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# Iterative Solution of Non-Linear Equations in the Design of Slot Arrays

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## Abstract

This paper describes the design of a waveguide-fed planar slot array with asymmetric excitations in the E-plane. The radiating waveguides are center-fed with centered-inclined coupling slots. The feed waveguide containing the coupling slots is also center-fed from an input waveguide with a shunt series coupling slot. Simultaneous non-linear design equations are solved by an iterative technique using a combination of quasi Newton technique and the secant method. Example designs demonstrate a highly convergent solution process.

## 1 Introduction

Fig. 1 shows an example of a resonant waveguide-fed planar slot array consisting of four radiating waveguides, each containing four radiating slots. Underneath the radiating waveguides, an orthogonal feed waveguide consisting of four centered-inclined coupling slots excites the radiating waveguides. The feed waveguide is excited, in turn, by an orthogonal input waveguide through a shunt series coupling slot that is longitudinal offset in the input waveguide and transverse to the feed waveguide. In a large planar array consisting of many sub-arrays in the E-plane, excitation of slots in the E-plane is expected to be asymmetric. Equation (1) relates the active admittance of $j^{th}$ radiating slot in the $i^{th}$ radiating waveguide relative to that of a reference slot 11[1, 2].

$$\frac{Y^a_{ij}}{Y^a_{11}} = \frac{|f_{ij}|V^s_{ij}}{|f_{11}|V^s_{11}}\,\frac{\chi_1}{\chi_i}\,\frac{|I_1|}{|I_i|} \qquad (1)$$

$$\chi_i^2 = S_{11i}/(1-S_{11i}) \qquad (2)$$

$V^s_{ij}$ and $V^s_{11}$ are voltages of slots ij and 11 respectively. $f_{ij}$ (and $f_{11}$ ) are related to the slot length, frequency, propagation constant of the $TE_{10}$ mode and the slot offset from the broad wall centerline. The coupling coefficients $\chi_i$ and $\chi_1$ of the coupling slots in waveguides i and 1 are related to the scattering parameter $S_{11i}$ ( $S_{111}$ ) of the four-port resonant slot coupler as given by (2). $I_i$ and $I_1$ are the mode currents in the series impedances of coupling slots i and 1 respectively. The active admittances are related to self-admittances and external and internal mutual coupling terms [1, 2]. Admittance equations for all slots form a set of simultaneous non-linear equations.

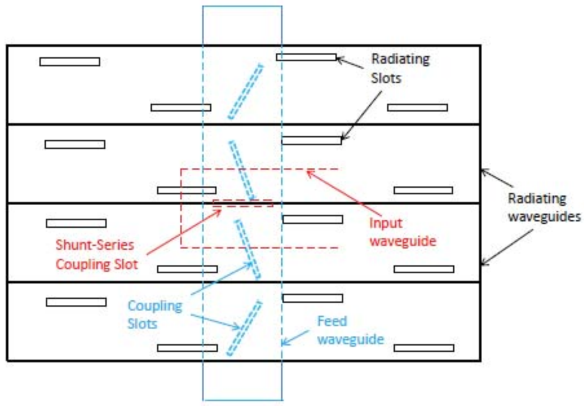


Figure 1. A view of 3-layer planar waveguide-fed slot array with center-fed radiating slots and center-fed coupling slots

In previous array designs consisting of center-fed coupling slots, slot excitations in the E-plane were symmetric [3, 4]. In such cases, whether the feed waveguide is center-fed or end-fed, all the mode currents are equal in magnitude at the design frequency since the slot spacing is one half of a guide wavelength. Thus the ratio of mode currents on the right side of (1) does not appear in such designs. In large arrays consisting of many sub-arrays in the E-plane, the E-plane excitation is expected to be asymmetric, and hence the mode current magnitudes on the two sides of the input slot in the feed waveguide, $I_A$ and $I_B$ in Fig. 2 are unequal. Since the mode currents are not known until the design is complete, it exacerbates the solution process.

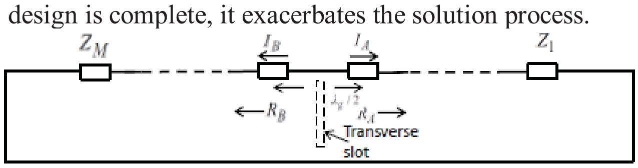


Figure 2. Transmission line equivalent of feed waveguide is shown with impedance representation of coupling slots. Mode currents and total resistances seen on slots on either side of the transverse slot are shown.

