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Stimulation of a Layered Medium by N External Dipoles

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Abstract

A layered spherical medium is excited by a number of external electric dipoles. Boundary-value problems for the involved individual and overall electromagnetic fields are formulated. The direct scattering problem is solved by a T-Matrix method. Explicit formulas for the far-field, the overall cross section and the total interaction cross sections are derived. Some preliminary numerical results on the variations of these cross sections are presented.

1 Introduction

Electromagnetic scattering problems for fields generated by more than one sources are motivated by a variety of applications over different disciplines, including, e.g. optical diffusion [1], electromagnetic activity of the brain [2], cancer-treatment techniques [3], and antenna design [4]. Besides, stimulation by multiple incident fields has been used in inverse scattering problems for isotropic [5], layered [6], and anisotropic [7] media.

In this work, we present a variant of the T-Matrix method for a layered sphere, excited by N dipoles located in the sphere's exterior. Specifically, we distinguish the individual fields and the overall field by using suitable *excitation operators*, and, in this way, express the overall field by coefficients given as sums of the individual scattering coefficients. This is particularly important in problems involving spherical waves, since it aids the determination of the additivity of the individual energy fluxes; such topics were studied for light scattering by particles in [8] and [9] and for acoustic waves in [10]. Furthermore, by introducing *boundary-transition vectors*, we obtain a separable form for the unknown coefficients, which allows the fast and efficient computation of the overall and individual fields' coefficients. This can also prove useful for inverse-problems schemes.

2 Mathematical Formulation

We consider a spherical scatterer V of radius R_1, which is divided by $P-1$ spherical surfaces S_p into P nested, concentric spherical shells V_p $(p=1,\ldots,P)$. The first $P-1$ layers V_p (i.e. $R_{p+1}<r<R_p$ for $p=1,\ldots,P-1$) are homogeneous and isotropic and are characterized by wavenumber k_p, dielectric permittivity ε_p and magnetic permeability μ_p. The core V_P $(0\le r<R_P)$ can be perfectly electric conducting (PEC) or dielectric. The exterior $V_0=\mathbb{R}^3\setminus V$ of the scatterer is homogeneous and isotropic, characterized by wavenumber k_0, dielectric permittivity ε_0 and magnetic permeability μ_0.

The scatterer is excited by N electric dipoles arbitrarily distributed in the sphere's exterior V_0. The position of each dipole with respect to the sphere's center is identified by the vector $\mathbf{r}_j$. These dipoles emit spherical waves, with their *individual primary fields* given by [11]

$$\mathbf{E}_j^{\mathrm{pr}}(\mathbf{r})=\mathrm{i}\omega\mu_0\widetilde{\mathbf{G}}(\mathbf{r},\mathbf{r}_j)\cdot\mathbf{p}_j, \tag{1}$$

where $\widetilde{\mathbf{G}}(\mathbf{r},\mathbf{r}_j)$ is the free-space dyadic Green's function and $\mathbf{p}_j$ is the moment of each dipole.

Each individual primary field interacts with the scatterer, generating *individual secondary fields* $\mathbf{E}_{p,j}^{\mathrm{sec}}(\mathbf{r})$ in V_p. The *individual total field* in V_p due to a source at $\mathbf{r}_j$ is denoted by $\mathbf{E}_{p,j}^{\mathrm{t}}(\mathbf{r})$. In V_0, it holds

$$\mathbf{E}_{0,j}^{\mathrm{t}}(\mathbf{r})=\mathbf{E}_j^{\mathrm{pr}}(\mathbf{r})+\mathbf{E}_{0,j}^{\mathrm{sec}}(\mathbf{r}). \tag{2}$$

The superpositions of all individual primary fields in V_0 and all individual total fields in V_p will be denoted by $\mathbf{E}^{\mathrm{pr}}(\mathbf{r})$ and $\mathbf{E}_p(\mathbf{r})$ and called *0-excitation primary field* and *overall field of V_p*, respectively.

