Scattering and Diffraction of a Uniform Complex-Source Beam by a Circular Aperture in a Plane Screen

Ludger Klinkenbusch Kiel University, Kaiserstr. 2, D-24143 Kiel, Germany



The scattering and diffraction of acoustic and electromagnetic waves by a circular aperture in an infinitely extended plane belongs to the classical and well investigated canonical problems in field theory. Via the Babinet principle this problem is closely related to that one of the disc. An overview on the works, the results, and the corresponding literature can be found in [1]. For the exact solution of the disc problem usually the Helmholtz equation is solved in the oblate spheroidal coordinate system where the disc is one of the coordinate surfaces. As discussed in [2] and [3] the disc and the circular aperture in a screen are also excellent candidates to compare the results of asymptotic methods (such as the evaluation of the Kirchhoff integral) with exact ones.

In this contribution we will derive an exact analytic solution directly for the circular aperture in a screen, i.e., not going via the solution of the disc problem and a subsequent application of the Babinet principle. This is achieved by solving a corresponding three-domain boundary-value problem in spherical coordinates. We will describe the field in each of the domains by a spherical-multipole expansion. The multipole amplitudes will be found by enforcing the field-continuity at the boundaries. We will exemplarily treat the scalar case for an circular aperture in a screen made from acoustically soft material. Moreover, we will focus on the case that the incident field is a uniform complex-source beam (CSB) Such a beam can be obtained by adding a converging and a diverging CSB with the same location of the waists and the same value for the Rayleigh lengths. In contrast to a standard CSB [4] the uniform CSB is analytic everywhere - particularly also in the waist. Moreover at the waist the uniform CSB represents a local plane wave which can be used to probe a desired area of a scattering object - in the present case the aperture or part of it. Thus we may avoid a full plane wave as the incident field.

References

- F.B. Slater, "The Disc." in: J.J. Bowman, T.B.A. Senior, and P.L.E. Uslenghi, *Electromagnetic and acoustic scattering by simple shapes (revised printing) Chapter 14*, New York: Hemisphere Publishing Cooperation, 1987.
- [2] C.J. Bouwkamp, "Diffraction theory," Rep. Progr. Phys., 17, pp. 35–100, 1954.
- [3] R.D. Spence, "A note on the Kirchhoff approximation in diffraction theory," J. Acoust. Soc. Am., 21, pp. 98-100, 1949.
- [4] L.B. Felsen, "Complex-source-point solutions of the field equations and their relation to the propagation and scattering of Gaussian beams," in: *Symposia Mathematica, Istituto Nazionale di Alta Matematica*. London, U.K.: Academic, vol. XVIII, pp. 40-56, 1976.