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Numerical method electromagnetic waves propagation problem in an inhomogeneous chiral media

Yury Shestopalov[(1)], Maxim Snegur*[(2)] and Eugene Smolkin*[(2)]
(1) Faculty of Engineering and Sustainable Development, University of Gävle, Gävle, Sweden;
(2) Penza State University, Penza, Russia.

Abstract

In this work, we study electromagnetic waves propagating in an inhomogeneous waveguide of circular cross section filled with a chiral medium. For the numerical solution of the problem, a Galerkin method is proposed. The results of numerical study of the spectrum of propagating surface waves of the considered open waveguide are presented.

1 Introduction

Recently, in connection with the progress in the field of polymer technologies, new synthesized chiral materials have appeared, which served as an incentive for research on the relevant problems of the wave propagation theory. The analysis of waves in chiral media is performed in [1]–[5] to name the few. One of the most significant properties has been revealed that chirality exhibits itself when the system of waves in a waveguide with a chirality parameter equal to zero degenerates. In an infinite medium, chirality removes the degeneracy that exists between plane waves with different directions of linear polarization. In addition, normal waves in a chiral waveguide acquire different propagation constants and definite polarization states.

The waves in open waveguide structures filled with chiral media was studied using the method of operator pencils and operator-valued functions. proposed by Smirnov and Smolkin in [6]–[9] where various results of analytical and numerical investigations of the spectrum of propagating surface waves are presented.

2 Statement of the problem

Consider three-dimensional space $\mathbb{R}^3$ equipped with cylindrical coordinate system $O\rho\varphi z$ and filled with an isotropic source-free medium having permittivity ε_0 and permeability μ_0 (vacuum).

Consider the determination of polarized surface waves in a cylindrical circular waveguide. The cross-section of the waveguide will be either a circle of radius $r > 0$ or a ring with internal radius $r_0 > 0$ (perfectly conducting cylinder) and external radius $r > r_0$. We will consider both sructures simultaneously setting in the first case $r_0 = 0$. The geometry of the problem is shown in Fig. 1.

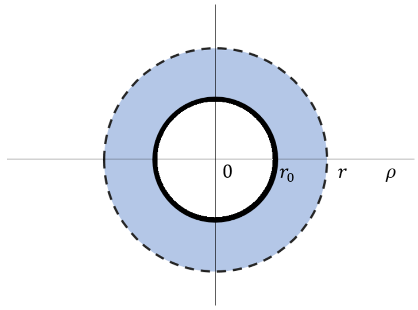

Figure 1. Geometry of the problem.

Determination of electromagnetic waves in a waveguide is the problem of finding nontrivial propagating wave solutions to the homogeneous system of Maxwell's equations, i.e., the solutions with dependence $e^{i\gamma z}$ on coordinate z [10],

$$\begin{cases} \operatorname{rot}\mathbf{H} = -i\omega\widetilde{\varepsilon}\mathbf{E} - \omega\widetilde{\chi}\mathbf{H}, \\ \operatorname{rot}\mathbf{E} = i\omega\widetilde{\mu}\mathbf{H} - \omega\widetilde{\chi}\mathbf{E}, \end{cases} \tag{1}$$

and

$$\mathbf{E} = \left(E_\rho(\rho)\,\mathbf{e}_\rho + E_\varphi(\rho)\,\mathbf{e}_\varphi + E_z(\rho)\,\mathbf{e_z}\right)e^{i\gamma z}, \tag{2}$$

$$\mathbf{H} = \left(H_\rho(\rho)\,\mathbf{e}_\rho + H_\varphi(\rho)\,\mathbf{e}_\varphi + H_z(\rho)\,\mathbf{e_z}\right)e^{i\gamma z}, \tag{3}$$

with the boundary conditions for the tangential electric field components on perfectly conducting surfaces,

$$E_\varphi\big|_{\rho=r_0} = 0, \quad E_z\big|_{\rho=r_0} = 0, \tag{4}$$

the transmission conditions for the tangential electric and magnetic field components on the surfaces of "breaks" of permittivity, permeability and chirality,

$$\begin{matrix} [E_\varphi]\big|_{\rho=r} = 0, & [E_z]\big|_{\rho=r} = 0, \\ [H_\varphi]\big|_{\rho=r} = 0, & [H_z]\big|_{\rho=r} = 0, \end{matrix} \tag{5}$$

where $[f]|_{\rho_0} = \lim\limits_{\rho\to\rho_0-0} f(\rho) - \lim\limits_{\rho\to\rho_0+0} f(\rho)$; and the radiation condition at infinity: the electromagnetic field decays as $O(1/\rho)$ for $\rho \to \infty$.