On the boundaries of layers V_p $(p=1,\ldots,P-1)$, holds

$$\hat{\mathbf{r}}\times\mathbf{E}_{p-1}(\mathbf{r})=\hat{\mathbf{r}}\times\mathbf{E}_p(\mathbf{r}),\quad r=R_p \tag{3}$$

$$\frac{1}{\mu_{p-1}}\hat{\mathbf{r}}\times\nabla\times\mathbf{E}_{p-1}(\mathbf{r})=\frac{1}{\mu_p}\hat{\mathbf{r}}\times\nabla\times\mathbf{E}_p(\mathbf{r}),\quad r=R_p. \tag{4}$$

The individual and overall fields in V_0 satisfy the Silver-Müller radiation condition. For a PEC core, we have

$$\hat{\mathbf{r}}\times\mathbf{E}_{P-1}(\mathbf{r})=\mathbf{0},\quad r=R_P, \tag{5}$$

whereas for a dielectric core, (3)-(4) hold for $p=P$ as well.

The *individual far-fields* $\mathbf{g}_j(\hat{\mathbf{r}})$ and *overall far-field* $\mathbf{g}(\hat{\mathbf{r}})$ are defined, respectively, by

$$\mathbf{E}_{0,j}^{\mathrm{sec}}(\mathbf{r})=\mathbf{g}_j(\hat{\mathbf{r}})h_0(k_0r)+O(r^{-2}),\quad r\to\infty, \tag{6}$$

$$\mathbf{E}_0^{\mathrm{sec}}(\mathbf{r})=\mathbf{g}(\hat{\mathbf{r}})h_0(k_0r)+O(r^{2}),\quad r\to\infty, \tag{7}$$

where h_0 is the 0-order spherical Hankel function of the first kind. *Individual* and *overall scattering cross sections* are defined, respectively, by

$$\sigma_j = \frac{1}{k_0^2}\int_{S^2} |\mathbf{g}_j(\hat{\mathbf{r}})|^2 \mathrm{d}s(\hat{\mathbf{r}}), \tag{8}$$

$$\sigma = \frac{1}{k_0^2}\int_{S^2} |\mathbf{g}(\hat{\mathbf{r}})|^2 \mathrm{d}s(\hat{\mathbf{r}}). \tag{9}$$

The interactions between individual fields generate additional energy fluxes, which are quantified by the *total interaction scattering cross section*, defined as the difference between the overall cross section and the sum of the individual cross sections, and given by [10]

$$\begin{aligned}\sigma^{\mathrm{T}} &= \sigma - \sum_{j=1}^{N}\sigma_j \\ &= \frac{2}{k_0^2}\sum_{j=1}^{N-1}\sum_{\nu=j+1}^{N}\int_{S^2}\mathbf{g}_j(\hat{\mathbf{r}})\cdot\overline{\mathbf{g}_\nu(\hat{\mathbf{r}})}\mathrm{d}s(\hat{\mathbf{r}}).\end{aligned} \tag{10}$$

3 Excitation Operators and Overall Superposition Method

3.1 Excitation Operators

The individual primary fields are given by (1), where

$$\widetilde{\mathbf{G}}(\mathbf{r},\mathbf{r}_j) = \frac{\mathrm{i}k_0}{4\pi}\sum_{n,m,s}\frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!}\varepsilon_m \times \begin{cases}\mathbf{M}_s^3(\mathbf{r},k_0)\mathbf{M}_s^1(\mathbf{r}_j,k_0)+\mathbf{N}_s^3(\mathbf{r},k_0)\mathbf{N}_s^1(\mathbf{r}_j,k_0), r>r_j\\ \mathbf{M}_s^1(\mathbf{r},k_0)\mathbf{M}_s^3(\mathbf{r}_j,k_0)+\mathbf{N}_s^1(\mathbf{r},k_0)\mathbf{N}_s^3(\mathbf{r}_j,k_0), r<r_j,\end{cases} \tag{11}$$

where $\mathbf{r}_j = (r_j,\theta_j,\phi_j)$, with $r_j > R_1$, for $j = 1,\dots,N$, the position vectors of the dipoles, $\sum_{n,m,s}$ denotes the triple sum with respect to $n\in\mathbb{N}$, $m = 0,\dots,n$, and $s\in\{e,o\}$, while ε_m is the Neumann factor.

