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Pseudo Combs – the Liquid State of Mode-Locking

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Abstract

Today the generation of frequency combs nearly exclusively relies on passive mode-locking, requiring a phase lock between the longitudinal modes of a laser. In order to overcome the non-equidistance of the cold cavity modes, it is generally considered mandatory to include an effective saturable absorption mechanism in the laser cavity. However, there exist a number of experimental demonstrations of mode-locking or comb formation in which saturable absorption was clearly absent. Here we show that four-wave mixing may equally well lead to a mode-locking effect. However, the resulting pulse trains are only partially coherent, and the comb structures lack perfect equidistance. Operation of lasers in the pseudo mode-locked regime can easily be confused with traditional mode-locking. We discuss indications and characterization approaches for unveiling pseudo mode-locking as well as limitations for application of pseudo combs.

1 Introduction

Mode-locking instabilities have a long history of fooling researchers into believing that they generated a coherent pulse train. Early mode-locking methods like synchronous pumping or slow absorber mode-locking of dye lasers are infamous for giving rise to a coherent artifact in autocorrelation measurements [1,2]. This coherent artifact has frequently been interpreted as evidence for the presence of stable mode-locking, but may equally well arise if the laser is only partially mode-locked and even if it operates as a simple multimode continuous wave laser. With the advent of mode-locked solid-state lasers with their long upperstate lifetimes, the coherent artifact has been considered a problem of the 1980s --- until the first reports of mode-locking of semiconductor lasers appeared that were clearly lacking a saturable absorption mechanism [3,4]. As saturable absorption is required to stabilize the mode-locking mechanism [5], the observation of self mode-locking gave rise to an extensive debate, which has not been resolved to date [6].

Specifically, self mode-locking was observed in a number of different laser systems, including vertical-cavity semiconductor lasers (VECSELs) and quantum cascade lasers (QCLs [7]). In semiconductor lasers, in general, upperstate lifetimes of the lasing transition are in the picosecond regime or even below, that is, they are much shorter than the cavity roundtrip time by orders of magnitude. In addition, these lasers include highly dispersive semiconductor materials inside the cavity, which lead to significant deviations of the cold-cavity modes from equidistance, in particular for the QCL case. This appears to be a rather hopeless case; if a short coherent pulse train is injected into any of these highly dispersive cavities, one would expect rapid dephasing and a resulting degradation of coherence. On the other hand, however, experimental demonstrations of QCL self mode-locking indicate an intermode beat width of less than a kilohertz, which is substantially narrower than what one would expect in simple multimode continuous operation of a QCL. Moreover, self mode-locking exhibits the characteristic threshold-like behavior that is also observed for other passive mode-locking methods.

So far, no convincing explanation for this peculiar form of mode-locking has been reported. A Kerr-lensing mechanism has been suspected in self mode-locked VECSELs, and four-wave mixing effects are often quoted as the source of QCL comb formation [7,8], but neither of these explanations appears to satisfactorily explain how the extremely strong gain saturation can be overcome in any of these lasers.

2. Haus Master Equation with a Four-Wave Mixing Nonlinearity

In order to explore how four-wave mixing can possibly lead to a mode-locking effect, we followed the concept of the Haus master equation [5]. Haus master equation is typically written as a single partial differential equation. One then seeks stable solutions (or fundamental solitons) of this equation. However, the existence of a stable solution is only a necessary yet not a sufficient criterion for the stability of mode-locking, i.e., the soliton is not necessarily an attractor of the system, and small perturbations may then lead to destabilization of the pulse formation process. As four-wave mixing cannot be treated in a single equation, we resorted to writing the master equation as a system of nonlinearly coupled ordinary equations [9]. In the simplest three mode variant, this is written as

$$\frac{\partial A_1}{\partial z} = i\gamma\left[\left(|A_1|^2+2|A_2|^2+2|A_3|^2\right)A_1+2A_2^2A_3^*\right]+i\frac{\beta_2}{2}\Delta\omega^2 A_1 \quad (1)$$

$$\frac{\partial A_2}{\partial z} = i\gamma\left[\left(2|A_1|^2+|A_2|^2+2|A_3|^2\right)A_2+4A_1A_2^*A_3\right] \quad (2)$$

$$\frac{\partial A_3}{\partial z} = i\gamma\left[\left(2|A_1|^2+2|A_2|^2+|A_3|^2\right)A_3+2A_1^*A_2^2\right]+i\frac{\beta_2}{2}\Delta\omega^2 A_3 \quad (3)$$

Here we assume perfect phase matching of the mixing process, yet include dispersive effects with the β_2 term.

