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Nonlinear TE-polarized electromagnetic waves in a metal-dielectric plane waveguide

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Abstract

The paper treats a problem of TE-wave propagation in a shielded dielectric layer. The permittivity of the layer is described by Kerr nonlinearity. The propagation problem is reduced to a nonlinear boundary eigenvalue problem for an ordinary differential equation. Spectral parameters of the problem are propagation constants of the waveguide. For the determination of approximate eigenvalues an approach is proposed based on the earlier developed Integral Dispersion Equation Method. Numerical results are presented.

1 Introduction

Analysis of the wave propagation in planar dielectric waveguides constitutes an important class of electromagnetic problems. A dielectric layer is the simplest type of such guiding structures. At the same time, such a structure is widely used in practice (planar optical waveguides) [1, 2, 3].

In this paper we investigate electromagnetic TE-wave propagation in a shielded nonlinear dielectric layer. The nonlinearity in the waveguide is described by the Kerr law [4]. Problems of TE-waves propagation in a layer with Kerr nonlinearities were considered in strict electromagnetic statement in [5] and investigated in [6, 7, 8, 9].

The physical setting under study can be stated as a nonlinear boundary eigenvalue problem for an ordinary differential equation. One of the ways to investigate this problem is to derive an equation with respect to the spectral parameter. This equation is called the dispersion equation. The Integral Dispersion Equation Method was suggested for the first time in [7]. Using the obtained dispersion equation we can calculate approximate solutions of the eigenvalue problem. In addition, this approach allows us to find eigenvalues of the problem that can not be obtained using any perturbation method [9, 10].

Numerical results are also presented for a broad set of parameters and different values of the nonlinearity coefficient.

2 Statement of the problem

Consider three-dimensional space $\mathbb{R}^3$ equipped with Cartesian coordinate system $Oxyz$. The space is filled with an isotropic source-free nonmagnetic medium having permittivity $\varepsilon_2 \equiv const$, $\varepsilon_2 > 0$. We consider electromagnetic waves propagating through a nonlinear dielectric layer located between two half-spaces $x < 0$ and $x > h$:

$$\Sigma := \{(x, y, z) : 0 \leqslant x \leqslant h\}.$$

The boundaries $x = 0$, $x = h$ are the projections of the surfaces of perfectly conducting screens. The geometry of the problem is shown in Fig. 1.

The waveguide Σ is filled with a nonlinear isotropic nonmagnetic medium characterised by the permittivity $\varepsilon + \alpha|\mathbf{E}|^2$, where $\varepsilon > 0$, $\alpha > 0$ are constants.

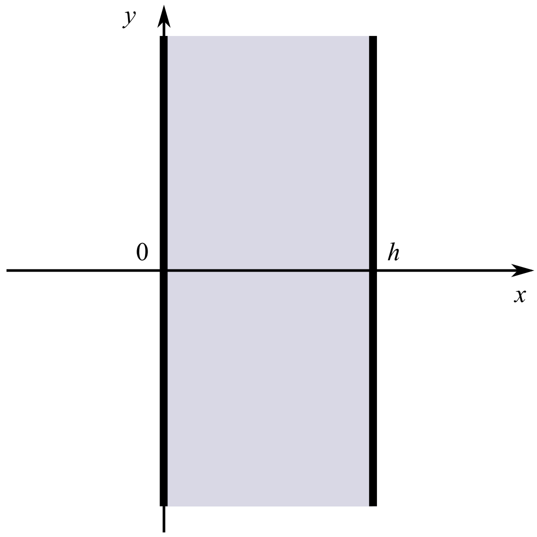

Figure 1. Geometry of the problem.

We assume that the fields depend harmonically on time as $\exp(-i\omega t)$, where $\omega > 0$ is the circular frequency.

Determination of normal TE-polarized waves reduces to finding nontrivial running-wave solutions of the homogeneous system of Maxwell's equations depending on the coordinate z along which the structure is regular in the form $e^{i\gamma z}$,

$$\begin{cases} \operatorname{rot}\mathbf{H} = -i\omega\left(\varepsilon + \alpha|\mathbf{E}|^2\right)\mathbf{E}, \\ \operatorname{rot}\mathbf{E} = i\omega\mathbf{H}, \end{cases} \tag{1}$$

$$\mathbf{E} = \left(0,\ E_y(x)e^{i\gamma z},\ 0\right),\ \mathbf{H} = \left(H_x(x)e^{i\gamma z},\ 0,\ H_z(x)e^{i\gamma z}\right), \tag{2}$$

with the boundary conditions for the tangential electric component on the perfectly conducting screens ($x = 0$ and $x = h$)

$$E_y(0) = 0,\ E_y(h) = 0. \tag{3}$$

Note that the problem on normal waves is an eigenvalue problem for the Maxwell equations with spectral parameter γ which is the propagation constant of the waveguide.