The individual fields in V_p are expanded as [12]

$$\begin{aligned}\mathbf{E}_{p,j}(\mathbf{r}) = -\frac{\omega\mu_p k_0}{4\pi}\sum_{n,m,s}\frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!}\varepsilon_m\times \\ \Big[\mathbf{M}_s^1(\mathbf{r},k_p)\left(a_{n,p}^j\mathbf{M}_s^1(\mathbf{r}_j,k_0)+b_{n,p}^j\mathbf{M}_s^3(\mathbf{r}_j,k_0)\right) \\ +\mathbf{N}_s^1(\mathbf{r},k_p)\left(c_{n,p}^j\mathbf{N}_s^1(\mathbf{r}_j,k_0)+d_{n,p}^j\mathbf{N}_s^3(\mathbf{r}_j,k_0)\right) \\ +\mathbf{M}_s^3(\mathbf{r},k_p)\left(\tilde{a}_{n,p}^j\mathbf{M}_s^1(\mathbf{r}_j,k_0)+\tilde{b}_{n,p}^j\mathbf{M}_s^3(\mathbf{r}_j,k_0)\right) \\ +\mathbf{N}_s^3(\mathbf{r},k_p)\left(\tilde{c}_{n,p}^j\mathbf{N}_s^1(\mathbf{r}_j,k_0)+\tilde{d}_{n,p}^j\mathbf{N}_s^3(\mathbf{r}_j,k_0)\right)\Big]\cdot\mathbf{p}_j,\end{aligned} \tag{12}$$

where $\mathbf{M}_s^\ell, \mathbf{N}_s^\ell$ with $\ell\in\{1,3\}$ are the spherical vector wave functions.

Now, we define the following *excitation operators*

$$\begin{aligned}\mathscr{M}_{n,m,s}^q(\mathbf{x}) = &\frac{\mathrm{i}k_0}{4\pi}\frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!}\varepsilon_m\times \\ &\sum_{j=1}^{N}\left(x_j\mathbf{M}_s^q(\mathbf{r}_j,k_0)\cdot\mathbf{p}_j\right),\end{aligned} \tag{13}$$

$$\begin{aligned}\mathscr{N}_{n,m,s}^q(\mathbf{x}) = &\frac{\mathrm{i}k_0}{4\pi}\frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!}\varepsilon_m\times \\ &\sum_{j=1}^{N}\left(x_j\mathbf{N}_s^q(\mathbf{r}_j,k_0)\cdot\mathbf{p}_j\right),\end{aligned} \tag{14}$$

where $q\in\{1,3\}$, and $\mathbf{x} = (\mathrm{x}_1,\dots,\mathrm{x}_N)$ an arbitrary vector of $\mathbb{R}^N$. Moreover, we introduce the notations

$$\begin{array}{ll}\mathscr{A}_{n,m,s}^p = \mathscr{M}_{n,m,s}^1(\mathbf{a}_n^p), & \tilde{\mathscr{A}}_{n,m,s}^p = \mathscr{M}_{n,m,s}^1(\tilde{\mathbf{a}}_n^p)\\ \mathscr{B}_{n,m,s}^p = \mathscr{M}_{n,m,s}^3(\mathbf{b}_n^p), & \tilde{\mathscr{B}}_{n,m,s}^p = \mathscr{M}_{n,m,s}^3(\tilde{\mathbf{b}}_n^p)\\ \mathscr{C}_{n,m,s}^p = \mathscr{N}_{n,m,s}^1(\mathbf{c}_n^p), & \tilde{\mathscr{C}}_{n,m,s}^p = \mathscr{N}_{n,m,s}^1(\tilde{\mathbf{c}}_n^p)\\ \mathscr{D}_{n,m,s}^p = \mathscr{N}_{n,m,s}^3(\mathbf{d}_n^p), & \tilde{\mathscr{D}}_{n,m,s}^p = \mathscr{M}_{n,m,s}^3(\tilde{\mathbf{d}}_n^p)\end{array}$$

where $\mathbf{x}_n^p = (x_{1,n}^p,\dots,x_{N,n}^p)$ with $x\in\{a,b,c,d\}$ are the vectors with components the coefficients of the individual secondary fields. Considering (11) and (12), we obtain the following expansions for the 0-excitation primary field

$$\mathbf{E}^{\mathrm{pr}}(\mathbf{r}) = \mathrm{i}\omega\mu_0\times \begin{cases}\sum_{n,m,s}\mathbf{M}_s^3(\mathbf{r},k_0)\mathscr{M}_{n,m,s}^1(\mathbf{1})+\mathbf{N}_s^3(\mathbf{r},k_0)\mathscr{N}_{n,m,s}^1(\mathbf{1}),\\ r>\max\{r_1,\dots,r_N\}\\ \sum_{n,m,s}\mathbf{M}_s^1(\mathbf{r},k_0)\mathscr{M}_{n,m,s}^3(\mathbf{1})+\mathbf{N}_s^1(\mathbf{r},k_0)\mathscr{N}_{n,m,s}^3(\mathbf{1}),\\ r<\min\{r_1,\dots,r_N\}\end{cases} \tag{15}$$

