

## Dielectric Measurement of Liquids via the Cut-off Waveguide Adapted Rectangular Waveguide Reflection Method

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### Abstract

The author proposes a method for dielectric measurement of a liquid using the open-ended cut-off waveguide loaded rectangular waveguide with rectangular dielectric plug technique. For this purpose, the mode-matching technique (MMT) was applied for exact calculation of the analytical model. Moreover, this method has the advantage of eliminating errors caused by jig removal because it is wholly unnecessary to remove the measurement system when calibration and measurement are performed. The complex permittivity of certain types of high-loss liquid materials was actually measured in the 3 GHz band as a fundamental study. The effectiveness of the presented method of measuring a liquid phantom with high permittivity and high loss was also confirmed by comparing the results of measurement with those obtained using the  $TM_{010}$  circular cavity resonator method.

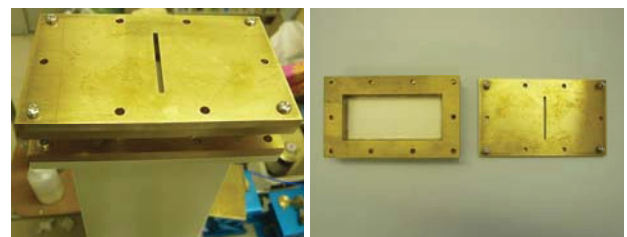
### 1 Introduction

The impact of electromagnetic (EM) wave exposure on the human body has become a matter of concern recently. As a result, tests on health effects using liquid phantoms that imitate human body tissues have been carried out [1]. In order to establish the effects of EM wave exposure on health, a faithful mock-up of human body tissue is needed, and the complex permittivity of these materials was therefore measured. Against this background, Shibata (2010) previously outlined the effectiveness of high-precision broadband dielectric measurement for small amounts of certain liquids based on a reflection constant using a coaxial-feed-type open-ended cut-off circular waveguide [2]. To develop this method, the potential for dielectric measurement in liquids in the low frequency band, estimation using a simple formula, calculation for uncertainty and ways to improve measurement accuracy have been presented [3] – [9]. Calibration of  $S_{11}$  at the front of the sample using three reference materials and SOM (short/open conditions and a known material) with a jig in the measurement system using a VNA (vector network analyzer) and before dielectric measurement has also been proposed [10], [11]. After calibration using the above methods,  $S_{11}$  calculated with various liquids inserted and the dielectric constant estimated from  $S_{11}$  were compared with those from the conventional method and similar approaches, with results showing the validity of related  $S_{11}$  calibration and dielectric constant estimation [11], [12]. However, this method assumes that the sample material insertion space reaches cut-off mode at the measurement frequency, and that this waveguide part reaches propagation mode at frequencies higher than those in the millimeter band. Accordingly, there are

concerns that the measurement results it produces may contain errors arising from the effects of reflective waves. In this study, the author proposes a method for measuring the permittivity of a liquid using the open-ended cut-off rectangular waveguide loaded rectangular waveguide technique to overcome these issues. This method has the advantage of eliminating errors caused by jig removal because it is wholly unnecessary to remove the measurement system when calibration and measurement are performed. For this purpose, the mode-matching technique (MMT) was applied for exact calculation of the analytical model. The permittivity of certain types of high-loss liquid materials was measured in the 3 GHz band. From the results, the effectiveness of this method was confirmed through comparison with measurements made using the open-ended cut-off circular waveguide reflection method [2] – [12].