Substituting (2) into system (1), we obtain

$$\begin{cases} i\gamma H_x - H_z' = -i\omega\left(\varepsilon + \alpha E_y^2\right) E_y, \\ \quad -i\gamma E_y = i\omega H_x, \\ \qquad E_y' = i\omega H_z, \end{cases} \tag{4}$$

From (4) we have

$$H_x = -\frac{\gamma}{\omega} E_y,\ H_z = -\frac{iE_y'}{\omega}. \tag{5}$$

The field in the waveguide can be represented using one scalar function

$$u := E_y(x). \tag{6}$$

We assume that $u \in C^2(0,h) \cap C^1[0,h]$.

The propagation problem is reduced to the following eigenvalue problem for the tangential electric field component u: find $\gamma \in \mathbb{R}$ such that there exist nontrivial solutions of the differential equation

$$u'' - \left(\gamma^2 - \omega^2\varepsilon\right) u + \alpha\omega^2 u^3,\ 0 \le x \le h, \tag{7}$$

satisfying the boundary conditions

$$u(0) = 0,\ u(h) = 0. \tag{8}$$

We introduce the additional boundary condition

$$u'(0) = A, \quad A > 0. \tag{9}$$

3 Numerical method

First let us introduce the auxiliary parameter $\lambda = \gamma^2 - \omega^2\varepsilon > 0$. Multiplying equation (7) by $u'(x)$ and integrating, we obtain

$$(u')^2 - \lambda u^2 + \frac{\alpha\omega^2}{2} u^4 = C_0,$$

where C_0 is a constant. Using conditions (8), (9), we find

$$(u'(0))^2 = A^2 > 0, A^2 = C_0.$$

This implies that

$$(u')^2 - \lambda u^2 + \frac{\alpha\omega^2}{2} u^4 = A^2. \tag{10}$$

From the latter equation we obtain

$$u' = \pm\sqrt{A^2 + \lambda u^2 - \frac{\alpha\omega^2}{2} u^4}. \tag{11}$$

Introduce the following notation

$$z_1 = \frac{\lambda + \sqrt{\lambda^2 + 2\alpha\omega^2 A^2}}{\alpha\omega^2},\ z_2 = \frac{\lambda - \sqrt{\lambda^2 + 2\alpha\omega^2 A^2}}{\alpha\omega^2}.$$

Then

$$A^2 + \lambda u^2 - \frac{\alpha\omega^2}{2} u^4 = -\frac{\alpha\omega^2}{2}\left(u^2 - z_1\right)\left(u^2 - z_2\right). \tag{12}$$

Integrating (11), we get *the integral dispersion equation* in the form

$$N \int_{-\sqrt{z_1}}^{\sqrt{z_1}} \frac{du}{\sqrt{A^2 + \lambda u^2 - \frac{\alpha\omega^2}{2} u^4}} = h. \tag{13}$$

It follows from

$$z_1 z_2 = -\frac{2A^2}{\alpha\omega^2},$$

that

$$2\sqrt{2} \int_0^{\frac{\pi}{2}} \frac{\sqrt{z_1}\, dt}{\sqrt{\alpha\omega^2 z_1^2 \sin^2 t + 2A^2}} = \frac{h}{N}. \tag{14}$$

The result is

$$\Delta(\gamma) := \int_0^{\frac{\pi}{2}} \frac{dt}{\sqrt{\alpha\omega^2 z^2 \sin^2 t + 2A^2}} - \frac{h}{2\sqrt{2} N \sqrt{z}} = 0, \tag{15}$$

where $N = 1, 2, \ldots$,

$$z = \frac{\gamma^2 - \omega^2\varepsilon + \sqrt{\left(\gamma^2 - \omega^2\varepsilon\right) + 2\alpha\omega^2 A^2}}{\alpha\omega^2}.$$

Let $\gamma = \tilde{\gamma}$ be such that $\Delta(\gamma) = 0$. This yields that $\tilde{\gamma}$ is a solution (propagation constant) of the problem (7)–(9).

Theorem 1 *Suppose* $[\underline{\gamma}, \overline{\gamma}]$ *is a segment such that* $\Delta(\underline{\gamma})\Delta(\overline{\gamma}) < 0$. *Then there exists at least one propagation constant (one eigenvalue) of the problem* (7)–(9) $\tilde{\gamma} \in (\underline{\gamma}, \overline{\gamma})$.