The permittivity, permeability, and chirality in the whole space are determined as follows

$$\widetilde{\varepsilon} = \begin{cases} \varepsilon, \ \rho \le r, \\ \varepsilon_0, \ \rho > r, \end{cases} \quad \widetilde{\mu} = \begin{cases} \mu, \ \rho \le r, \\ \mu_0, \ \rho > r, \end{cases} \quad ,\widetilde{\chi} = \begin{cases} \chi, \ \rho \le r, \\ 0, \ \rho > r, \end{cases}$$

where $\varepsilon(\rho) \in C^1[r_0, r]$, $\min\limits_{[r_0, r]} \varepsilon(\rho) > \varepsilon_0$, $\mu(\rho) \in C^1[r_0, r]$, $\min\limits_{[r_0, r]} \mu(\rho) > \mu_0$ and χ is a real constant.

The weak formulation[9] of problem (1)–(5) leads us to the following variational relation:

$$\gamma^2 \int_{r_0}^{r} (u_e \overline{v}_e + u_m \overline{v}_m) d\rho + \int_{r_0}^{r} (u'_e \overline{v}'_e + u'_m \overline{v}'_m) d\rho -$$
$$- \int_{r_0}^{r} (g_e u_e \overline{v}_e + g_m u_m \overline{v}_m) d\rho - \int_{r_0}^{r} (h_e u'_e \overline{v}_e + h_m u'_m \overline{v}_m) d\rho +$$
$$+ \int_{r_0}^{r} (f_e (\rho u_e)' \overline{v}_m + f_m (\rho u_m)' \overline{v}_e) d\rho + \int_{r_0}^{r} (k_e u_e \overline{v}_m + k_m u_m \overline{v}_e) d\rho +$$
$$+ \kappa \left(\left(\frac{\mu}{\varepsilon_0} F(\gamma) - \frac{1}{r} \right) u_e(r) + \frac{\chi}{\varepsilon_0} F(\gamma) u_m(r) \right) \overline{v}_e(r) +$$
$$+ \kappa \left(\left(\frac{\varepsilon}{\mu_0} F(\gamma) - \frac{1}{r} \right) u_m(r) + \frac{\chi}{\mu_0} F(\gamma) u_e(r) \right) \overline{v}_m(r) = 0,$$

where $u_e := iE_\varphi(\rho)$, $u_m := H_\varphi(\rho)$; v_e and v_m are sufficiently smooth test functions;

$$h_e = \frac{\varepsilon \mu'}{\chi^2 - \varepsilon\mu} + \frac{1}{\rho}, \text{ and } h_m = \frac{\varepsilon' \mu}{\chi^2 - \varepsilon\mu} + \frac{1}{\rho},$$
$$g_e = \omega^2(\chi^2 + \varepsilon\mu) - \frac{1}{\rho^2} + \frac{1}{\rho} \frac{\varepsilon \mu'}{\chi^2 - \varepsilon\mu},$$
$$g_m = \omega^2(\chi^2 + \varepsilon\mu) - \frac{1}{\rho^2} + \frac{1}{\rho} \frac{\varepsilon' \mu}{\chi^2 - \varepsilon\mu},$$
$$f_e = \frac{\chi \varepsilon'}{\rho(\chi^2 - \varepsilon\mu)}, \text{ and } f_m = \frac{\chi \mu'}{\rho(\chi^2 - \varepsilon\mu)},$$
$$k_e = -2\omega^2 \chi \varepsilon, \text{ and } k_m = -2\omega^2 \chi \mu,$$
$$\kappa^2 = \gamma^2 - \omega^2 \varepsilon_0 \mu_0,$$
$$F(\gamma) = -\frac{K_0(\kappa r)}{K_1(\kappa r)}.$$

3 Numerical Method and Results

Using the projection method we reduce the addressed variational relation to a system of algebraic equations.