where $\mathbf{1}$ denotes the N-dimensional vector $(1,1,\dots,1)$. Similarly, the expansion of the overall field of V_p is

$$\begin{aligned}\mathbf{E}_p(\mathbf{r}) = \mathrm{i}\omega\mu_p\sum_{n,m,s}\Big[&\mathbf{M}_s^1(\mathbf{r},k_p)\left(\mathscr{A}_{n,m,s}^p+\mathscr{B}_{n,m,s}^p\right)+ \\ &\mathbf{N}_s^1(\mathbf{r},k_p)\left(\mathscr{C}_{n,m,s}^p+\mathscr{D}_{n,m,s}^p\right)+\mathbf{M}_s^3(\mathbf{r},k_p)\left(\tilde{\mathscr{A}}_{n,m,s}^p+\tilde{\mathscr{B}}_{n,m,s}^p\right)+ \\ &\mathbf{N}_s^3(\mathbf{r},k_p)\left(\tilde{\mathscr{C}}_{n,m,s}^p+\tilde{\mathscr{D}}_{n,m,s}^p\right)\Big].\end{aligned} \tag{16}$$

3.2 Scattering Coefficients

Imposing successively the boundary conditions, we arrive at

$$\begin{bmatrix}\mathscr{A}_{n,m,s}^{P-1} & \mathscr{B}_{n,m,s}^{P-1}\\ \tilde{\mathscr{A}}_{n,m,s}^{P-1} & \tilde{\mathscr{B}}_{n,m,s}^{P-1}\end{bmatrix} = \mathbf{T}_n^{(0\to P-1)}\begin{bmatrix}0 & \mathscr{M}_{n,m,s}^3(\mathbf{1})\\ \tilde{\mathscr{A}}_{n,m,s}^0 & \tilde{\mathscr{B}}_{n,m,s}^0\end{bmatrix}, \tag{17}$$

$$\begin{bmatrix}\mathscr{C}_{n,m,s}^{P-1} & \mathscr{D}_{n,m,s}^{P-1}\\ \tilde{\mathscr{C}}_{n,m,s}^{P-1} & \tilde{\mathscr{D}}_{n,m,s}^{P-1}\end{bmatrix} = \mathbf{S}_n^{(0\to P-1)}\begin{bmatrix}0 & \mathscr{N}_{n,m,s}^3(\mathbf{1})\\ \tilde{\mathscr{C}}_{n,m,s}^0 & \tilde{\mathscr{D}}_{n,m,s}^0\end{bmatrix}, \tag{18}$$

where $\mathbf{T}_n^{(0\to P-1)}$ and $\mathbf{S}_n^{(0\to P-1)}$ are the *transition matrices* from layer V_0 to layer V_{P-1}, given by

$$\mathbf{A}_n^{(0\to P-1)} = \mathbf{A}_n^{P-1}\mathbf{A}_n^{P-2}\cdots\mathbf{A}_n^1, \tag{19}$$

for $\mathbf{A} \in \{\mathbf{T}, \mathbf{S}\}$. Matrices $\mathbf{T}_n^p$ and $\mathbf{S}_n^p$ are the transition matrices from layer V_{p-1} to layer V_p [12].

Applying the boundary conditions for each core type, we get

$$\tilde{\mathscr{A}}^0_{n,m,s} = 0, \quad \tilde{\mathscr{B}}^0_{n,m,s} = \mathbb{K}_n \mathscr{M}^3_{n,m,s}(\mathbf{1}) \tag{20}$$

$$\tilde{\mathscr{C}}^0_{n,m,s} = 0, \quad \tilde{\mathscr{D}}^0_{n,m,s} = \mathbb{L}_n \mathscr{N}^3_{n,m,s}(\mathbf{1}) \tag{21}$$

where

$$\mathbb{K}_n = -\begin{cases} \frac{\Psi^1_{n,P-1}(y_P)}{\Psi^2_{n,P-1}(y_P)}, & \text{PEC} \quad \text{core} \\ \frac{T^{(0\to P)}_{n,21}}{T^{(0\to P)}_{n,22}}, & \text{dielectric} \quad \text{core} \end{cases} \tag{22}$$

