Ill-defined Topologies and Energy Sinks

D. E. Fernandes[(1)], Mário G. Silveirinha*[(2)],
(1) University of Coimbra and Instituto de Telecomunicações, Pólo II, 3030-Coimbra, Portugal
(1) University of Lisbon – Instituto Superior Técnico and Instituto de Telecomunicações, Avenida Rovisco Pais 1, 1049-001 Lisbon, Portugal, e-mail: mario.silveirinha@co.it.pt

The net number of unidirectional edge modes at the boundary of a topological system is determined by the bulk-edge correspondence principle [1]. Importantly, there are physical systems which cannot be topologically classified even though their spectrum has a full band-gap [2]. To geometrically illustrate this idea, consider the mathematical objects represented in Fig. 1b. The objects on the left and right panels have a well-defined topology determined by the genus of the corresponding surface. In contrast, the object in the middle panel has an ill-defined topology. Indeed, the cross-sectional cut of a torus with vanishing inner radius consists of two-kissing circles, and, thereby the number of holes of the surface is indefinite. Examples of physical systems with an ill-defined topology arise naturally in the study of the wave propagation in magnetically-biased gyrotropic media and other electromagnetic continua with a continuous translational symmetry [2]. In such systems, the underlying wave vector space is the unbounded Euclidean plane. Generically, the topological classification of continuum models requires the regularization of the system response with some high-spatial frequency cut-off [2]. Without the spatial cut-off, the topology is ill-defined in the same manner as the topology of the torus with vanishing inner radius is ill-defined. A practical consequence of this is that the wave propagation in such systems is *not* constrained by the bulk-edge correspondence principle.

In this article, we will show how by exploiting ill-defined topologies it is possible to stop a wave in carefully designed electromagnetic waveguides, leading to the formation of hotspots wherein the electromagnetic fields are massively enhanced (Fig 1a) [3]. It is envisioned that such systems can be useful for energy harvesting and to enhance nonlinearities.

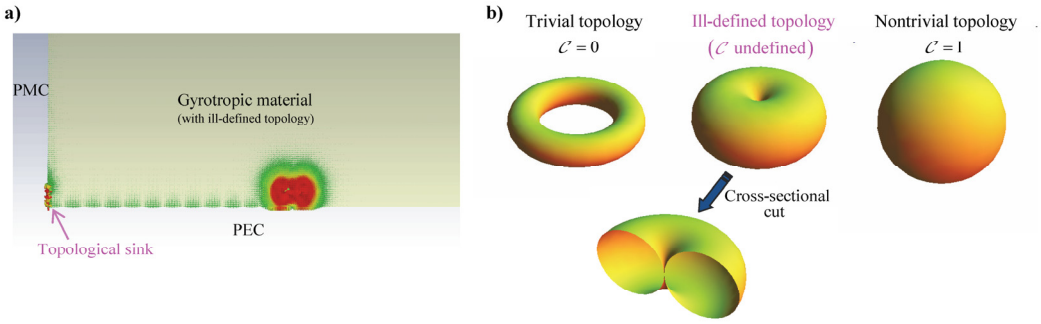

Figure 1 a) A nonreciprocal material with a continuous translational symmetry has an ill-defined topology, and can be used to realize a topological energy sink that drags towards its interior all the radiation generated in the surroundings. b) Geometrical illustration of the concept of an ill-defined topology. The torus and the sphere on the left and right panels have well defined topologies determined the number holes of each surface. In contrast a torus with a vanishingly small inner radius (middle panel) has an ill-defined topology.

References

[1] M. G. Silveirinha, "Proof of the bulk-edge correspondence through a link between topological photonics and fluctuation-electrodynamics", *Phys. Rev. X*, **9**, 011037, (2019).
[2] M. G. Silveirinha, "Bulk-edge correspondence for topological photonic continua", *Phys. Rev. B*, **94**, 205105, (2016).
[3] D. E. Fernandes, M. G. Silveirinha, "Topological origin of electromagnetic energy sinks", *Phys. Rev. Appl.*, **12**, 014021, (2019).