## 2 Expressions for the mode currents

Let us consider the four-port coupler containing the shunt series coupling slot. Ports 3 and 4 are planes passing through the center of the transverse slot in the feed waveguide whereas ports 1 and 2 are planes passing through the center of the longitudinal slot in the input waveguide. The coefficient of the wave leaving port 3 is given by

$$b_3 = S_{31}a_1 + S_{32}a_2 + S_{33}a_3 + S_{34}a_4$$
$$= S_{31}a_1 + S_{32}a_2 + S_{33}\Gamma_3 b_3 + S_{34}\Gamma_4 b_4$$
$$\therefore b_3(1 - S_{33}\Gamma_3) - S_{34}\Gamma_4 b_4 = S_{31}a_1 + S_{32}a_2 = A \quad (3)$$

Similarly we can show that

$$-S_{43}\Gamma_3 b_3 + b_4(1 - S_{44}\Gamma_4) = S_{41}a_1 + S_{42}a_2 = -A \quad (4)$$

From (3) and (4) we can show that

$$b_3 / b_4 = -(1 - \Gamma_4)/(1 - \Gamma_3) \quad (5)$$

where and are reflection coefficients of the loads at ports 3 and 4 respectively. The total voltage at ports 3 and 4 are related by (6).

$$V_3 / V_4 = b_3(1 + \Gamma_3) / \{b_4(1 + \Gamma_4)\} = -R_B / R_A \quad (6)$$

since $\Gamma_3 = (1 - R_A)/(1 + R_A)$ and $\Gamma_4 = (1 - R_B)/(1 + R_B)$, where $R_A$ and $R_B$ are total resistances seen at the slots on either side closest to the transverse input slot (see Fig. 2). The power ratio is

$$P_A / P_B = I_A^2 R_A / (I_B^2 R_B) = V_3^2 R_A / (V_4^2 R_B)$$
$$\therefore \; |I_A / I_B| = P_A / P_B = R_B / R_A \quad (7)$$

## 3 Solution of Non-Linear Equations

Each radiating slot has two unknowns, the slot offset and length. The total number of unknowns for each radiating waveguide except the first one is 2N+1 where N is the number of radiating slots, with the last unknown being the normalized coupling coefficient for that waveguide. The number of equations is also 2N+1 for these waveguides since (1) is solved for each slot twice, once for the real part and then for the imaginary part. For the last equation, we specify the total normalized slot admittance of all slots in each waveguide, e.g., $\bar{Y}^a_{tot,i} = 2$. For the slot 11, eqn. (1) is not relevant. However, for this slot we make the active slot admittance zero, assuming that excitations are pure real. Thus there are 2N equations for the first waveguide. The number of unknowns is also 2N since the normalized coupling coefficient is assumed to be 1 for this waveguide.

We used the quasi-Newton technique, although the conjugate gradient method would have been fine too. The initial solution assumed values for slot offsets and lengths somewhere in the middle of practical range of values, including alternating characteristics of slot offsets on either side of the broad wall center line. There are two values of the mode current, one for each side of the transverse slot in the feed waveguide. The mode current ratio is 1, when radiating waveguide i is on the same side of waveguide 1. Otherwise, the ratio of the magnitudes of mode currents is the ratio of power radiated by slots on either side of the input transverse slot in the feed waveguide.

Initially the power ratio in (7) is determined approximately by ignoring mutual coupling between slots. We solve equations for each radiating waveguide starting from the first. After solving the equations for the last radiating waveguide, we equate the normalized total impedance $\bar{Z}_{in}$ in the feed waveguide, given by (8), to a value, e.g., 2, by choosing the normalization for the coupling coefficients.

$$\bar{Z}_{in} = \chi_1^2 \sum_{i=1} \bar{Y}^a_{tot,i} \; \bar{\chi}_i^2 \quad (8)$$

where $\bar{\chi}_i = \chi_i / \chi_1$ is the normalized coupling coefficient of the coupling slot i relative to that of the coupling slot 1. All the impedances are assumed to be pure real, without loss of generality. The total normalized resistances $R_A$ and $R_B$ on the two sides of the coupling slots are also determined and thus a new value for power ratio in (7) is obtained. We continue the iteration process through all radiating waveguides. This process was found to be unstable no matter what the initial design values were.

Subsequently we used the secant method to update the power ratio by looking for zero of the error function, which is the difference between the assumed power ratio during any iteration and the value of the power ratio obtained at the end of that iteration. Once again quasi-Newton iterations were employed through all radiating waveguides. This method is found to be highly stable, converging to a relative error in the order of $10^{-5}$ in single precision arithmetic. The design is completed by synthesizing the input shunt series resonant input slot to realize the non-symmetric loads on ports 3 and 4, and also by designing the resonant coupling slots' tilt angles and lengths to realize the required coupling coefficients needed in the design.

## 4 Example Designs

Two planar arrays were designed using the successful quasi-Newton and secant method. The first is a 4x4 array and the second a 6x9 array. In both cases excitations in the H-plane are assumed to be constant. The E-plane excitations in the first case are 0.7, 0.8, 0.9, and 1.0 while the second one had values of 0.742, 0.754, 0.789, 0.844, 0.916 and 1.0. The arrays designed at X-band were analyzed by a previously validated method-of-moments code. The analysis showed that the realized amplitude distributions in both cases were within $\pm 1\%$ of the specified values while phase errors were within $\pm 1.5^{\circ}$.

## 6 References


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