### 2 Measurement method

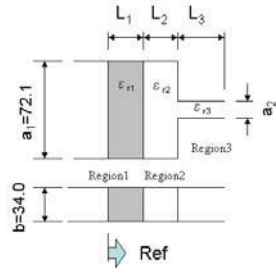
With this approach, a measurement jig is first connected at the measurement reference plane of the measurement system as shown in Fig. 1. The jig is assembled by inserting a ceramic block of  $L_1 = 10$  mm in thickness inside a rectangular waveguide (WRJ-3) with inside dimensions of  $a_1 = 72.1$  mm and  $b = 34.0$  mm and a thickness of 20 mm. Next, a reflective metal plate with a slot of  $a_2 = 2$  mm in width is placed on the jig.  $S_{11}$  is then measured using a vector network analyzer (VNA) at the front of the ceramic plug by inserting a liquid through the slot into the spacer. When the actual measurement is made, a waveguide attenuator (6 dB attenuation) is connected as shown in Fig. 6 in front of the spacer to reduce the influence of reflection waves on the measurement results when using this setup. Next, the permittivity of the liquid material is estimated by comparing the calculated results with those of EM analysis via the MMT (mode-matching technique) and the measurement results for  $S_{11}$ . In addition, the electrical constant of the sample materials must be estimated from the measured  $S_{11}$  as an inverse problem. This work is performed using the 2D Newton-Raphson method.



**Figure 1.** Measurement jig (the outside appearance (left) and the inside appearance (right))



**Figure 2.** Measurement setup (left)



**Figure 3.** Analytical model (right)

### 3 S-parameter calculation

For accurate evaluation of the dielectric property of liquids using the above jig in the microwave band, the exact calculation of  $S_{11}$  for the analytical model is needed. In this chapter, the calculation procedure for the analytical via EM analysis is proposed. The analytical model for  $S_{11}$  here is a cut-off waveguide adapted rectangular waveguide with rectangular alumina plug (Figs. 1 to 2). Region 1 is the alumina block in the analytical model shown in Fig. 3, and therefore does not contain liquid. On the other hand, the waveguide of Region 2 and the reflective plate with the open-ended slot of Region 3 are filled with liquid. Moreover, we assumed that EM waves propagate in propagation mode in Region 3. In this case, the mode-matching technique (MMT) [13], [14] can be applied to this analytical model. Accordingly, the S parameter for a single h-plane discontinuity of the rectangular waveguide shown in Fig. 4 can first be calculated as follows:

$$S_{11} = [L_E L_H + I]^{-1} [L_E L_H - I] \quad \dots (1)$$

$$S_{12} = 2[L_E L_H + I]^{-1} L_E \quad \dots (2)$$

$$S_{21} = L_H \left\{ I - [L_E L_H + I]^{-1} [L_E L_H - I] \right\} \quad \dots (3)$$

$$S_{22} = I - 2L_H [L_E L_H + I]^{-1} L_E \quad \dots (4)$$

where  $L_E = (L_E)_{mn}$ ,  $L_H = (L_H)_{nm}$  and these matrix element become

$$(L_E)_{mn} = (L_H)_{nm} = 2 \sqrt{\frac{k_m^{(1)}}{a_1 a_2 k_n^{(2)}}} \cdot \int_{\frac{a_1 - a_2}{2}}^{\frac{a_1 + a_2}{2}} \sin\left(\frac{m\pi}{a_1} x\right) \sin\left[\frac{n\pi}{a_2} \left(x - \frac{a_1 - a_2}{2}\right)\right] dx \quad \dots (5)$$

Where, the above integral is commonly known in analytical calculation [13], [14]. Moreover, the waveguide shown in Fig. 5 can be defined using the following S parameters:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 0 & D \\ D & 0 \end{bmatrix} \quad \dots (6)$$

where

$$D = D_{mn} = e^{-j\beta_n L_n} \delta_{mn} \quad (m, n=1, 2, \dots, N) \quad \dots (7)$$

In addition,  $\delta_{mn}$  is the Kronecker delta function,  $L_n$  is the length of the waveguide, and  $\beta_n$  is the propagation constant of the rectangular waveguide. The propagation constants are as follows:

$$\beta_n = -j \sqrt{\left(\frac{n \cdot \pi}{a}\right)^2 - k_n^2}, \quad \left(\frac{n \cdot \pi}{a}\right)^2 - k_n^2 > 0$$

