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Motion Induced Transformation of the Dispersion and Loss Properties of an Electromagnetic Medium

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Abstract

We investigate how the dispersion and loss properties of a medium are altered when the medium is set into motion. Dispersive moving media is shown to give rise to unidirectional propagation in certain frequency ranges. Lossy moving media is shown to be maximally lossy for contra-directional propagation, at the velocity where the real part of the refractive index crosses zero. This work will serve to develop emerging spacetime metamaterials.

1 Introduction

Bulk moving media have been abundantly studied since the pioneering works of Fizeau [1] and Minkowski [2]. They have different upstream and downstream velocities, an effect called the Fizeau drag, and are hence nonreciprocal. Moreover, they exhibit a velocity-dependent bianisotropic response that may be radically different from their rest response [3] and exhibit a negative index of refraction at high velocities [4].

Moving dispersive media were first studied by Lorentz, who extended the Fizeau drag formula to include dispersion [5]. His theory was verified experimentally by Zeeman [6]. There were many works modelling moving plasmas in waveguides, including dispersion and loss, in the 1960s [7–9]. More recently, Leonhardt and Piwnicki showed that a dispersive moving medium mimicks a gravitational field [10].

In this work, we first briefly review the Fizeau drag for nondispersive and lossless moving media. We then derive the laboratory-frame dispersion relation of a dispersive moving medium and show that within some frequency range, all the solutions must propagate downstream. We finally calculate the refractive index of a lossy medium as a function of velocity, without any approximation, and find that for co-directional propagation loss decreases, while for contra-directional propagation loss is enhanced, in particular in a specific velocity range.

2 Lossless Nondispersive Medium

We start by calculating the refractive index of lossless and nondispersive moving media in the laboratory frame. We will limit ourselves throughout the paper to one-dimensional propagation, with the wave propagating in a direction parallel to the motion of the medium.

The velocity of a wave propagating in a moving medium, v_w, is not the sum of the moving media velocity, v, and the wave velocity in the stationnary medium, v'_w, as suggested by Fizeau [1], but rather reads

$$v_\mathrm{w} = \frac{v'_\mathrm{w} + v}{1 + v'_\mathrm{w} v/c^2}. \tag{1}$$

Equation (1) is the relativistic velocity addition rule, which is typically derived in the direct space (z, ct) by substituting the Lorentz transformations $z = \gamma(z' + vt')$ and $t = \gamma(t' + v/c^2 z')$, where $\gamma = (1 - v^2/c^2)^{-1/2}$, into the expression of velocity $v_\mathrm{w} = dz/dt$. Here, we rather derive this formula in the inverse space $(k, \omega/c)$, so as to avoid the derivative and introduce the methodology for the next sections.

The dispersion relation in the moving frame, where the medium appears stationary, reads

$$k' = \omega' \frac{n'}{c}. \tag{2}$$

To transform this relation into the laboratory frame, we use the frequency Lorentz transformations [11]

$$\omega' = \gamma(\omega - vk), \qquad k' = \gamma\left(k - \frac{v}{c^2}\omega\right). \tag{3}$$

Inserting (3) into (2), we obtain

$$k - \frac{v}{c^2}\omega = (\omega - vk)\frac{n'}{c}, \tag{4}$$

whose resolution for k provides

$$k = \omega\left(\frac{v/c^2 + n'/c}{1 + vn'/c}\right) = \omega\frac{n}{c}. \tag{5}$$

Substituting $v_w = c/n$ and $v'_w = c/n'$ into (5) and solving for v_w indeed retrieves the relativistic velocity addition (1).

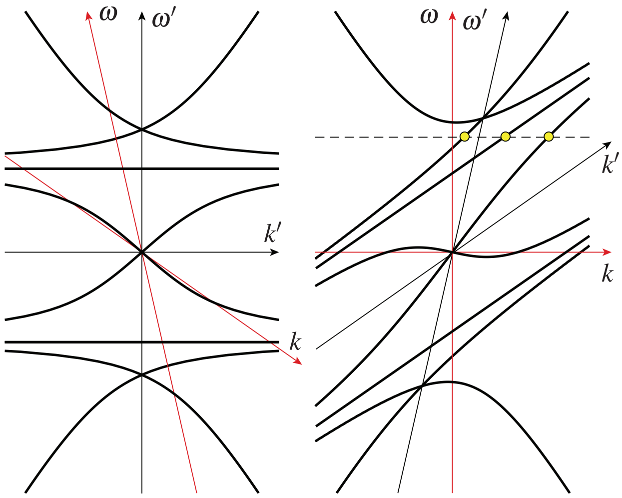

Figure 1. Dispersion diagram for a Lorentz-dispersive lossless medium. (a) As seen in the moving frame [Eq. 6]. (b) As seen in the laboratory frame [Eq. 8]. There are overall six solutions (the three in the continuation of the bottom curves would appear farther at the right of the graph and are not shown here), and these solutions have all a positive wavenumber in some frequency range.