$$\mathbb{L}_n = -\begin{cases} \frac{\Omega^1_{n,P-1}(y_P)}{\Omega^2_{n,P-1}(y_P)}, & \text{PEC} \quad \text{core} \\ \frac{S^{(0\to P)}_{n,21}}{S^{(0\to P)}_{n,22}}, & \text{dielectric} \quad \text{core} \end{cases} \tag{23}$$

with $\Psi^\ell_{n,P-1}, \Omega^\ell_{n,P-1}$, for $\ell \in \{1,2\}$, being the components of the *boundary transition vectors*

$$\mathbf{\Psi}_{n,P-1}(y_P) = \begin{bmatrix} j_n(y_P) \\ h_n(y_P) \end{bmatrix}^{\mathrm{T}} (\mathbf{T}_n^{(0\to P-1)}), \tag{24}$$

$$\mathbf{\Omega}_{n,P-1}(y_P) = \begin{bmatrix} \hat{j}_n(y_P) \\ \hat{h}_n(y_P) \end{bmatrix}^{\mathrm{T}} (\mathbf{S}_n^{(0\to P-1)}), \tag{25}$$

while $y_P = k_{P-1}R_P$ and $\hat{j}_n, \hat{h}_n$ denote the Riccati-Bessel functions.

Now, we are able to obtain directly the coefficients of the individual fields. Precisely, it holds

$$b^j_{n,0} = \mathbb{K}_n \mathscr{M}^3_{n,m,s}(\mathbf{m}_j), \quad d^j_{n,0} = \mathbb{L}_n \mathscr{N}^3_{n,m,s}(\mathbf{n}_j), \tag{26}$$

where

$$\mathbf{m}_j = \frac{\mathbf{e}_j}{\mathbf{M}^3_s(\mathbf{r}_j, k_0)\cdot \mathbf{p}_j}, \quad \mathbf{n}_j = \frac{\mathbf{e}_j}{\mathbf{N}^3_s(\mathbf{r}_j, k_0)\cdot \mathbf{p}_j}, \tag{27}$$

with $\mathbf{e}_j$ being the vectors of the standard base of $\mathbb{R}^N$. We note that for $N = 1$, and $\theta_j = 0$, Eqs. (20), (21) are reduced to (3.6) and (3.9) of [13].

Lastly, utilizing the asymptotic relations of the spherical vector wave functions for $r \to \infty$ [12], we obtain the far-fields expressions

$$\mathbf{g}(\hat{\mathbf{r}}) = \omega\mu_0 \sum_{n,m,s} \sqrt{n(n+1)}(-\mathrm{i})^{n-1} \times \left[\mathbf{K}_{smn}(\theta,\phi)\mathscr{M}^3_{n,m,s}(\mathbf{1}) + \mathrm{i}\mathbf{L}_{smn}(\theta,\phi)\mathscr{N}^3_{n,m,s}(\mathbf{1})\right], \tag{28}$$

$$\mathbf{g}_j(\hat{\mathbf{r}}) = \omega\mu_0 \sum_{n,m,s} \sqrt{n(n+1)}(-\mathrm{i})^{n-1} \times \left[\mathbf{K}_{smn}(\theta,\phi)\mathscr{M}^3_{n,m,s}(\mathbf{e}_j) + \mathrm{i}\mathbf{L}_{smn}(\theta,\phi)\mathscr{N}^3_{n,m,s}(\mathbf{e}_j)\right], \tag{29}$$

with

$$\mathbf{K}_{smn}(\theta,\phi) = \mathbb{K}_n \mathbf{C}_{smn}(\theta,\phi), \quad \mathbf{L}_{smn}(\theta,\phi) = \mathbb{L}_n \mathbf{B}_{smn}(\theta,\phi)$$

where $\mathbf{B}_{smn}, \mathbf{C}_{smn}$ denote the vector spherical harmonics.