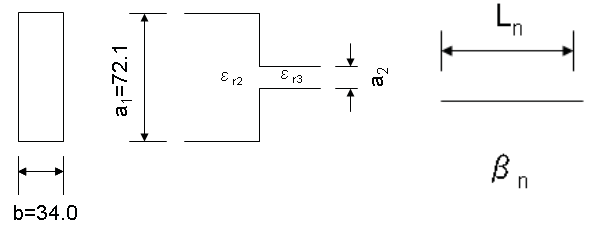
$$\beta_n = -j \sqrt{k_n^2 - \left(\frac{n \cdot \pi}{a}\right)^2}, \quad \left(\frac{n \cdot \pi}{a}\right)^2 - k_n^2 \leq 0$$

Moreover, the S parameter when the tip of the slot is terminated by the open can be calculated using

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} +1 & 0 \\ 0 & +1 \end{bmatrix} \quad \dots (8)$$

Additionally, the S parameter can be calculated if it is terminated by the short as follows:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \dots (9)$$



**Figure 4.** Analytical model for a single h-plane discontinuity (left)

**Figure 5.** Analytical model for a single waveguide (right)

Using each given S parameter from the above study, the total S parameter value under the condition that each adjacent matrix is cascade-connected can be calculated [13], [15] using

$$S_{T11} = S_{L11} + S_{L12} \cdot S_{R11} \cdot W \cdot S_{L12} \quad \dots (10)$$

$$S_{T12} = S_{L12} \cdot (I + S_{R11} \cdot W \cdot S_{L22}) \cdot S_{R12} \quad \dots (11)$$

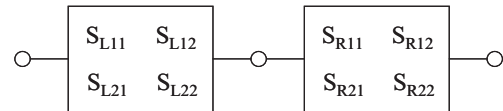
$$S_{T21} = S_{R21} \cdot W \cdot S_{L21} \quad \dots (12)$$

$$S_{T22} = S_{R22} + S_{R21} \cdot W \cdot S_{L22} \cdot S_{R12} \quad \dots (13)$$

where  $W$  is

$$W = [I - S_{L22} \cdot S_{R12}]^{-1} \quad \dots (14)$$

Accordingly, the total  $S_{11}$  value at the reference plane shown in Fig. 3 as seen from the right-hand side can be determined to calculate the cascade connection of the S parameter at each connecting point of this analytical model in consecutive segments.

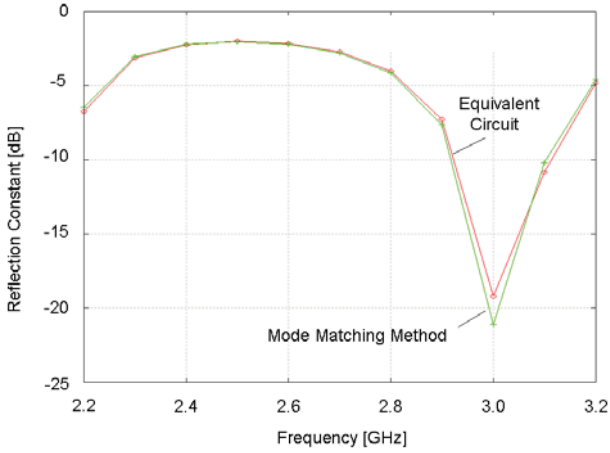


**Figure 6.** Concatenation model for two S parameters

### 4 Determination of Alumina plug dimensions

For estimation of the complex permittivity using the above theorem, first the dimensions of the alumina block as the matching layer must be determined. In this case, the complex permittivity ( $\epsilon_{r2}$ ) of the liquid in the jig was set to  $\epsilon_{r2} = 72 - j 7.5$  assuming that tap water was chosen. In addition, the alumina block length and the insertion length of the sample material was set to  $L_1 = L_2 = 10$  mm in the