We further adopt the convention $\omega_1<\omega_2<\omega_3$. Energy conservation then requires $\omega_1+\omega_3=2\omega_2$. The strength of the nonlinear effects is parametrized with γ; $\Delta\omega=\omega_2-\omega_1=\omega_3-\omega_2$ is the mode spacing. Using real-valued parameters γ and β_2, the master equation suitably describes cavity soliton formation in passive microring resonators. Complex values of γ allow for the inclusion of fast saturable absorption (or gain); an imaginary part of β_2 describes gain dispersion effects.

In this simplistic version, we only consider degenerate mixing effects, and we end up with a three-mode description of the suspected mode-locking mechanism. Numerically solving the system of nonlinearly coupled differential equations with an Adams predictor-corrector method, we can in fact immediately observe a mode-locking effect, which tends to lock two neighboring modes at a phase offset of $\pi/2$. Moreover, this locking effect can overcome dispersive effects as long as $\beta_2\,\Delta\omega^2 << \gamma$, i.e., for weak dispersive effects. Seeding the model with random phases, we found that the locking effect is typically connected to the appearance of slow phase and amplitude oscillations, which appear at a period $z_0\sim 1/\gamma$. Inserting typical material parameters, we convinced ourselves that z_0 correspond to millions of roundtrips, which seems to match experimental observations [10]. Consequently, a peculiar situation arises, in which the standard definition of temporal coherence results in a value smaller than unity, that is, we only have partial coherence despite a mode-locking effect, that is, the phases of A_1 and A_3 are perfectly locked in our simple three-mode model. Four-wave-mixing based mode-locking may therefore provide perfect intrapulse coherence while displaying a coherent artifact in all typical pulse characterization approaches. This finding appears to be key to understanding four-wave-mixing (FWM) mode-locking.

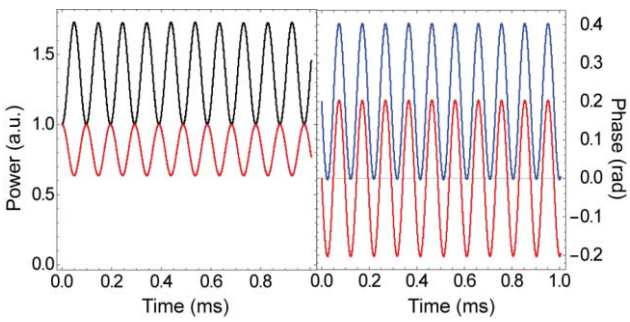

Figure 1. Three-mode model. **(a)** Temporal evolution of the power in the central mode at frequency ω_2 (black) compared to the energy in either sideband ω_1 and ω_3 (red). The total power is conserved. **(b)** Phases of the two resulting intermode beats at $\omega_2-\omega_1$ (red) and $\omega_3-\omega_2$ (blue). Both beats exhibit a synchronous phase modulation at kHz frequency. In this simulation γ was set to 10^{-6}/roundtrip, and a small dispersion was included in Eqs. (1-3). Phases have been shifted by 0.2 rad for clarity

It is now relatively straightforward to expand the model to a larger number of modes, considering both, degenerate and non-degenerate mixing processes. Using an 11-mode model, we searched for stable eigensolutions (or solitons) of the master equation. For anomalous dispersion, we find the solutions with bell-shaped spectra that strongly resemble fundamental solitons of the Nonlinear Schrödinger Equation. Quite interestingly, however, we also find soliton solutions in the absence of dispersion, for higher-order dispersion, and for normal dispersion. The latter case is depicted in Fig. 2 and exhibits a concave spectral shape that is reminiscent of a gray soliton. In the time domain, this peculiar spectral shape translates into a sinc-like pulse shape. It is important to note that the solution is only stable when the spectral shape exactly matches to the soliton solution found for a given dispersion. Even small deviations from the exact shape immediately results in breather solutions of the equation.

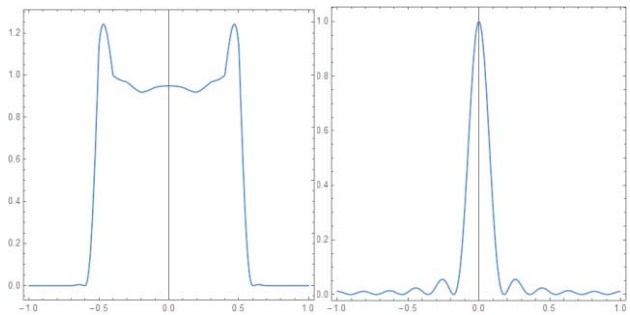

Figure 2. Characteristic soliton solustions of the FWM master equation for the normal dispersion regime in the spectral domain (left) and time domain (right).