The overall, individual, and total interaction cross sections are, respectively, given by

$$\sigma = \frac{2\pi}{k_0^2} \sum_{n,m,s} (2n+1) \left(\left|\mathbb{K}_n \mathscr{M}^3_{n,m,s}(\mathbf{1})\right|^2 + \left|\mathbb{L}_n \mathscr{N}^3_{n,m,s}(\mathbf{1})\right|^2 \right) \tag{30}$$

$$\sigma_j = \frac{2\pi}{k_0^2} \sum_{n,m,s} (2n+1) \left(\left|\mathbb{K}_n \mathscr{M}^3_{n,m,s}(\mathbf{e}_j)\right|^2 + \left|\mathbb{L}_n \mathscr{N}^3_{n,m,s}(\mathbf{e}_j)\right|^2 \right) \tag{31}$$

$$\sigma^{\mathrm{T}} = \frac{4\pi}{k_0^2} \sum_{n,m,s} (2n+1) \times \left(|\mathbb{K}_n|^2 \mathrm{Re}\left(\sum_{j=1}^{N-1} \sum_{\nu=j+1}^{N} \mathscr{M}^3_{n,m,s}(\tilde{\mathbf{e}}_j) \overline{\mathscr{M}^3_{n,m,s}(\tilde{\mathbf{e}}_\nu)} \right) + |\mathbb{L}_n|^2 \mathrm{Re}\left(\sum_{j=1}^{N-1} \sum_{\nu=j+1}^{N} \mathscr{N}^3_{n,m,s}(\tilde{\mathbf{e}}_j) \overline{\mathscr{N}^3_{n,m,s}(\tilde{\mathbf{e}}_\nu)} \right) \right), \tag{32}$$

where $\tilde{\mathbf{e}}_j = \mathbf{1} - \mathbf{e}_j$.

4 Numerical Results

In Figs. 1 and 2, we depict the variations of the overall cross section σ and the total interaction cross section σ^{T} versus k_0R_1. The layered sphere has a dielectric core, with $R_1 = 2R_2$, and relative parameters $\varepsilon_{r1} = 2, \varepsilon_{r2} = 3, \mu_{r1} = 1.5, \mu_{r2} = 2.5$. Three different distributions of $N = 4$ exciting dipoles lying on the z-axis are considered: $r_j = (r_1 + d(j-1))R_1$ with $r_1 = 1.25$ and $d = 0.25, 0.75, 1.25$, respectively. Both cross sections follow a similar pattern: for lower frequencies ($k_0R_1 \leq 1.5$) they exhibit a smooth, ascending behavior and the dipole distributions seem to leave the cross-sections values unaffected, while for higher frequencies ($k_0R_1 > 1.5$) oscillations appear which are accompanied by differences in the values for each distribution.

Furthermore, Table 1 shows the 100% percentages of the ratios of the individual cross sections over the overall scattering cross section. It is observed that, for $k_0R_1 > 1$, the contributions of each dipole to the overall cross section do not exhibit large deviations. On the contrary, for $k_0R_1 < 1$ the dipoles closer to the sphere (corresponding to σ_1, σ_2) contribute significantly more than the dipoles away from the sphere (corresponding to σ_3, σ_4).

References

[1] J. L. Hollmann and L. V. Wang, "Multiple-source optical diffusion approximation for a multilayer scattering medium," *Applied Optics*, **46**, 2007, pp. 6004–6009, doi.org/10.1364/AO.46.006004.

[2] G. Dassios and A. S. Fokas, "Electro-magneto-encephalography for a three-shell model: dipoles and beyond for the spherical geometry," *Inverse*

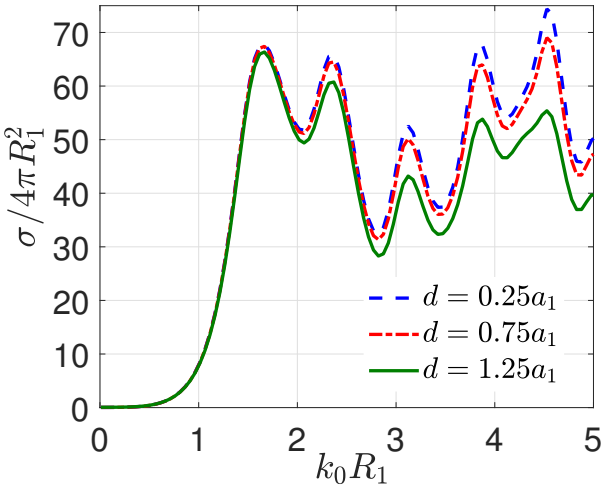

Figure 1. Overall cross section σ versus k_0R_1 for a layered sphere with a dielectric core excited by $N = 4$ dipoles.

