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# Application of an extended Sommerfeld-Malyuzhinets technique to diffraction of a surface wave by an impedance polygon

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The problem of scattering of a surface wave on a polygonal impedance cylinder is discussed. We consider a three-dimensional cylinder whose cross-section is a polyline consisting of two rays and two finite segments. The scattering of a surface wave coming from infinity along one semi-infinite side of the cylinder and orthogonal to the edges of the cylinder is studied. The aim of the study is to construct the asymptotics of the wave field at large distances. The problem is reduced to the study of solution of the Helmholtz equation in a plane domain with a polygonal boundary that satisfies boundary conditions of the Robin type. Classical formulation of the problem implies conditions for the behaviour of the solution at the corner points (the Meixner conditions) and radiation condition of the Sommerfeld type at infinity in the integral (Rellich) form. Uniqueness of the classical solution of the scattering problem is proved. It is shown that the scattered field consists of the surface waves propagating to infinity along the boundary and a cylindrical (circular) wave. The diffraction coefficient of the cylindrical wave and the excitation coefficients of the surface (reflected and transmitted) waves are calculated. Numerical computation of the far scattered field is carried out. The wave field is sought in the form of Sommerfeld integrals which solve the Helmholtz equation in polar coordinate systems associated with three vertices of a polygonal boundary consisting of two semi-infinite sides and two finite segments. Sommerfeld transformants, three unknown meromorphic functions in the integrand of each integral, must be chosen so that the boundary and other conditions in the problem statement are met. Using the boundary conditions, we derive six related functional-difference equations (Malyuzhinets) with entire coefficients that are the products of the exponents of the cosine by the polynomials of the sine. The method of deriving the equations is new compared to the one known in the literature (Bernard, 2006). The solution of the system of Malyuzhinets equations is sought in a special class of meromorphic functions, which ensures the fulfilment of the Meixner conditions at the vertices and the radiation condition. It is assumed that a transformant has a pole with the prescribed residue, which specifies the incident surface wave in the asymptotics of the wave field at large distances. Technically the most challenging part of the research is connected with reduction of the problem for systems of Malyuzhinets equations to a system of integral equations on the imaginary axis and with study of the latter using the analytic Fredholm alternative. The reduction exploits a well-known special function, i.e. a meromorphic solution of an auxiliary first-order difference equation (the Malyuzhinets function). With its help, the system of functional equations is, first, simplified, and then, by means of the Fourier transform along the imaginary axis, is reduced to a system of integral equations of the second kind along the imaginary axis. On this way, we obtain integral representations for the transformants that enable us to analytically extend them from the imaginary axis onto the whole complex plane. The study of the system of integral equations along the imaginary axis is based on the analytical Fredholm alternative, since the corresponding integral operator analytically depends on the wave number, which is a characteristic parameter. The applicability of the analytical Fredholm theorem to the equation is shown. It is found that the equation is uniquely solvable for all values of the characteristic parameter in a rectangle in the upper half-plane with one side coinciding with an arbitrary segment of the positive real half-axis, with the exception of a discrete set of characteristic values. The unique solution of the integral equation obtained for positive wave numbers is continued from the imaginary axis onto the complex plane and enables us to construct Sommerfeld transformants from the corresponding class of meromorphic functions. The asymptotic behaviour of the wave field at large distances is computed by applying the standard technique of the steepest descent method to the Sommerfeld integrals. The integration contour is deformed into the steepest descent ones. At the same time, the contribution from the crossed poles determines the incident and reflected surface waves, and the saddle points give rise to the cylindrical wave propagating to infinity, which ensures the fulfilment of the radiation conditions. The corresponding diffraction coefficient of this wave, as well as the excitation coefficients of the reflected (or transmitted) surface waves, are expressed in terms of the Sommerfeld transformants, and therefore their calculation requires solution of the integral equations. Based on the obtained analytical results, numerical computations were carried out. A numerical angular diagram of the scattered field at large distances is obtained. To do this, using quadrature formulas of Gauss-Laguerre, the integral equations are solved by reducing to a linear algebraic system. Stability and convergence of the numerical method are ensured by the Fredholm properties of the equations at hand.