First, split interval $[r_0, r]$ into n subintervals with the length

$$l = \frac{r_0 - r}{n}.$$

Define a set of n subintervals

$$\Phi_i = [r_0 + (i-1)l,\ r_0 + (i+1)l],\ i = 1, ..., n-1$$

and

$$\Phi_n = [r_0 + (n-1)l,\ h],$$

and set of $n+1$ subintervals

$$\Psi_1 = [r_0,\ r_0 + l],$$

$$\Psi_j = [r_0 + (i-2)l,\ r_0 + il],\ j = 2, ..., n$$

and

$$\Psi_{n+1} = [r_0 + (n-1)l,\ h].$$

These subintervals are called *base finite elements*.

In accordance with the scheme of the projection method, it is necessary to introduce *basis functions* ϕ_i and ψ_j in order to approximate the solution. The basis functions are defined on each subinterval Φ_i and Ψ_j (ϕ_i and ψ_j vanishes outside intervals Φ_i and Ψ_j, respectively).

Basis functions ϕ_i defined on Φ_i are

$$\phi_i = \begin{cases} \dfrac{\rho - r_0 - (i-1)l}{l}, & \rho < r_0 + il, \\ -\dfrac{\rho - r_0 - (i+1)l}{l}, & \rho > r_0 + il, \end{cases} , i = \overline{1, n-1}$$

and

$$\phi_n = \frac{\rho - r + l}{l};$$

Basis functions ψ_i defined on Φ_i are

$$\psi_1 = -\frac{\rho^2 - 2r_0\rho + r_0^2 - l^2}{l^2},$$

$$\psi_2 = \begin{cases} \dfrac{\rho^2 - 2r_0\rho + r_0^2}{l^2}, & \rho < r_0 + l, \\ -\dfrac{\rho - r_0 - 2l}{l}, & \rho > r_0 + l, \end{cases}$$

$$\psi_j = \begin{cases} \dfrac{\rho - r_0 - (i-2)l}{l}, & \rho < r_0 + (i-1)l, \\ -\dfrac{\rho - r_0 - il}{l}, & \rho > r_0 + (i-1)l, \end{cases} , j = \overline{3, n}$$

and

$$\psi_{n+1} = \frac{\rho - r + l}{l}.$$

Such basis functions take into account the physical nature of the problem under consideration.

We look for an approximate solution with real coefficients α_i and β_j such that

$$\Pi = \sum_{i=1}^{n} \alpha_i \phi_i, \quad \Phi = \sum_{j=1}^{n+1} \beta_j \psi_j. \tag{6}$$

Substituting functions u_e and u_m with representations (6) into the variational relation, we obtain a system of linear equations with respect to α_i and β_j (for a fixed value of γ)

$$A(\gamma)x = 0, \tag{7}$$

where matrices $A(\gamma)$ and x have the form

$$A = \begin{pmatrix} A_{ee}^{1,1} & \cdots & A_{ee}^{1,n} & A_{em}^{1,1} & \cdots & A_{em}^{1,n+1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ A_{ee}^{n,1} & \cdots & A_{ee}^{n,n} & A_{em}^{n,1} & \cdots & A_{em}^{n,n+1} \\ A_{me}^{1,1} & \cdots & A_{me}^{1,n} & A_{mm}^{1,1} & \cdots & A_{mm}^{1,n+1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ A_{me}^{n+1,1} & \cdots & A_{me}^{n+1,n} & A_{mm}^{n+1,1} & \cdots & A_{mm}^{n+1,n+1} \end{pmatrix},$$

and

$$x = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \\ \beta_1 \\ \vdots \\ \beta_{n+1} \end{pmatrix},$$

where

$$A_{ee}^{i,j} = \gamma^2 \int\limits_{\Phi_i} \phi_i \phi_j d\rho + \int\limits_{\Phi_i} \phi_i' \phi_j' d\rho - \\ - \int\limits_{\Phi_i} g_e \phi_i \phi_j d\rho - \int\limits_{\Phi_i} h_e \phi_i' \phi_j d\rho + \\ + \kappa \left(\frac{\mu(r)}{\varepsilon_0} F(\gamma) - \frac{1}{r} \right) \phi_i(r) \phi_j(r),\ i,j = \overline{1,n};$$