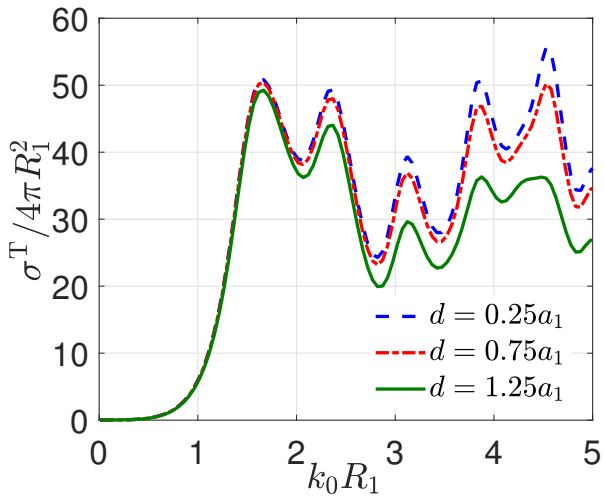

Figure 2. Total interaction cross section σ^{T} versus k_0R_1 for the same setup as Fig. 1.

k_0R_1	σ_1/σ	σ_2/σ	σ_3/σ	σ_4/σ
0.05	14.81	7.16	3.88	2.28
0.1	14.68	7.16	3.92	2.33
0.25	13.59	7.11	4.24	2.80
1	7.16	6.41	6.05	5.85
2.5	6.69	6.41	6.29	6.26
5	7.05	6.71	6.61	6.58
7.5	7.58	7.09	6.97	6.91
10	8.04	7.52	7.39	7.35

Table 1. Percentages of individual cross sections' contributions to the overall scattering cross section

Problems, **25**, 2009, 035001, doi.org/10.1088/0266-5611/25/3/035001.

[3] P. Gas, "Optimization of multi-slot coaxial antennas for microwave thermotherapy based on the S_{11}-parameter analysis," *Biocybernetics and Biomedical Engineering*, **37**, 2017, pp. 78–93, doi.org/10.1016/j.bbe.2016.10.001.

[4] C. A. Valagiannopoulos and N. L. Tsitsas, "On the resonance and radiation characteristics of multi-layered spherical microstrip antennas," *Electromagnetics*, **28**, 2008, pp. 243–264, doi.org/10.1080/02726340802040096.

[5] R. Potthast, *Point Sources and Multipoles in Inverse Scattering Theory*, Chapman and Hall/CRC, 2001.

[6] X. Liu, B. Zhang, and J. Yang, "The inverse electromagnetic scattering problem in a piecewise homogeneous medium," *Inverse Problems*, **26**, 2010, 125001, doi.org/10.1088/0266-5611/26/12/125001.

[7] K. Wang, J. J. Laurin, Q. Zhang, Q. Zhang, and K. Wu, "Three-Dimensional Scattering from Uniaxial Objects with a Smooth Boundary Using a Multiple Infinitesimal Dipole Method," *IEEE Access*, **8**, 2020, pp. 80842–80854, doi: 10.1109/ACCESS.2020.2990702.

[8] M. I. Mishchenko and M. A. Yurkin, "Additivity of integral optical cross sections for a fixed tenuous multiparticle group," *Optics Letters*, **44**, 2019, pp. 419–422, doi.org/10.1364/OL.44.000419.

[9] A. E. Moskalensky and M. A. Yurkin, "Energy budget and optical theorem for scattering of source-induced fields," *Physical Review A*, **99**, 2019, 053824, doi.org/10.1103/PhysRevA.99.053824.

[10] A. Kalogeropoulos and N. L. Tsitsas, "Excitation of a Layered Medium by N Sources: Scattering relations, Interaction Cross Sections and Physical Bounds," *Quarterly of Applied Mathematics*, 2020, 22 pages, doi.org/10.1090/qam/1581.

[11] C. T. Tai, *Dyadic Green Functions in Electromagnetic Theory*, IEEE-Press, 1994.

[12] P. Prokopiou and N. L. Tsitsas, "Electromagnetic Excitation of a Spherical Medium by an Arbitrary Dipole and Related Inverse Problems," *Studies in Applied Mathematics*, **140**, 2018, pp. 438–464, doi.org/10.1111/sapm.12206.

[13] N. L. Tsitsas, "Direct and inverse dipole electromagnetic scattering by a piecewise homogeneous sphere," *Zeitschrift für die Angewandte Mathematik und Mechanik (ZAMM)*, **89**, 2009, pp. 833–849, doi.org/10.1002/zamm.200900272.