - qW_2'(t)} e^{-jXt}\, dt \ , \tag{1}$$

$$g(X,q) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-jXt}}{W_2'(t) - qW_2'(t)}\, dt \ . \tag{2}$$

Fock-type integrals are difficult to integrate because of the presence of both rapidly- and slowly-varying factors. In the 60's, a limited tabulation was prepared by Logan [1], presumably through the use of mechanical calculators of that era. With the explosion of numerical computation in the 80's, a strong pull arose for efficient computer routines for evaluation of Fock-type integrals.

The computation of the integration is obscured by Fock's Airy function notation, which appears in V and W_2 in expressions (1) and (2). The key construction of a clean quadrature is to replace the Fock-Airy functions with Olver's notation. The Oliver functions can be transformed among their three forms through rotations of 120 degrees. The integrals are then readily sorted into contributions along 120 degree rays in the complex plane. On properly chosen rays, the Oliver Airy functions are slowly varying, monotonically decaying functions that allow any numerical quadrature that one chooses.

It is noteworthy that this scheme is amenable to complex valued surface impedance in the Fock integrals, a significant expansion of the embrace beyond that of mechanical calculations.

References

[1] N. A. Logan, "General research in diffraction theory," *Lockheed Missiles and Space Div.Rep.LMSD-288087 (Nat.Tech. Info. Service AD No. 241228 and 243182)*, Vols **1** and **2**, 1959.