$$A_{em}^{i,j} = \int\limits_{\Phi_i} \left(f_e(\rho \phi_i)' + k_e \phi_i \right) \psi_j d\rho + \\ + \kappa \frac{\chi}{\mu_0} F(\gamma) \phi_i(r) \psi_j(r),\ i = \overline{1,n}, j = \overline{1,n+1},$$

$$A_{me}^{i,j} = \int\limits_{\Psi_i} \left(f_m(\rho \psi_i)' + k_m \psi_i \right) \phi_j d\rho + \\ + \kappa \frac{\chi}{\varepsilon_0} F(\gamma) \psi_i(r) \phi_j(r),\ i = \overline{1,n+1}, j = \overline{1,n},$$

$$A_{mm}^{i,j} = \gamma^2 \int\limits_{\Psi_i} \psi_i \psi_j d\rho + \int\limits_{\Psi_i} \psi_i' \psi_j' d\rho - \\ - \int\limits_{\Psi_i} g_m \psi_i \psi_j d\rho - \int\limits_{\Psi_i} h_m \psi_i' \psi_j d\rho + \\ + \kappa \left(\frac{\varepsilon(r)}{\mu_0} F(\gamma) - \frac{1}{r} \right) \psi_i(r) \psi_j(r),\ i,j = \overline{1,n+1}.$$

Thus $A(\gamma)$ is a $(2n+1) \times (2n+1)$ matrix. Denote by $\Delta(\gamma)$ the determinant of $A(\gamma)$,

$$\Delta(\gamma) = \det A(\gamma). \qquad (8)$$

Definition of approximate solution. *If there exists* $\gamma = \widetilde{\gamma}$ *such that* $\Delta(\widetilde{\gamma}) = 0$*, then* $\widetilde{\gamma}$ *is an approximate eigenvalue of Problem P. In other words, if an interval* $[\underline{\gamma}, \overline{\gamma}]$ *is such that* $\Delta(\underline{\gamma}) \times \Delta(\overline{\gamma}) < 0$*, then this means that there exists* $\gamma = \widetilde{\gamma} \in [\underline{\gamma}, \overline{\gamma}]$ *which is a propagation constant of problem* (1)–(5)*. This value can be calculated with any prescribed accuracy.*

As a model problem, consider the following set of parameters: $r_0 = 2$, $r = 4$, $\varepsilon = 4 + \rho$, $\mu = 1$, $\varepsilon_0 = \mu_0 = 1$. Dispersion curves (graphs of the dependence of normalized propagation constant γ/ω on frequency ω) are shown in the figure. Red dots correspond to the chiral filling of the waveguide $\chi = 0.01$; blue curves correspond to a waveguide filled with an inhomogeneous dielectric $\chi = 0$.

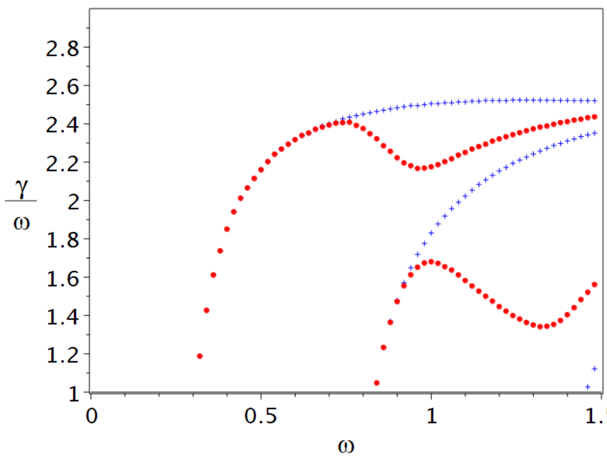

Figure 2. Dispersion curves.

We see that with increasing frequency ω, the curves corresponding to the chiral case differ from those in the dielectric case.

Thus, waves in chiral media has significant characteristic differences compared with the propagation in linear nonchiral dielectrics. Our numerical results confirm this significant property.

4 Conclusion

The developed numerical method is efficient for the analysis of the wave propagation in open waveguides filled with chiral media and can be applied to calculating propagation constants of polarized waves in cylindrical nonlinear circular waveguides. The obtained results of numerical modeling highlight characteristic features of waves in chiral open waveguides.

5 Acknowledgements

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