3. Breather Solutions

Solving the above equation with random seeded phases of the A_i, we observe, in fact and quite surprisingly, a tight phase lock between the modes. However, the lock is not stationary, and energy is periodically transferred from the spectral center to the wings and back. Similar oscillations appear in the phase structure and lead to the formation of breather solitons, with breathing periods on the order of thousands or even millions of cavity roundtrips, see Fig. 3. Given the temporal variation of the spectral phases, pulses in a breather pulse train are only partially coherent, that is we have mode-locking without perfect coherence. This previously undiscovered regime of mode-locking is therefore best described as pseudo mode-locking. This dynamic mode-locking process is probably best understood in the analogy of the rotating saddle lock, which is used to describe trapping of a charged particle in a Paul trap [11]. If a ball is placed not exactly in the center of the rotating saddle, it will also initially perform a periodic movement. Because of dissipation (friction), this oscillation is damped and will eventually force the ball into the stable central position in the rotating saddle. In a similar fashion, we also find stable soliton solutions for our equation. However, these solutions have a flat spectral phase and only exist in the absence of gain saturation.

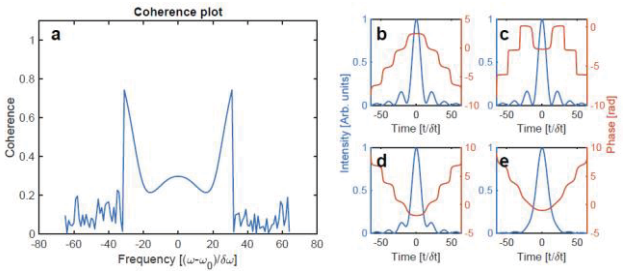

Figure 3. Interpulse coherence is plotted in (a). This shows that the pulses cannot be retrieved by conventional methods that assume perfect interpulse coherence. Sample pulses are plotted in (b-e), showing how the pulses vary from negative to positive chirp while maintaining an almost constant temporal width.

4. The Coherent Artifact of Pseudo Mode-Locking

As previous characterization attempts have apparently missed the partially coherent nature of the pulse train, we asked ourselves what would be necessary to unambiguously detect pseudo mode-locking. Previous characterization efforts nearly exclusively relied on autocorrelation variants, which have no built-in redundancy to point out contradictions from the assumption of a stable pulse train [2]. As frequency-resolved optical gating (FROG) is the most established technique that overcomes this shortcoming, we computed an averaged FROG trace from the simulated data of a breather soliton in Fig. 3. Quite characteristically, these FROG traces show a coherent artifact at delays of +/- 20 time units in the spectral centrum, see Fig. 4(a). This artifact region cannot correctly be reconstructed using standard FROG retrieval algorithms [2]. However, if we use a mixed-state reconstruction approach [12,13] (as established for the characterization of free-electron laser pulses), we obtain a perfect reconstruction that also clearly unveils the presence of a degraded coherence of the pulse train [Fig. 4(b)]

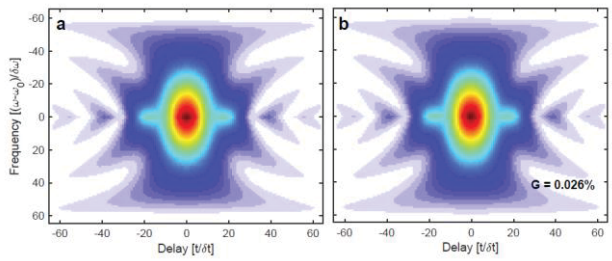

Figure 4 (a) Average FROG trace of the pulses in Fig. 1. This FROG trace shows a coherent artifact and cannot be retrieved with standard algorithms. **(b)** Retrieved FROG trace using mixed-states reconstruction.

5. Conclusions

In conclusion of our study it does not appear overly surprising that the partially coherent nature of pseudo mode-locked pulse trains has previously been frequently overlooked. Laser that show this peculiar mode-locking behavior show many indications of regular mode-locking, including a threshold-like onset of the mode-locking and narrow intermode beats. Even when using rather sophisticated measurement techniques like FROG, the resulting coherent artifact is rather subtle and clearly deviates from previously reported artifacts. Standard retrieval algorithms may therefore easily miss the coherence degradation in the pulse train. On the other hand, pseudo mode-locking shows superior coherence properties compared to multimode continuous-wave lasers. Using the analogy of transitions between the gas, liquid, and solid phase, pseudo mode-locking can be compared with the liquid state, which only exists for pressures and temperatures above the triple point. Solely regarding the aspect of density, liquids and solids are virtually indistinguishable, yet, liquids cannot provide the feats of continuum mechanics, either. Pseudo mode-locking therefore may serve as an intermediate conveyor mechanism in the mid-infrared, which is nevertheless limited to precisions in the 10^{-10} level or below. If we understand "equidistant" as an absolute term then microresonator or QCL combs are clearly not equidistant. Nevertheless, these effects do not seem to corrupt certain applications, e.g., in dual-comb spectroscopy as they do not require precisions below the 10^{-10} level.

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