3 Addition of Temporal Dispersion

We now investigate the effect of velocity on the temporal dispersion. We assume the medium in the moving frame has a Lorentz lossless dispersion, i.e.,

$$n'(\omega') = 1 + \frac{{\omega'_p}^2}{{\omega'_0}^2 - \omega'^2}, \tag{6}$$

with ω'_p, ω'_0, the plasma frequency and the resonance frequency. Inserting the frequency Lorentz transformation (3) into (6), we obtain a dispersion relation in terms of the frequency and wavenumber,

$$n(\omega - vk) = 1 + \frac{{\omega'_p}^2}{{\omega'_0}^2 - \gamma^2 (\omega - vk)^2}. \tag{7}$$

Substituting this relation into the dispersion relation (2), we obtain the following cubic relation

$$\alpha_3 k^3 + \alpha_2 k^2 + \alpha_1 k + \alpha_0 = 0, \tag{8a}$$

with

$$\alpha_3 = -\gamma^2 v^2 (c + v), \tag{8b}$$

$$\alpha_2 = \gamma^2 v \omega \left(2c + 3v + v^2/c\right), \tag{8c}$$

$$\alpha_1 = v\left({\omega'_0}^2 + {\omega'_p}^2\right) + c{\omega'_0}^2 - \gamma^2\omega^2\left(2v^2/c - 3v - c\right), \tag{8d}$$

$$\alpha_0 = -\omega\left({\omega'_0}^2 + {\omega'_p}^2\right) - v\omega{\omega'_0}^2/c + \gamma^2\omega^3(1 + v/c). \tag{8e}$$

Figure 1 presents the dispersion diagram of the Lorentz medium. In Fig. 1(a), the diagram is plotted in the moving frame, where we have the usual Lorentz dispersion relation. In Fig. 1(b), the diagram is plotted in the laboratory frame, using (8). In this laboratory frame, six solutions exist at every frequency, while in the moving frame only two solutions exist at each frequency. Also, in the laboratory frame there are some frequencies where all the solutions have a positive wavenumber k (one such situation is highlighted in Fig. 1(b)), meaning that at these frequencies, all the waves propagate downstream, with different velocities.

4 Addition of Loss

We now calculate the refractive index of a moving nondispersive lossy medium.

The dispersion relation (2) now consists of a complex refractive index, complex wavenumbers and complex frequencies. We apply the Lorentz transformations (7) to the dispersion relation (2), and write explicitly the real and imaginary parts of each component. This leads to

$$k_r + ik_i - \frac{v}{c^2}(\omega_r + i\omega_i) = (\omega_r + i\omega_i - v(k_r + ik_i))(n_r + in_i)/c. \tag{9}$$

We separate this equation into real and imaginary parts, and obtain

$$k_r - \frac{v}{c^2}\omega_r = (\omega_r - vk_r)\frac{n_r}{c} - (\omega_i - vk_i)\frac{n_i}{c}, \tag{10a}$$

$$k_i - \frac{v}{c^2}\omega_i = (\omega_r - vk_r)\frac{n_i}{c} + (\omega_i - vk_i)\frac{n_r}{c}. \tag{10b}$$

To plot the (ω, k_r) and (ω, k_i) curves, we set $\omega_i = 0$. Note that setting $\omega'_i = 0$ in the moving frame, which corresponds to $\omega_i - vk_i = 0$ in the laboratory frame, would yield the incorrect result that $\omega_i \neq 0$ for $k_i \neq 0$. We solve the system of equations for k_r and k_i, and find

$$k_r = \frac{\omega_r}{c}\frac{(1 + vn'_r/c)(n'_r + v/c) + vn'^2_i/c}{(1 + vn'_r/c)^2 + v^2 n'^2_i/c^2} = \frac{\omega_r}{c}n_r \tag{11a}$$

$$k_i = \frac{\omega_r}{c}\frac{n'_i\left(1 - v^2/c^2\right)}{(1 + vn'_r/c)^2 + v^2 n'^2_i/c^2} = \frac{\omega_r}{c}n_i, \tag{11b}$$

with (11a) reducing to (5) for $n_i = 0$, as expected.

The refractive indices in (11) are plotted in Fig. 2. For positive velocities, corresponding to co-directional propagation, both the real and imaginary parts of the refractive index decrease gradually, so that the wave's velocity increases and loss decreases. At the limit $v/c = 1$, the medium acts like free space. For contra-directional propagation, loss is highly amplified, with maximal amplification occuring when the real part of the refractive index changes sign. For lossless media, this would occur at $v = -c/n$.

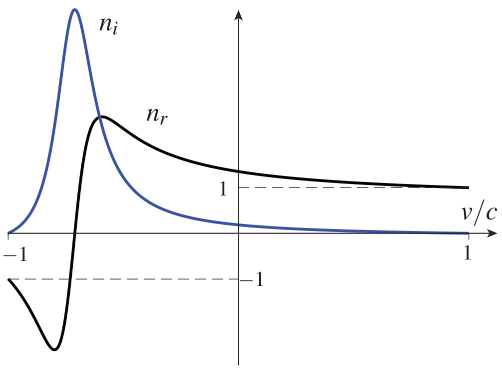

Figure 2. Refractive index as a function of velocity for a dispersionless lossy moving medium.

5 Conclusions

We have addressed the one-dimensional problems of a dispersive moving medium and a lossy moving medium. We found that perfect isolation occurs for certain frequencies in a moving dispersive medium. We also found that loss is maximally amplified near the velocity $v = -c/n$.

This work may help develop emerging spacetime metamaterials [12, 13]

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