analytical model as shown in Fig. 3 to match at frequencies of 3.0 GHz when filled with the sample material. In this case, the complex permittivity of the alumina block was set to  $\epsilon_{r1} = 9.8 - j 0.001$ . Moreover, the thickness of the reflective plate and the width of the slot were set to  $L_3 = 5$  mm and  $a_2 = 2$  mm, respectively, to suppress reflection waves from the aperture plane. Thus,  $S_{11}$  under these conditions at frequencies ranging from 2.2 to 3.2 GHz was calculated using the MMT [13], [14] and equivalent circuit analysis. In addition, the  $a_2$  value (the width of the slot in the reflective plate) was excluded in calculation of the reflection constant using equivalent circuit analysis. The calculation results are shown in Fig. 6. It can be seen that the reflection constant under the MMT [13], [14] and equivalent circuit analysis becomes about  $-20$  dB at a frequency of 3.0 GHz for each, indicating that the results showed excellent agreement. Accordingly, estimation of the complex permittivity of liquids will be performed on the assumption that the parameters of the alumina block are  $L_1 = 10$  mm and  $\epsilon_{r1} = 9.8 - j 0.001$  as of this study.



**Figure 6.** Comparison of calculated  $S_{11}$  results

## 5 Estimation results for the permittivity

The thickness of the alumina plug and the insertion length of the sample were thus set to  $L_1 = L_2 = 10$  mm. The thickness of the reflection plate was set to  $L_3 = 5$  mm, the slot width was set to  $a_2 = 2$  mm, and the permittivity ( $\epsilon_{r1}$ ) of the alumina block was set to  $\epsilon_{r1} = 9.8 - j 0.001$ . Moreover, the open part of the slot in the reflection plate was expressed as a perfect magnetic conductor (PMC) when numerical calculation was performed. Firstly, the input impedance was measured three times by inserting tap water at a frequency of  $f = 2.49806$  GHz under the above conditions. The results of estimating the permittivity from this method are shown in Table. 1 along with those of the five estimations made using the open-ended cut-off waveguide reflection method [2] and the  $TM_{010}$  circular cavity resonator method [16] as an inverse problem at a frequency of 2.49806 GHz in a similar way. The room temperatures for  $S_{11}$  measurement under this method, under the cut-off circular waveguide reflection method of [2] and under the cavity resonator method of [16] were 19.7°C, 22.4°C and 20.2°C, respectively.

Accordingly, the mean values of complex permittivity under the present method become  $\epsilon_{r2} = 74.43 \pm 0.83 - j 9.40 \pm 0.52$ . It was confirmed that the measurement results obtained by this approach agree well with the difference between the results from [2] and those from the cavity resonator method [16], corresponding to within 0.86 for real parts and 0.91 for imaginary parts, respectively.

**Table 1.** Dielectric measurement for tap water (2.49 GHz)

Present method	The method of [2]	The cavity method [16]
$74.43 \pm 0.83$	$73.57 \pm 0.93$	$73.20 \pm 0.40$
$-j9.40 \pm 0.52$	$-j8.44 \pm 0.41$	$-j8.28 \pm 1.50$

Moreover,  $S_{11}$  of methanol and ethanol was measured three times by inserting them as liquids, and the permittivity was estimated from each value. Tables 2 to 3 show the results measured under the present method, those under the cut-off circular waveguide [2], and those under the  $TM_{010}$  cavity resonator method [16]. In addition, the measurement frequency value under the present method was set to the frequency value of the cavity resonator method. It was confirmed that the measurement results for methanol under the present method agree well with the difference between the results of the method in [2] and those from the cavity resonator method [16], corresponding to within 0.10 for real parts and 0.77 for imaginary parts, respectively. It was also confirmed that the measurement results for ethanol under the present method also agree well with the difference between the results of the method in [2] and those from the cavity resonator method, corresponding to within 1.10 for real parts and 0.79 for imaginary parts, respectively.

**Table 2** Dielectric measurement for methanol (2.53 GHz)

Present method	The method of [2]	The cavity method [16]
$21.23 \pm 0.39$	$21.33 \pm 0.50$	$22.1 \pm 1.3$
$-j14.54 \pm 0.57$	$-j13.77 \pm 0.10$	$-j11.4 \pm 2.1$