Note that the condition $\Delta(\underline{\gamma})\Delta(\overline{\gamma}) < 0$ is only a sufficient condition for the existence of a propagation constant of the problem (7)–(9) $\tilde{\gamma} \in (\underline{\gamma}, \overline{\gamma})$.

4 Numerical results

Figures 2–4 display the calculated propagation constants for the problem of the TE-polarized wave propagation in a nonlinear shielded dielectric layer are shown. The following values of parameters are used in calculations: $\varepsilon = 4$, $A = 3$, $h = 3$ mm, and $N = \overline{1,7}$.

Numerical analysis of the behavior of dispersion curves (graphs of the dependence of $\frac{\gamma}{\omega}$ on frequency ω) is performed for different values of coefficient α. Blue and red curves correspond, respectively, to the nonlinear and linear cases (at $\alpha = 0$). All real solutions of the linear problem must satisfy the condition $0 < \gamma < \sqrt{\omega^2\varepsilon}$. The boundary $\gamma = \sqrt{\omega^2\varepsilon}$ in Figs. 2–4 is marked by a dashed horizontal line.

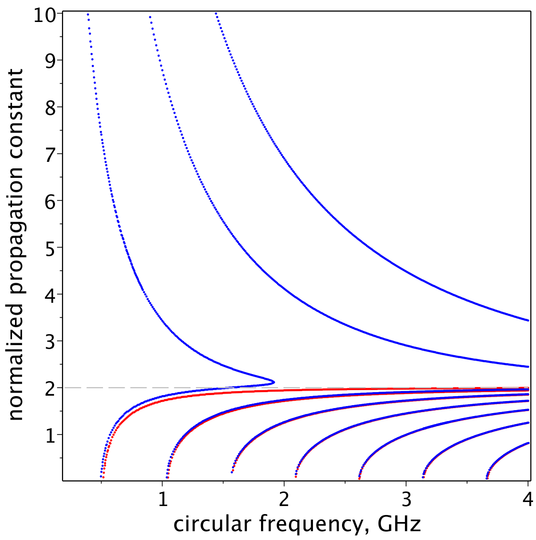

Figure 2. Dispersion curves: $\alpha = 0.05$ V^{-2}.

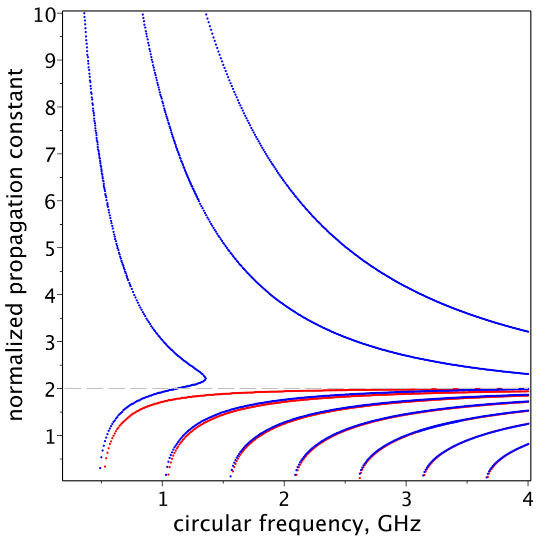

Figure 3. Dispersion curves: $\alpha = 0.1$ V^{-2}.

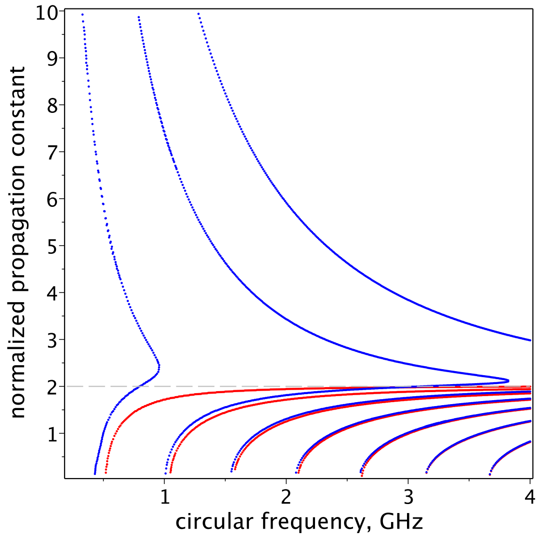

Figure 4. Dispersion curves: $\alpha = 0.2$ V^{-2}.

5 Acknowledgements

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