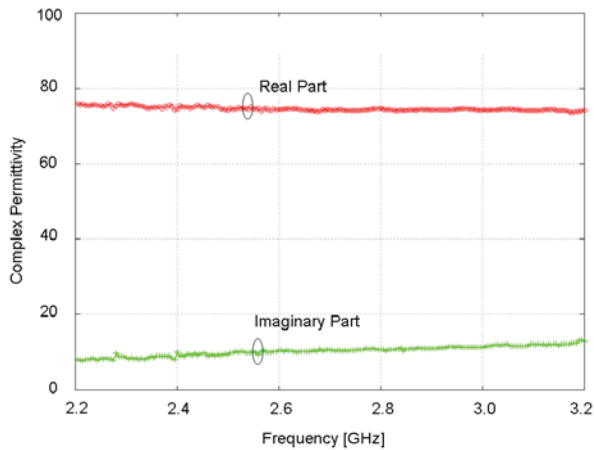
**Table 3.** Dielectric measurement for ethanol (2.53 GHz)

Present method	The method of [2]	The cavity method [16]
$7.36 \pm 0.57$	$6.26 \pm 0.18$	$6.30 \pm 0.3$
$-j7.92 \pm 0.53$	$-j7.13 \pm 0.18$	$-j6.80 \pm 0.3$

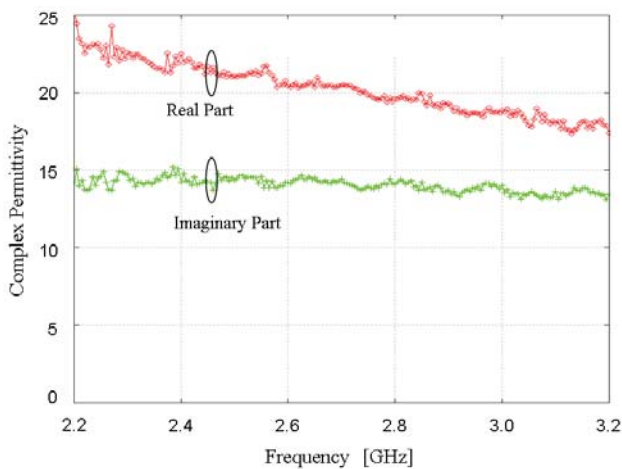
## 6 Frequency characteristics of complex permittivity for different types of liquids

From this study, the method presented here was found to be useful for measurement of complex permittivity for high-loss liquid materials. Next, the complex permittivity of certain types of liquid materials was measured under the conditions of  $L_1 = L_2 = 10$  mm,  $L_3 = 5$  mm,  $a_2 = 2$  mm and  $\epsilon_{r1} = 9.8 - j 0.001$  at frequencies ranging from 2.2 to 3.2 GHz. The measurement results are shown in Figs. 7 and 8. As a result, the real and imaginary parts of the complex permittivity of tap water become 76.7 to 74.6 and 7.9 to 13.1, respectively. Next, the real and imaginary parts of methanol become 25.6 to 17.4 and 13.9 to 13.4, respectively. It was therefore confirmed that the results of measurement show a similar tendency to those obtained using the open-ended cut-off circular waveguide reflection method [2].





**Figure 7.** Frequency characteristics of complex permittivity (tap water)



**Figure 8.** Frequency characteristics of complex permittivity (methanol)

## 7 Conclusions

This paper proposed a way of measuring the complex permittivity of liquid materials using the reflection method with a rectangular cut-off waveguide adapted rectangular waveguide. The permittivity of tap water, methanol and ethanol were estimated from the measured  $S_{11}$ . Moreover, comparison between the results of measurement using the present method and those obtained using the open-ended cut-off circular waveguide reflection method and the cylindrical cavity resonator method was carried out to confirm the effectiveness of the method presented here. Based on this fundamental study, the frequency characteristics of the complex permittivity of certain types of high-loss liquid materials were confirmed at frequencies ranging from 2.2 to 3.2 GHz based on the measurement procedure outlined above. The rectangular waveguide (WRJ-3) used in the present method requires a relatively large amount of liquid material due to its large capacity. However, this method is considered suitable for measuring small amounts of liquids in the millimeter band. Studies of other materials and their temperature characteristics will be made in the near future. Moreover, studies to extend the method to measurement of liquid materials in the millimeter band are also required.

## 8 Acknowledgements

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